ECE 6345

Spring 2015

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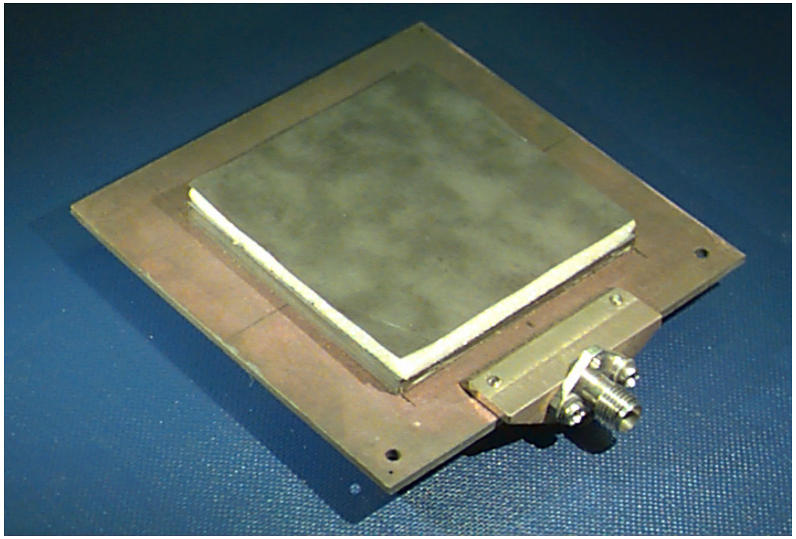
ECE Dept.

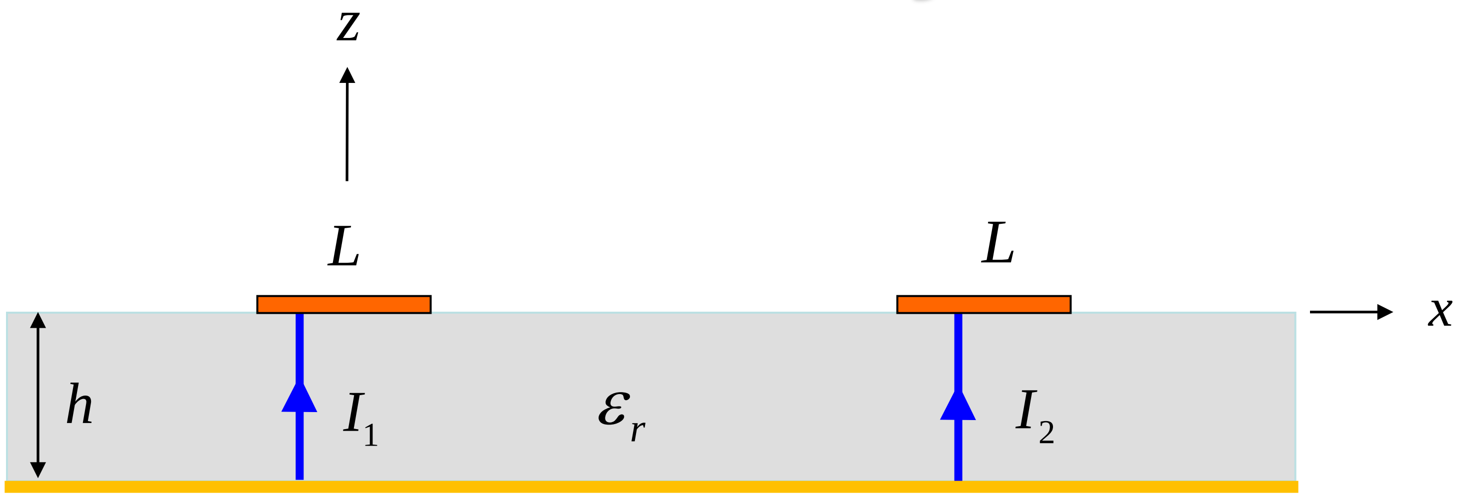


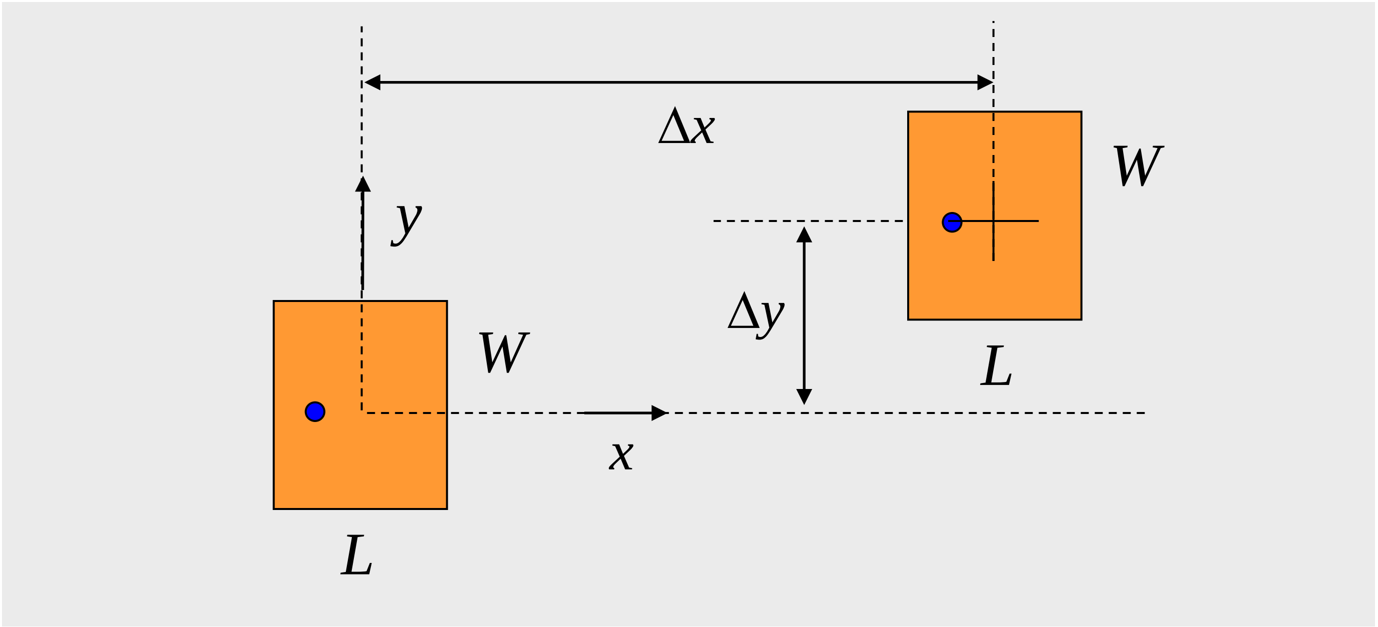
Notes 26

Overview

In this set of notes we use the spectral-domain method to find the mutual impedance between two rectangular patch antennas.

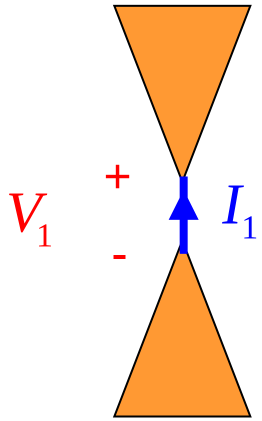
Geometry



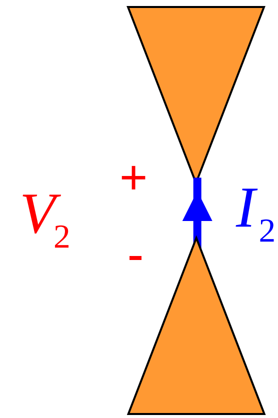


Mutual Impedance Formulation

Assume two arbitrary antennas, to be general.



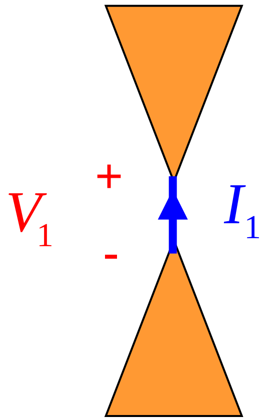
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



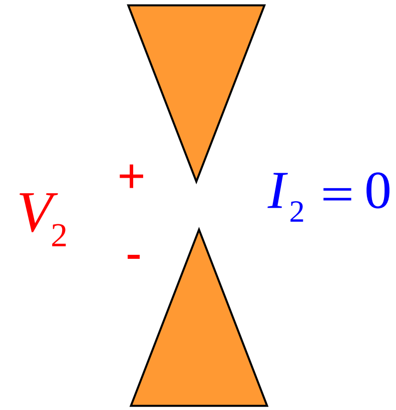
The two-port system is described by a 2×2 Z matrix.

Mutual Impedance Formulation (cont.)

The self impedance Z_{11} is calculated.



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0}$$

$$Z_{11} \approx Z_{in}$$

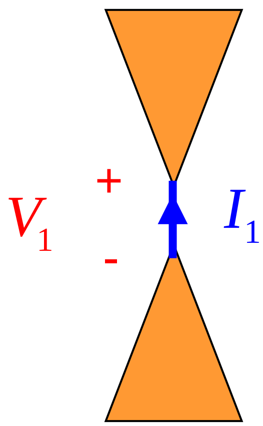
(The presence of open-circuited antenna 2 does not significantly affect the input impedance of antenna 1.)

Mutual Impedance Formulation (cont.)

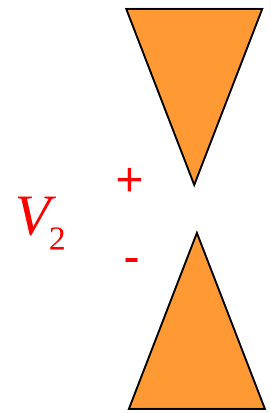
The mutual impedance Z_{21} is calculated.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$

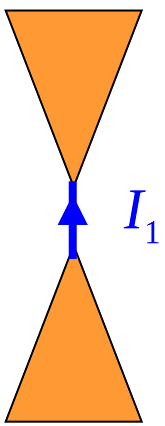


Note: $Z_{21} = Z_{12}$

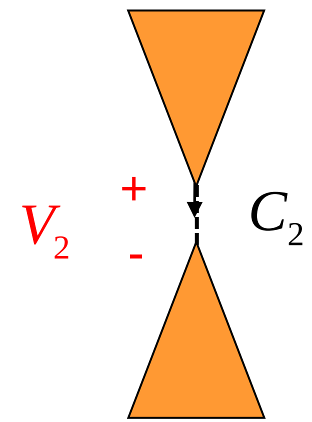
Mutual Impedance Formulation (cont.)

The open-circuit voltage V_2 is obtained by integrating the electric field produced from current I_1 over the path C_2.

$$V_2 = \int_{C_2} \underline{E}_1 \cdot d\underline{r}$$

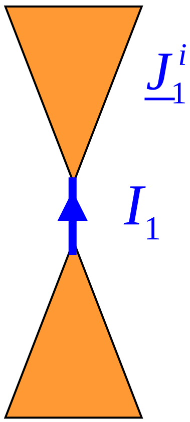


E_1 = electric field produced by the feed current I_1, in the presence of antenna 1 and antenna 2.



Mutual Impedance Formulation (cont.)

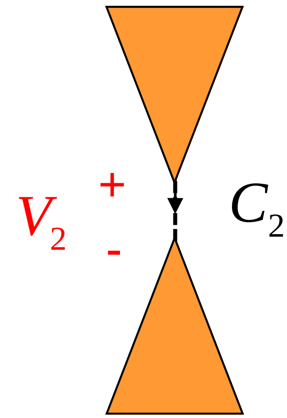
The open-circuit voltage V_2 is put in the form of a reaction.

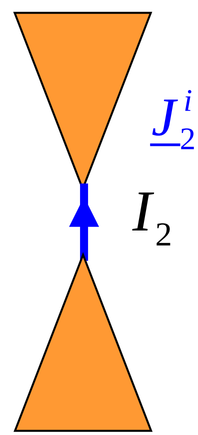


$$V_2 = \int_{C_2} \underline{E}_1 \cdot d\underline{r}$$

$$= -\frac{1}{I_2} \int_V \underline{E}_1 \cdot \underline{J}_2^{\,i} \, dV$$

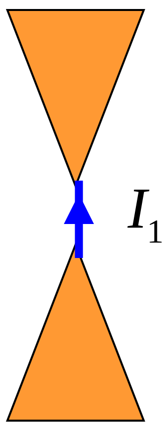
$$= -\frac{1}{I_2} \left\langle \underline{J}_1^{\,i}, \underline{J}_2^{\,i} \right\rangle$$





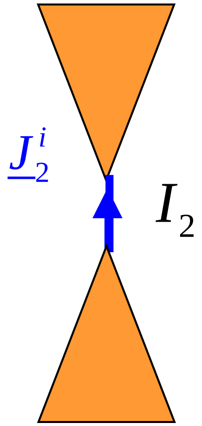
Mutual Impedance Formulation (cont.)

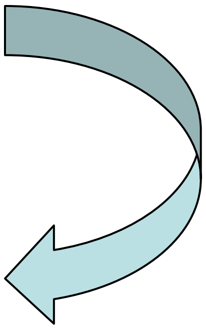
The equivalence principle is used to replace antenna 1 with its surface current.

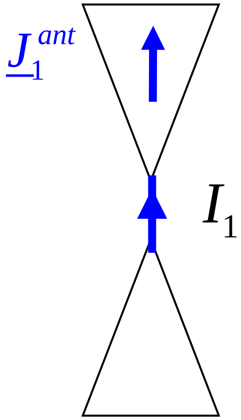


$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^{\,i}, \underline{J}_2^{\,i} \right\rangle$$

$$= -\frac{1}{I_2}\left\langle \underline{J}_1^{\,ant}, \underline{J}_2^{\,i} \right\rangle$$





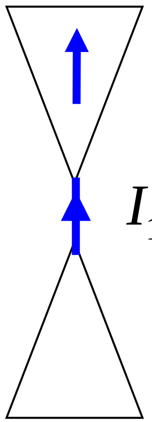


The field produced by current I_1 exciting antenna 1 is the same as that produced by the current on antenna 1 in free space.

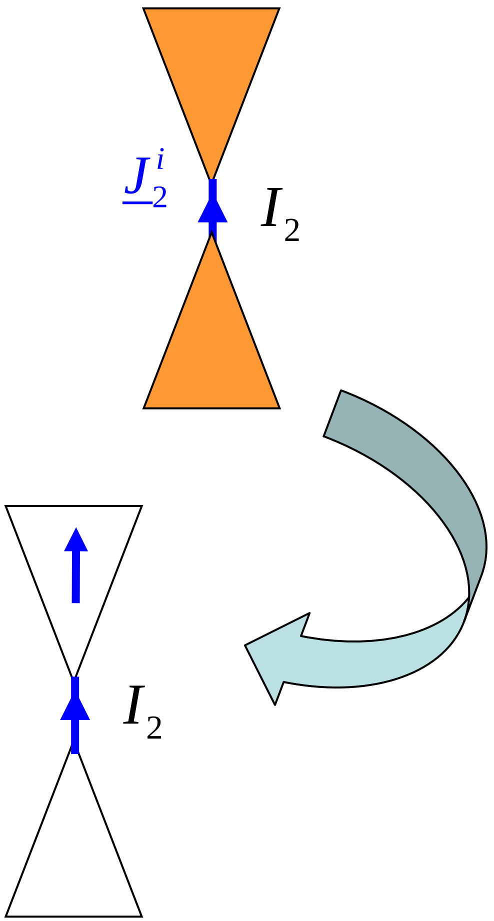
(The antenna current $\underline{J}_1^{\,ant}$ is that excited on antenna 1 when it is in the *presence* of open-circuited antenna 2.)

Mutual Impedance Formulation (cont.)

Reciprocity is invoked, and then the equivalence principle.



$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^{\,ant}, \underline{J}_2^{\,i} \right\rangle = -\frac{1}{I_2}\left\langle \underline{J}_2^{\,i}, \underline{J}_1^{\,ant} \right\rangle = -\frac{1}{I_2}\left\langle \underline{J}_2^{\,ant}, \underline{J}_1^{\,ant} \right\rangle$$

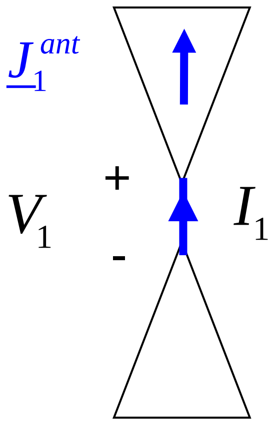


The field produced by current I_2 exciting antenna 2 is the same as that produced by the current on antenna 2 in free space.

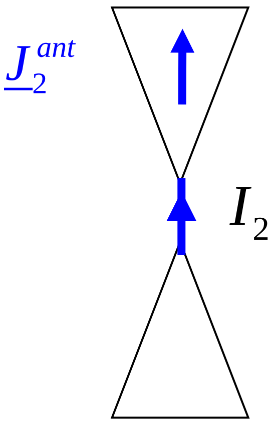
(The antenna current $\underline{J}_2^{\,ant}$ is that excited on antenna 2 when antenna 1 is absent.)

Mutual Impedance Formulation (cont.)

Reciprocity is invoked one more time.

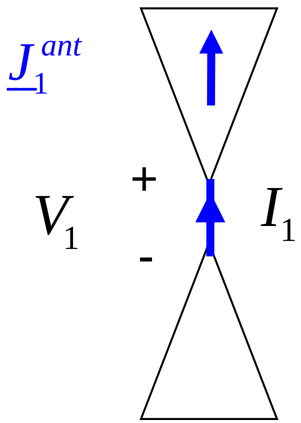


$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_2^{ant}, \underline{J}_1^{ant} \right\rangle$$
$$= -\frac{1}{I_2}\left\langle \underline{J}_1^{ant}, \underline{J}_2^{ant} \right\rangle$$



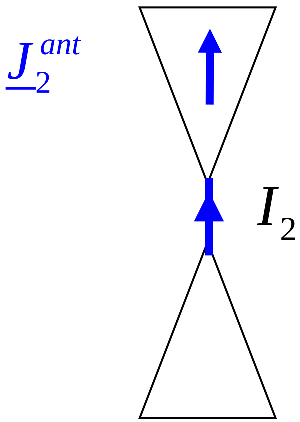
Mutual Impedance Formulation (cont.)

$$V_2 = -\frac{1}{I_2}\left\langle \underline{J}_1^{ant}, \underline{J}_2^{ant} \right\rangle$$



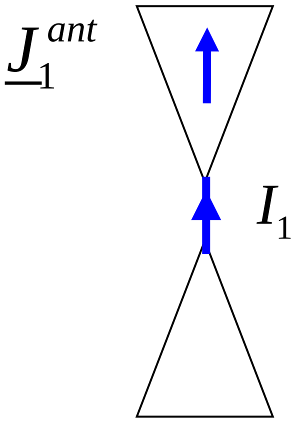
The mutual impedance is then

$$Z_{21} = -\frac{1}{I_1 I_2}\left\langle \underline{J}_1^{ant}, \underline{J}_2^{ant} \right\rangle$$

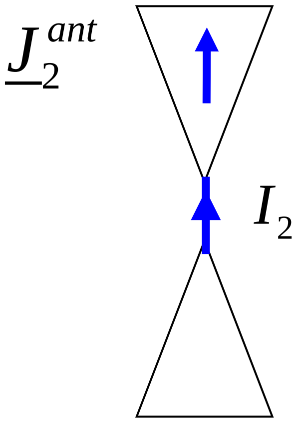


Mutual Impedance Formulation (cont.)

Summary



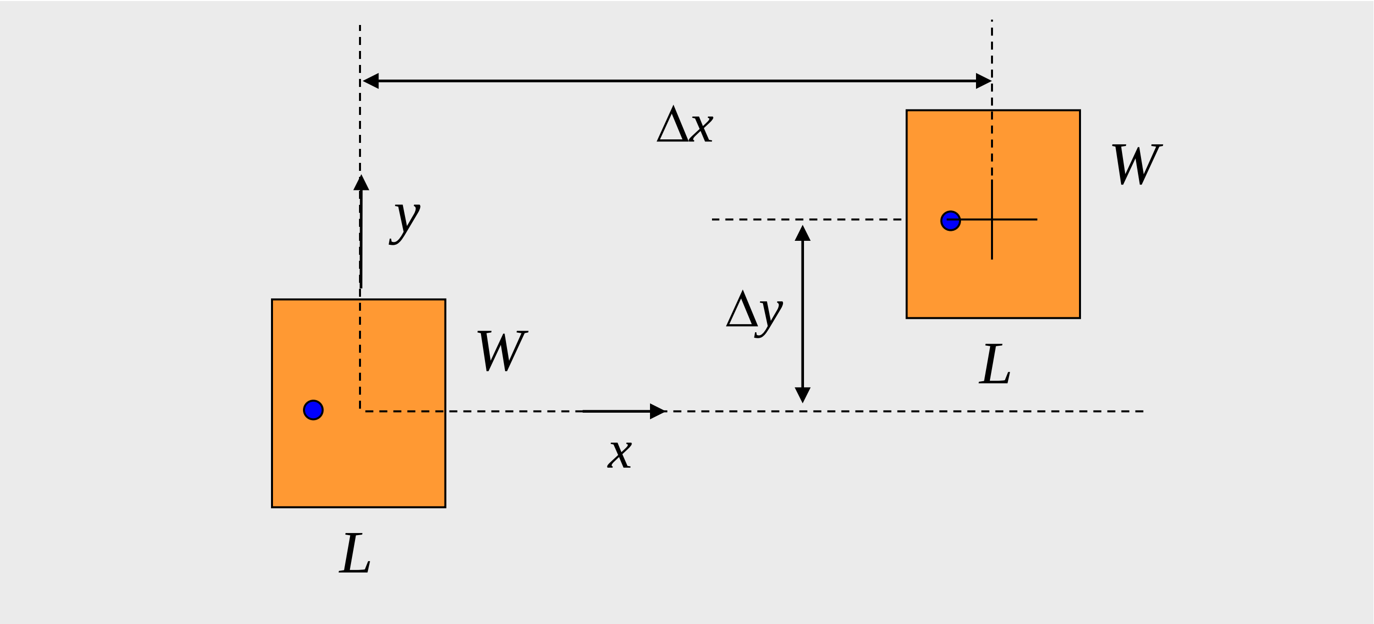
$$Z_{12} = -\frac{1}{I_1 I_2}\left\langle \underline{J}_1^{ant}, \underline{J}_2^{ant} \right\rangle$$



J_1^{ant} = current on antenna 1,
when excited by current I_1 in the *presence* of open-circuited antenna 2.

J_2^{ant} = current on antenna 2,
when excited by current I_2 in the *absence* antenna 1.

Mutual Impedance Between Patches



$$Z_{12} = -\frac{1}{I_1 I_2}\left\langle J_{sx}^{(1)}, J_{sx}^{(2)} \right\rangle$$

The two patches are assumed to be identical here.

Assume $I_1 = I_2 = 1\ [\mathrm{A}]$

Mutual Impedance Between Patches (cont.)

$$Z_{12} = -\left\langle J_{sx}^{(1)}, J_{sx}^{(2)} \right\rangle = -A_x^{(1)} A_x^{(2)} \left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -A_x^2 \left\langle B_x^{(1)}, B_x^{(2)} \right\rangle$$

Note : $A_x^{(1)} = A_x^{(2)} = A_x$

Denote $\quad Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle$

Then we have $\quad Z_{12} = A_x^2 Z_{xx}^{1,2}$

Recall that $\quad A_x = -\dfrac{Z_{zx}}{Z_{xx}}$

Note: Formulas for Z_{zx} and Z_{xx} were given previously in the analysis of the single patch.

Hence $$Z_{12} = \left(\frac{Z_{zx}}{Z_{xx}} \right)^2 Z_{xx}^{1,2}$$

Mutual Impedance Between Patches (cont.)

Calculation of reaction $Z_{xx}^{1,2}$ between patch basis functions:

$$\tilde{E}_x = \tilde{G}_{xx} \tilde{B}_x$$

$$E_x\left[B_{x1}\right] = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx} \tilde{B}_{x1} \, e^{-j\left(k_x x + k_y y\right)} \, dk_x dk_y$$

Hence, integrating over the surface of patch 2, we have

$$Z_{xx}^{1,2} = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^{(1)}\left(k_x, k_y\right) \tilde{B}_x^{(2)}\left(-k_x, -k_y\right) \, dk_x dk_y$$

Mutual Impedance Between Patches (cont.)

$$Z_{xx}^{1,2} = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}(k_x, k_y)\, \tilde{B}_x^{(1)}(k_x, k_y)\, \tilde{B}_x^{(2)}(-k_x, -k_y)\, dk_x dk_y$$

From the Fourier "shifting" theorem, we have

$$\tilde{B}_x^{(2)}(k_x, k_y) = \tilde{B}_x^{(1)}(k_x, k_y)\, e^{j(k_x \Delta x + k_y \Delta y)}$$

Note: The "1" superscript is dropped henceforth.

Hence we have

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_{xx}(k_x, k_y)\, \tilde{B}_x^2(k_x, k_y)\, e^{-j(k_x \Delta x + k_y \Delta y)} dk_x dk_y$$

Mutual Impedance Between Patches (cont.)

Converting to polar coordinates, we have

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) e^{-j\left(k_x \Delta x + k_y \Delta y\right)} k_t \, dk_t \, d\overline{\phi}$$

Since the integrand is an even function of k_x and k_y, we can write

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_0^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\overline{\phi}$$

Quadrant 1 Quadrant 2 Quadrant 3 Quadrant 4

Note:

$$e^{-j\left(k_x x_0\right)} e^{-j\left(k_y y_0\right)} + e^{+j\left(k_x x_0\right)} e^{-j\left(k_y y_0\right)} + e^{+j\left(k_x x_0\right)} e^{+j\left(k_y y_0\right)} + e^{-j\left(k_x x_0\right)} e^{+j\left(k_y y_0\right)}$$

$$= 2\cos\left(k_x x_0\right) e^{-j\left(k_y y_0\right)} + 2\cos\left(k_x x_0\right) e^{+j\left(k_y y_0\right)}$$

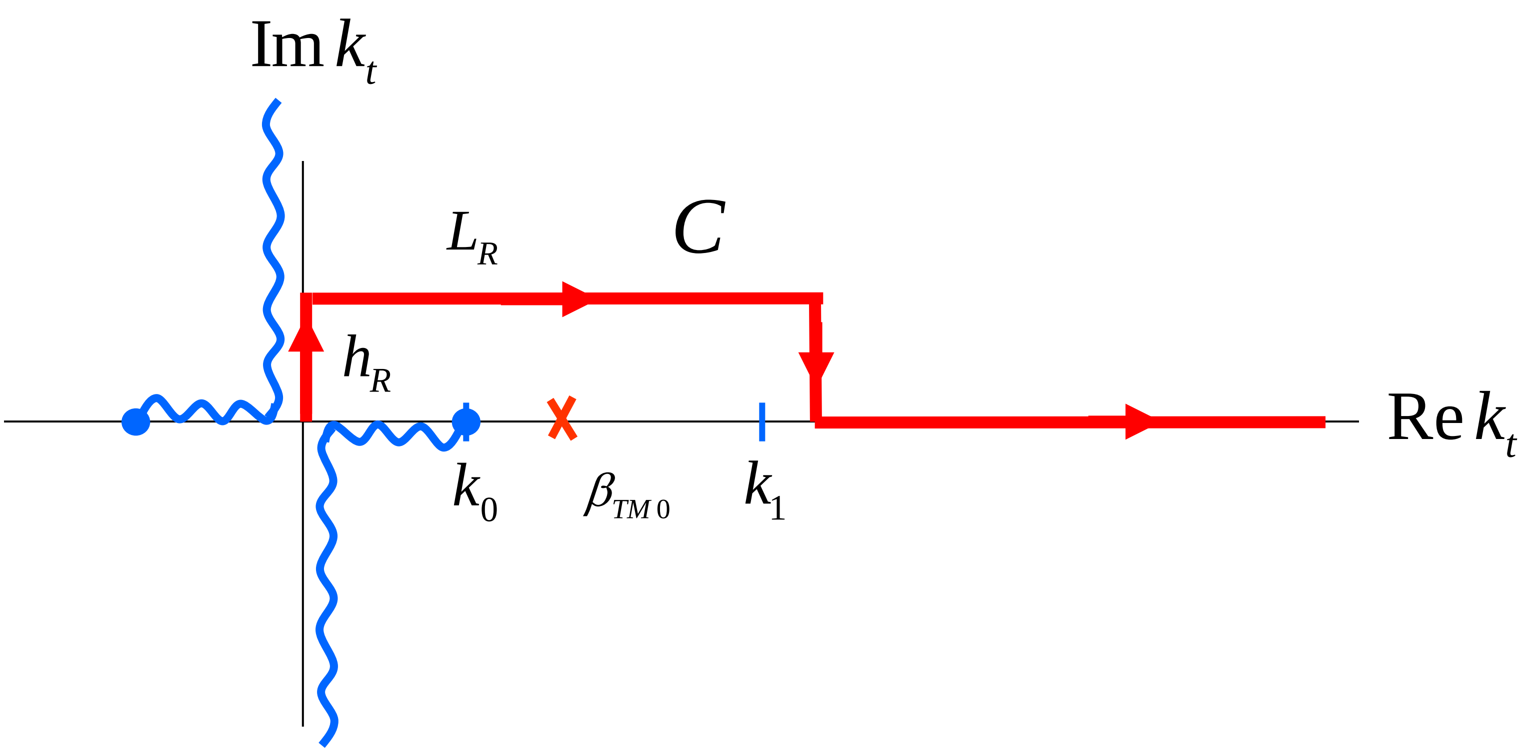
$$= 2\cos\left(k_x x_0\right) \left[e^{-j\left(k_y y_0\right)} + e^{+j\left(k_y y_0\right)} \right]$$

$$= 4\cos\left(k_x x_0\right) \cos\left(k_y y_0\right)$$

Mutual Impedance Between Patches (cont.)

Final form of mutual reaction

$$Z_{xx}^{1,2} = -\left\langle B_x^{(1)}, B_x^{(2)} \right\rangle = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\overline{\phi}$$



Summary

$$Z_{12} = \left(\frac{Z_{zx}}{Z_{xx}} \right)^2 Z_{xx}^{1,2}$$

$$Z_{xx}^{1,2} = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\overline{\phi}$$

$$Z_{xx} = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) k_t \, dk_t \, d\overline{\phi}$$

$$Z_{zx} = \frac{j}{\pi^2} \left(\frac{h}{\omega \varepsilon_1} \right) \int_0^{\pi/2} \int_C \left\{ k_t^2 \, I_i^{TM}(-h) \, \tilde{B}_x \cos\overline{\phi} \, \mathrm{sinc}\left(k_{z1} h\right) \right\} \cdot \sin\left(k_x x_0\right) \cos\left(k_y y_0\right) dk_t \, d\overline{\phi}$$

Results

D. M. Pozar, "Input Impedance and mutual coupling of rectangular microstrip antennas,“ *IEEE Trans. Antennas Propagat.*, vol. AP-30. pp. 1191-1196, Nov. 1982.

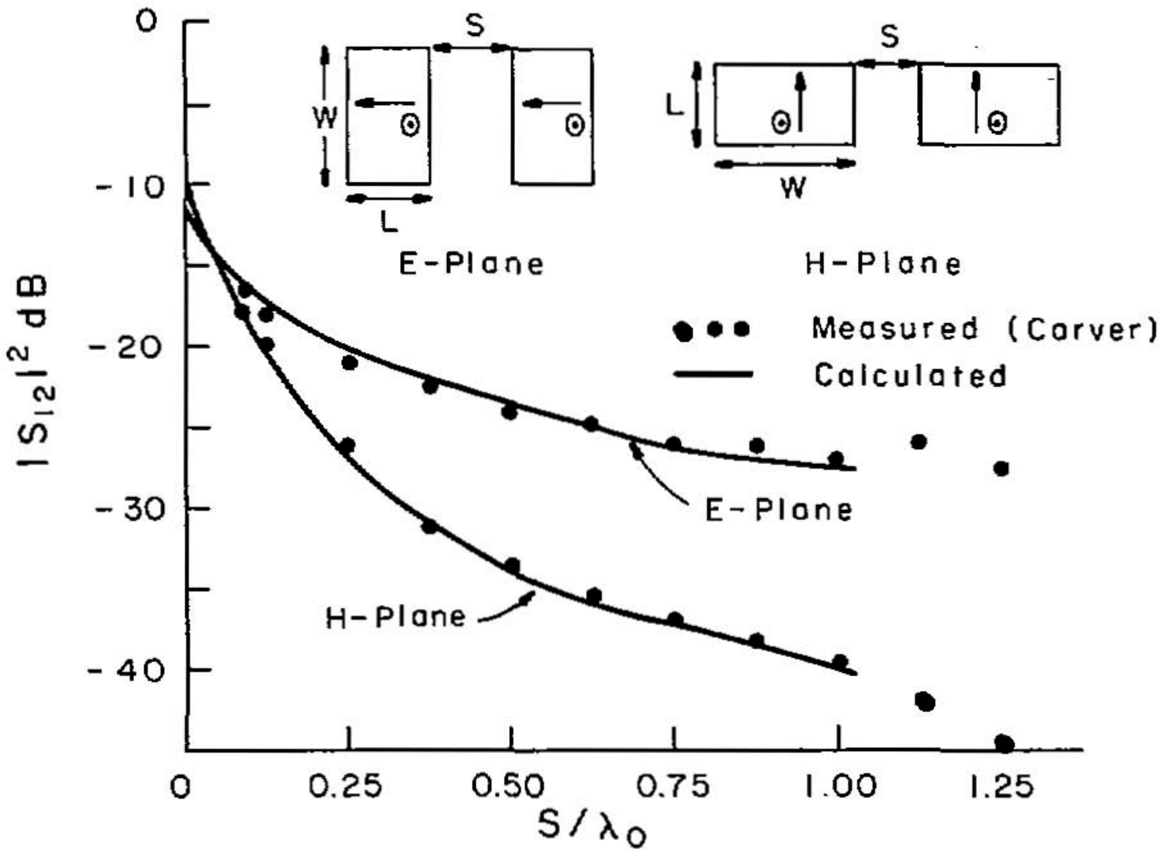
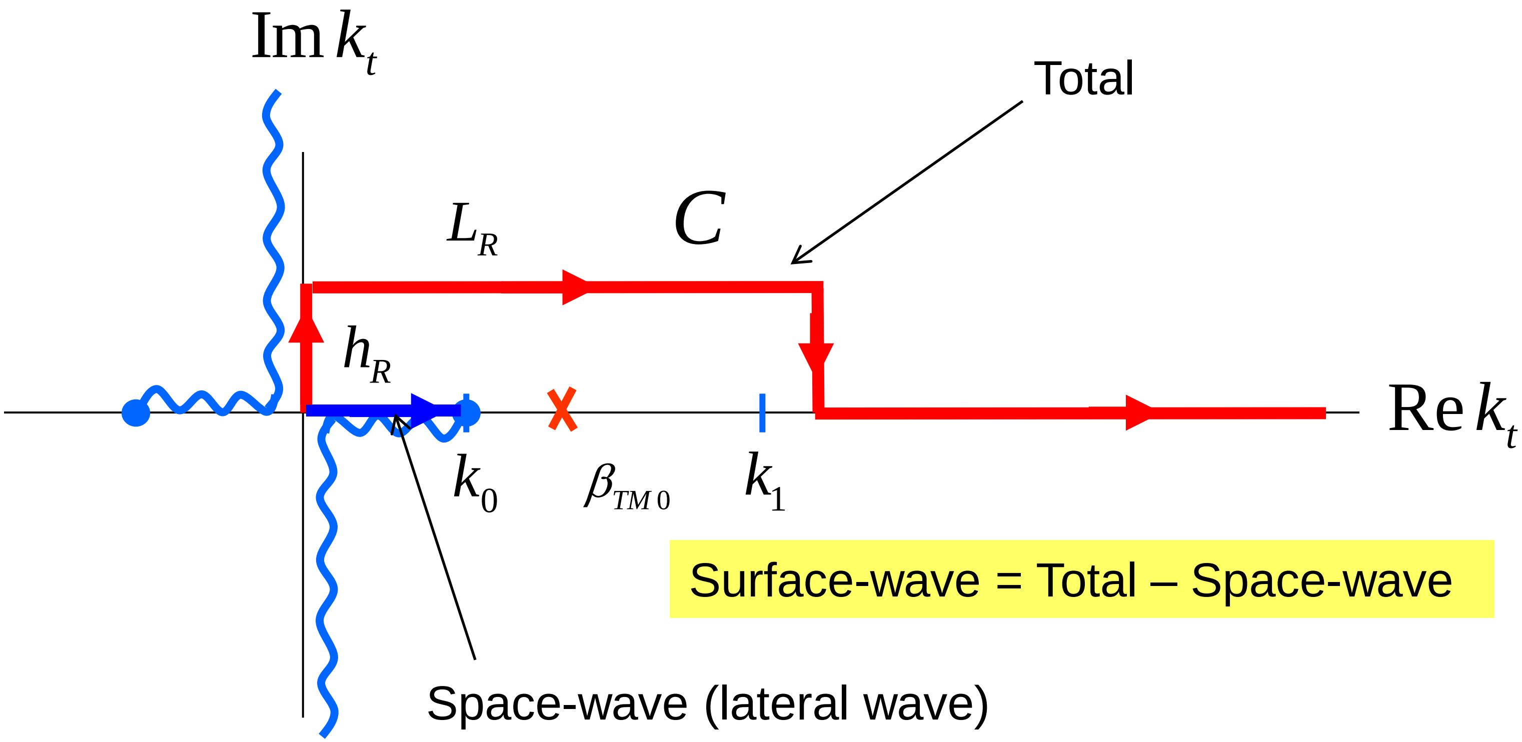


Fig. 8. Measured and calculated mutual coupling between two coax-fed microstrip antennas, for both $\bar{E}$-plane and $\bar{H}$-plane coupling. W = 10.57 cm, L = 6.55 cm, d = 0.1588 cm, ϵ_r = 2.55, f = 1410. MHz.

Components of Mutual Impedance

We can also consider the various components of Z_{12}

$$Z_{xx}^{1,2} = -\frac{1}{\pi^2} \int_0^{\pi/2} \int_C \tilde{G}_{xx}\left(k_x, k_y\right) \tilde{B}_x^2\left(k_x, k_y\right) \cos\left(k_x \Delta x\right) \cos\left(k_y \Delta y\right) k_t \, dk_t \, d\bar{\phi}$$



Surface-wave = Total – Space-wave

Components of Mutual Impedance (cont.)

Results for typical patches

Circular patches

$$\varepsilon_r = 2.94, \quad h / \lambda_0 = 0.01, \quad f = 2.0\,\text{GHz}$$