ECE 6345

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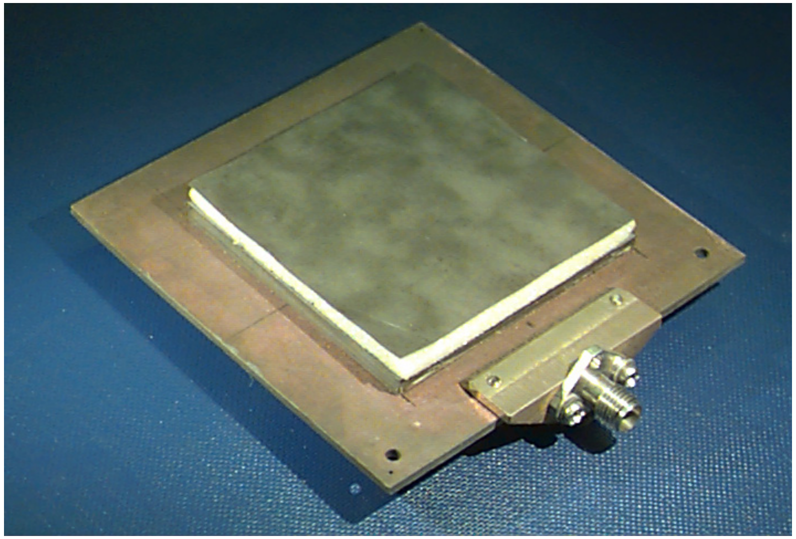

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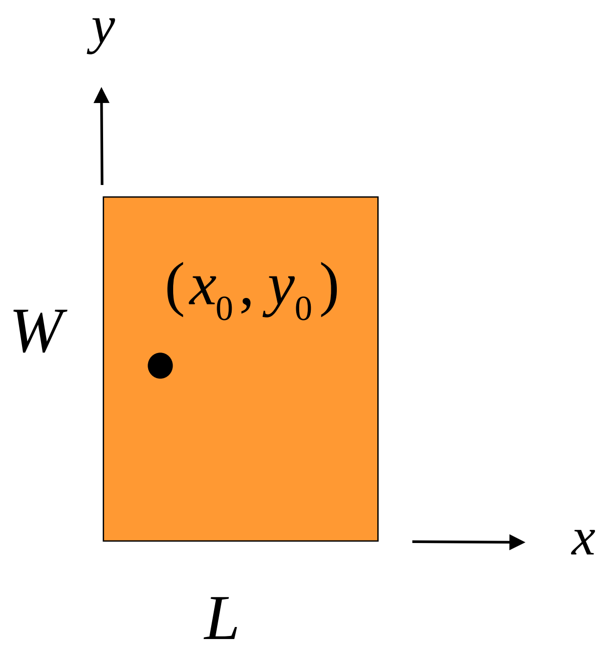
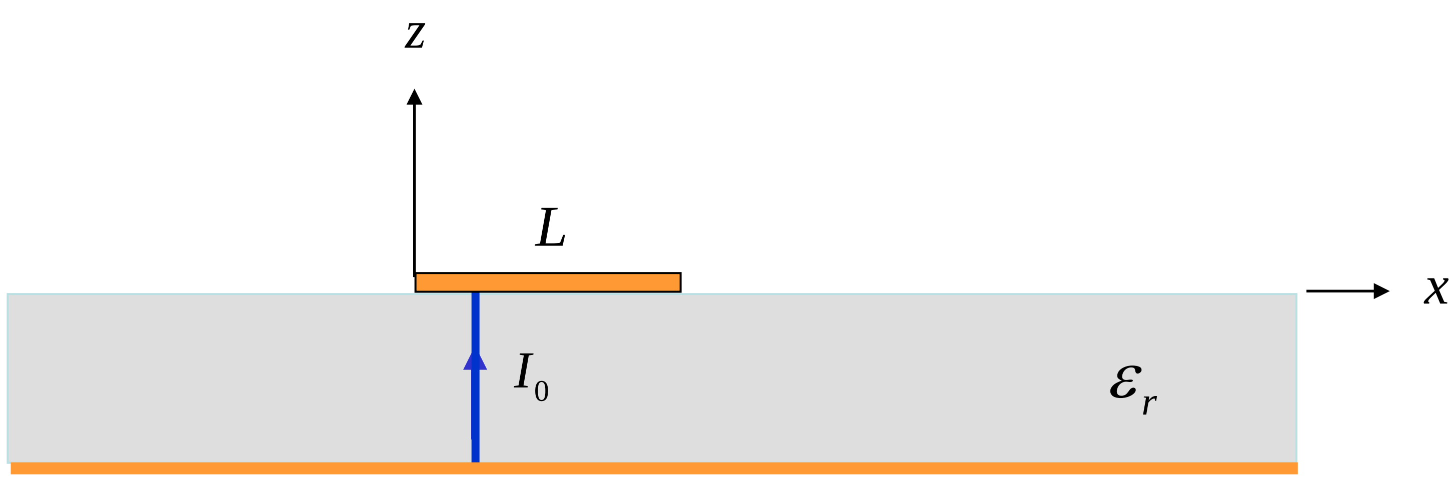
Overview

In this set of notes we investigate the transmission line (TL) model for the input impedance of a rectangular patch antenna.

Assumption: The patch is operating in the usual (1,0) mode: the patch acts as a wide microstrip line of width W.

TL Model

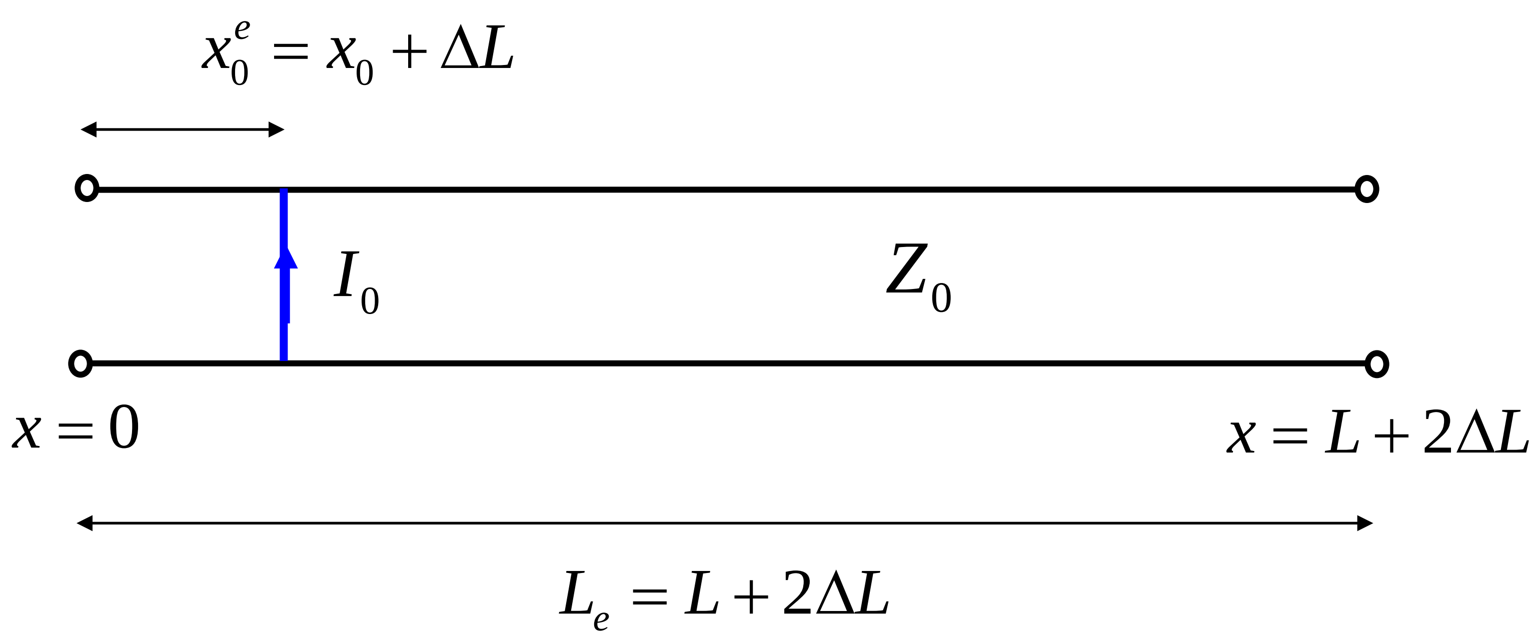




TL Model (cont.)

Ignore variation in the *y* direction: $\underline{J}_s \approx \underline{\hat{x}} \, J_{sx}(x)$ (TM$_{10}$ mode)

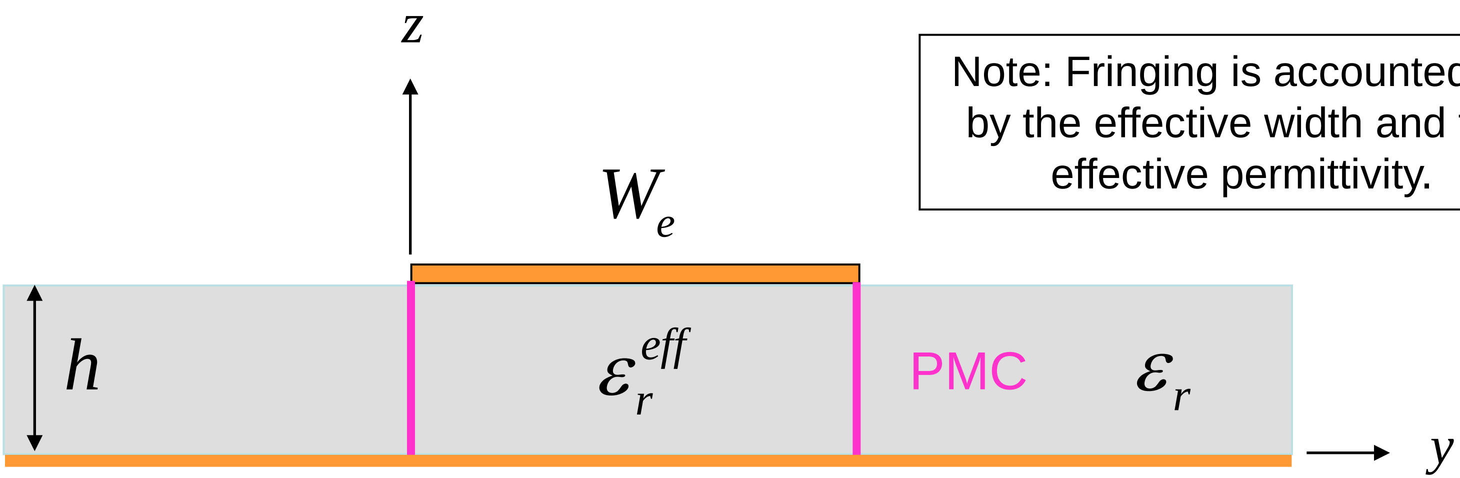
Fringing extensions are added at both ends, and x is now measured from the left edge after extending it.



Planar Waveguide Model

Planar waveguide model for wide microstrip line:

Note: Fringing is accounted for by the effective width and the effective permittivity.



$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

$$Z_0 = \frac{V}{I} = \frac{-E_z h}{J_{sx} W_e} = \frac{-E_z h}{H_y W_e}$$

so

$$Z_0 = \eta_{eff} \left(\frac{h}{W_e} \right)$$

Planar Waveguide Model (cont.)

The characteristic impedance and phase constant are given by

$$Z_0 = \eta_0 \frac{1}{\sqrt{\varepsilon_r^{eff}}}\left(\frac{h}{W_e}\right)$$

$$\beta = k_0 \sqrt{\varepsilon_r^{eff}}$$

From a knowledge of the characteristic impedance and the phase constant, we can then determine the effective width and the effective permittivity.

$$W_e = \eta_0 \frac{1}{Z_0}\left(\frac{h}{\sqrt{\varepsilon_r^{eff}}}\right)$$

$$\varepsilon_r^{eff} = \left(\beta / k_0\right)^2$$

Planar Waveguide Model (cont.)

Approximate formulas

The effective permittivity and characteristic impedance are given by:

$$Z_0 = \frac{\eta_0}{\sqrt{\varepsilon_r^{eff}}} \left[\frac{W}{h} + 1.393 + 0.667 \ln\left(\frac{W}{h} + 1.444 \right) \right]^{-1}$$

$$\varepsilon_r^{eff} = \left(\frac{\varepsilon_r + 1}{2} \right) + \left(\frac{\varepsilon_r - 1}{2} \right) \frac{1}{\sqrt{1 + 12\left(\frac{h}{W} \right)}}$$

These are accurate for $W / h > 1$

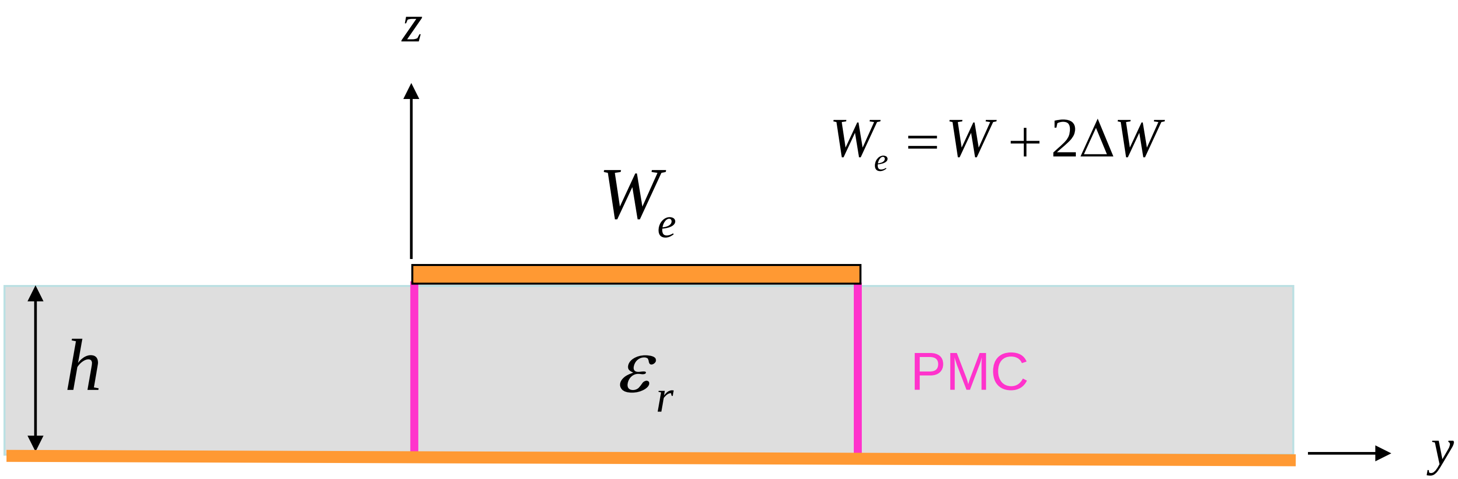
D. M. Pozar, *Microwave Engineering*, Wiley, 1998, p. 162

The two formulas allow us to find W_e.

Planar Waveguide Model (cont.)

Alternative (*the approach that we will use*):

We keep the *original permittivity*, and extend the edges.



Note: This is consistent with the Hammerstad formula:

$$f_0 = \frac{c}{2\sqrt{\varepsilon_r}\left(L + 2\Delta L\right)}$$

(The Hammerstad formula appears to work best when used with the actual permittivity instead of the effective permittivity in the frequency formula.)

Fringing Extensions

We then have the following results:

$$\Delta W = h\left(\frac{\ln 4}{\pi}\right) \qquad \text{(Wheeler formula)}$$

$$\Delta L = 0.412h\left[\frac{\varepsilon_r^{eff}+0.300}{\varepsilon_r^{eff}-0.258}\right]\left[\frac{\dfrac{W}{h}+0.262}{\dfrac{W}{h}+0.813}\right] \qquad \text{(Hammerstad formula)}$$

$$\varepsilon_r^{eff} = \left(\frac{\varepsilon_r+1}{2}\right)+\left(\frac{\varepsilon_r-1}{2}\right)\frac{1}{\sqrt{1+12\left(\dfrac{h}{W}\right)}}$$

Effective Loss Tangent

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}$$

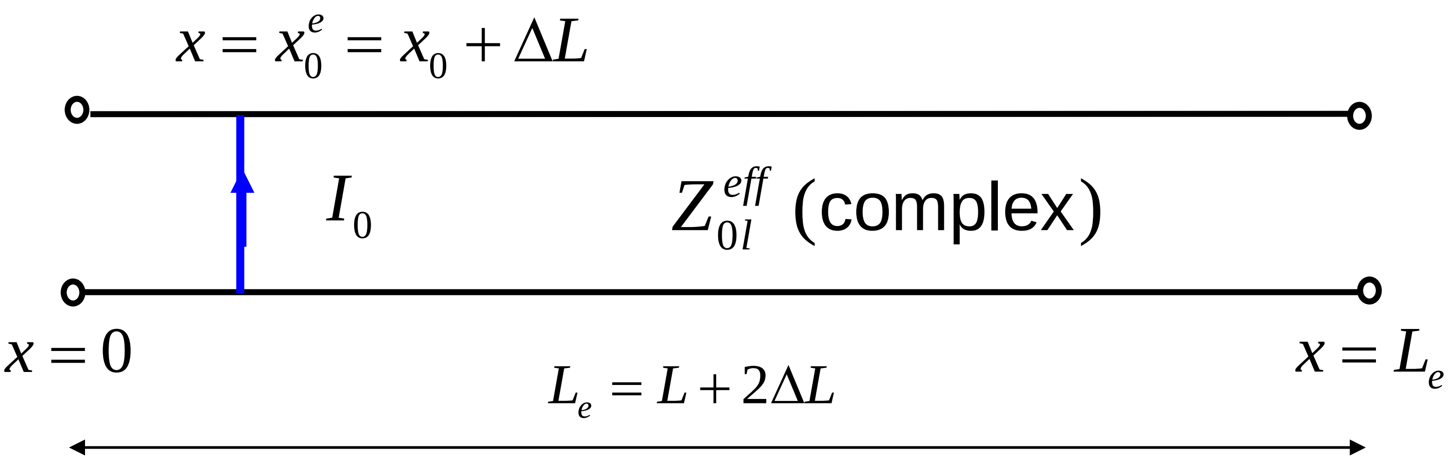
Recall that $\frac{1}{Q_d} = \tan\delta \equiv l$ (shorthand notation for loss tangent)

We account for all losses (and radiation) by means of an effective loss tangent:

$$l_{eff} \equiv (\tan\delta)_{eff} = \frac{1}{Q}$$

$$\varepsilon_{rl}^{eff} = \varepsilon_r' \left(1 - j l_{eff}\right)$$

Input Impedance



$$Z_{0l}^{eff} = \eta_0 \frac{1}{\sqrt{\varepsilon_{rl}^{eff}}} \left(\frac{h}{W_e} \right)$$

$$\varepsilon_{rl}^{eff} = \varepsilon_r' \left(1 - j l_{eff} \right)$$

Input Impedance (cont.)

From transmission-line theory we have that

$$Z_{in}^{TL} = 1 / Y_{in}^{TL}$$

$$Y_{in}^{TL} = jY_{0l}^{eff} \tan\left(k_l^{eff} x_0^e\right) + jY_{0l}^{eff} \tan\left(k_l^{eff}\left(L_e - x_0^e\right)\right)$$

where

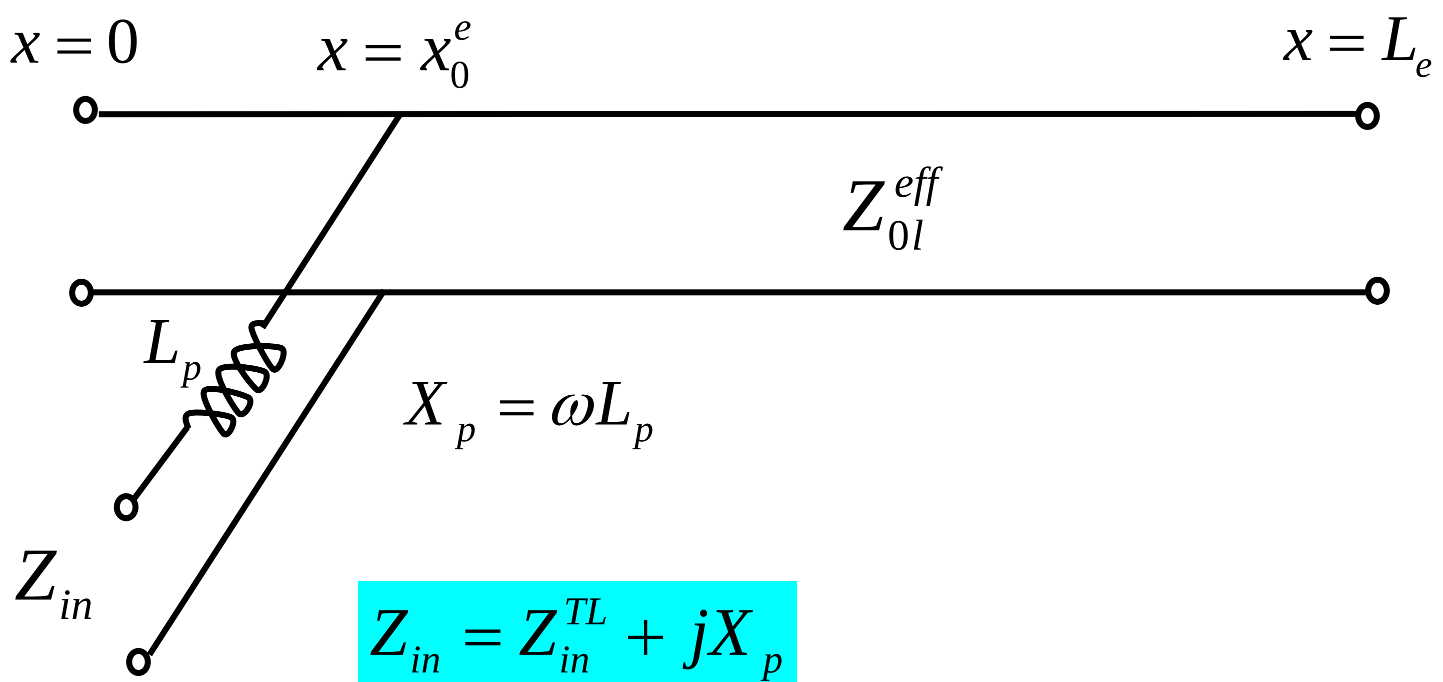
$$Y_{0l}^{eff} = \frac{1}{Z_{0l}^{eff}}$$

$$k_l^{eff} = k_0 \sqrt{\varepsilon_{rl}^{eff}}$$

$$l_{eff} \equiv \left(\tan\delta\right)_{eff} = \frac{1}{Q}$$

Note: The effective permittivity accounts for all material losses and radiation.

Probe Correction



$$Z_{in} = Z_{in}^{TL} + jX_p$$

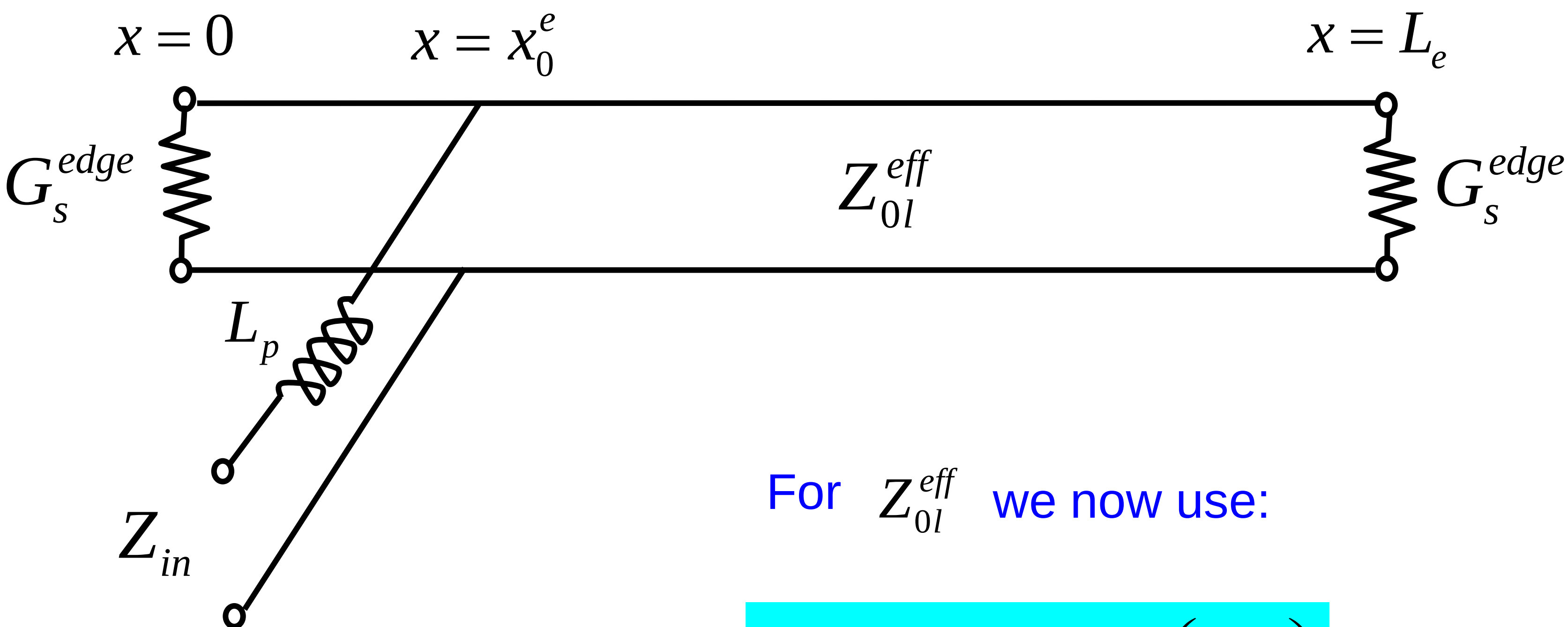
where

$$X_p = \eta_0 \mu_r \left(\frac{h}{\lambda_0} \right) \left[\ln\left(\frac{1}{a/\lambda_0} \right) - \gamma - \ln \pi - \ln \sqrt{\mu_r \varepsilon_r} \right]$$

$\gamma \doteq 0.57722$

(Euler's constant)

Alternative (Edge Admittance)



The edge admittances account for radiation effects (radiation into space and surface waves).

For Z_{0l}^{eff} we now use:

$$Z_{0l}^{eff} = \eta_0 \frac{1}{\sqrt{\varepsilon_{rl}^{eff}}} \left(\frac{h}{W_e} \right)$$

$$\varepsilon_{rl}^{eff} = \varepsilon_r' \left(1 - j l_{eff}^{diss} \right)$$

$$l_{eff}^{diss} = \frac{1}{Q_d} + \frac{1}{Q_c}$$

Alternative (Edge Admittance) (cont.)

G_s^{edge} models radiation (space-wave + surface-wave)

$$P_{rad} = 2\left(\frac{1}{2} G_s^{edge} \left|V(0)\right|^2\right) = G_s^{edge} \left|E_z(0)\right|^2 h^2$$

We assume that the two edge voltages are approximately the same in magnitude.

Also, we can express the radiated power in terms of the radiation *Q* as

$$Q_r = \frac{\omega_0 U_s}{P_{rad}} = 2\frac{\omega_0 U_E}{P_{rad}} \quad \Rightarrow \quad P_{rad} = 2\frac{\omega_0 U_E}{Q_r}$$

where

$$\frac{1}{Q_r} = \left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}\right)$$

Alternative (Edge Admittance) (cont.)

The time-average stored electric energy is given by

$$U_E = \int_V \frac{1}{4} \varepsilon_0 \varepsilon_r' \left| E_z \right|^2 dV$$

$$= W_e h \left(\frac{1}{4} \right) \varepsilon_0 \varepsilon_r' \int_0^{L_e} \left| E_z \right|^2 dx$$

Hence we have

$$P_{rad} = 2 \frac{\omega_0 U_E}{Q_r} \quad \Rightarrow \quad P_{rad} = \left(\frac{2\omega_0}{Q_r} \right) W_e h \left(\frac{1}{4} \right) \varepsilon_0 \varepsilon_r' \int_0^{L_e} \left| E_z \right|^2 dx$$

Alternative (Edge Admittance) (cont.)

The field inside the patch is approximately described by a cosine function.

Therefore, we have

$$P_{rad} = \frac{1}{2}\omega_0\left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}\right) W_e h \,\varepsilon_0 \varepsilon_r' \int_0^{L_e} \left|E_z(0)\right|^2 \cos^2\left(\frac{\pi x}{L_e}\right) dx$$

Evaluating the integral, we have

$$P_{rad} = \frac{1}{2} W_e h\,\omega_0\,\varepsilon_0 \varepsilon_r' \left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}\right)\left(\left|E_z(0)\right|^2 \frac{L_e}{2}\right)$$

The radiated power is now in terms of the edge field.

Alternative (Edge Admittance) (cont.)

We then equate these two expressions for the radiated power:

$$P_{rad} = G_s^{edge} \left| E_z(0) \right|^2 h^2$$

$$P_{rad} = \frac{1}{2} W_e h \, \omega_0 \varepsilon_0 \varepsilon_r' \left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} \right) \left(\left| E_z(0) \right|^2 \frac{L_e}{2} \right)$$

The result for the edge conductance is

$$G_s^{edge} = \frac{1}{4} \varepsilon_r' \left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} \right) \left(\omega_0 \varepsilon_0 \right) \left(\frac{W_e L_e}{h} \right)$$

Alternative (Edge Admittance) (cont.)

Using $\omega_0 \varepsilon_0 = k_0 / \eta_0$

the final result for the edge conductance is

$$G_s^{edge} = \frac{1}{\eta_0}\left(\frac{\varepsilon_r'}{4}\right)\left(\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}\right)(k_0 h)\left(\frac{W_e L_e}{h^2}\right)$$

Alternative (Edge Admittance) (cont.)

The input impedance is

$$Z_{in} = jX_p + \left\{ Y_{0l}^{eff} \left[\frac{G_s^{edge} + jY_{0l}^{eff} \tan\left(k_l^{eff} x_0^e\right)}{Y_{0l}^{eff} + jG_s^{edge} \tan\left(k_l^{eff} x_0^e\right)} \right] + Y_{0l}^{eff} \left[\frac{G_s^{edge} + jY_{0l}^{eff} \tan\left(k_l^{eff} \left(L_e - x_0^e\right)\right)}{Y_{0l}^{eff} + jG_s^{edge} \tan\left(k_l^{eff} \left(L_e - x_0^e\right)\right)} \right] \right\}^{-1}$$

$$Y_{0l}^{eff} = \frac{1}{Z_{0l}^{eff}} \qquad Z_{0l}^{eff} = \eta_0 \sqrt{\frac{1}{\varepsilon_{rl}^{eff}}} \left(\frac{h}{W_e} \right)$$

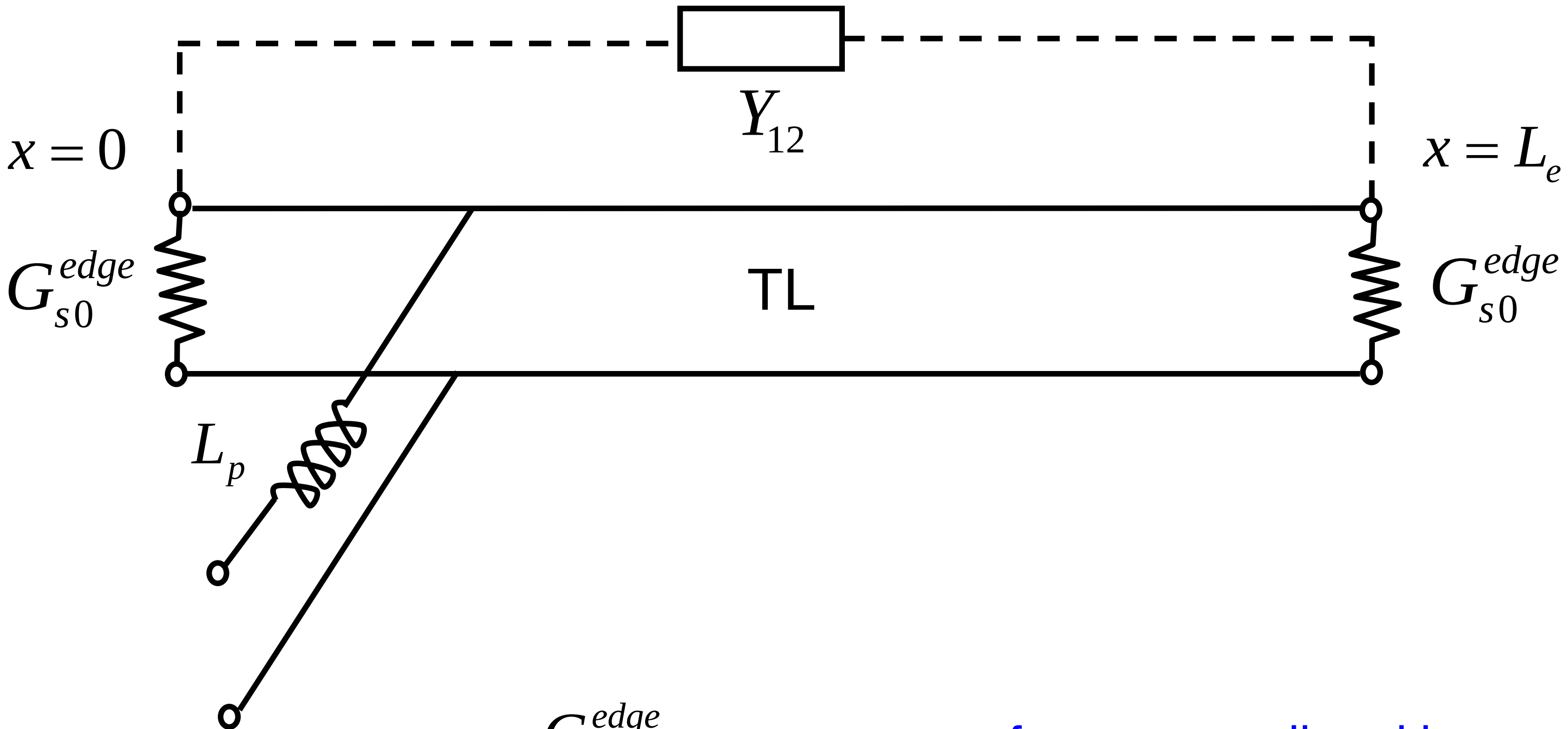
$$\varepsilon_{rl}^{eff} = \varepsilon_r' \left(1 - j l_{eff}^{diss} \right)$$

$$k_l^{eff} = k_0 \sqrt{\varepsilon_{rl}^{eff}}$$

$$l_{eff}^{diss} = \frac{1}{Q_d} + \frac{1}{Q_c}$$

Note: The effective permittivity accounts for only material losses (not radiation).

Another Alternative (Edge Admittance Network)



G_{s0}^{edge} now accounts for power radiated by a *single edge* (without mutual coupling).