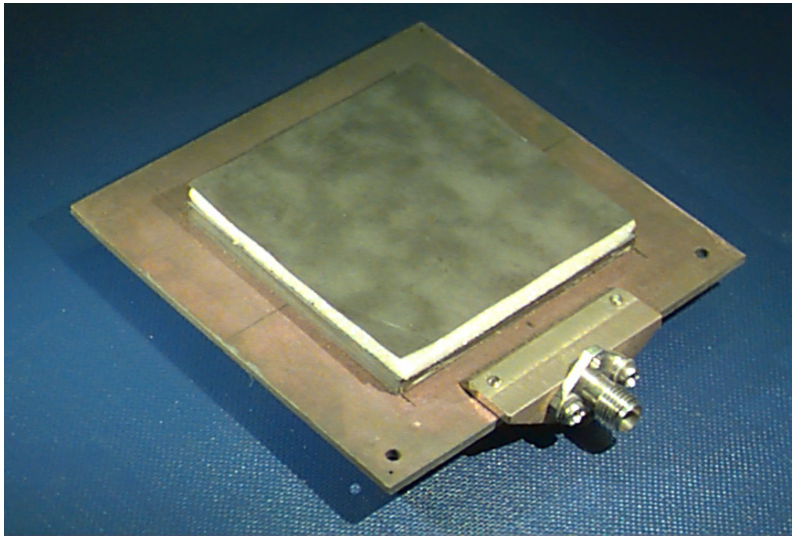
ECE 6345

Spring 2015

Prof. David R. Jackson
ECE Dept.



Notes 29

Overview

In this set of notes we develop a CAD formula for the resonant input resistance of a rectangular patch antenna, based on the cavity model.

CAD Formula for R_{in}

From Notes 28 we have the following formula:

$$R_{mn} = \left(\frac{P_{mn}}{k_{mn}^2 l_{eff}} \right) \omega_{mn} \qquad P_{mn} = \mu h \frac{1}{\left| I_{in} \right|^2} \frac{\left| \left\langle \psi_{mn}, J_z^i \right\rangle \right|^2}{\left\langle \psi_{mn}, \psi_{mn} \right\rangle}$$

Hence, for the (1,0) mode we have

$$R_{10} = \left(\frac{\omega_{10}}{k_{10}^2 l_{eff}} \right) P_{10} \qquad P_{10} = \mu h \frac{1}{\left| I_{in} \right|^2} \frac{\left| \left\langle \psi_{10}, J_z^i \right\rangle \right|^2}{\left\langle \psi_{10}, \psi_{10} \right\rangle}$$

Since all the other (non-resonant) modes mainly contribute to the probe reactance, and not the input resistance, we have

$$R_{in} \approx R_{10}$$

CAD Formula for R_{in} (cont.)

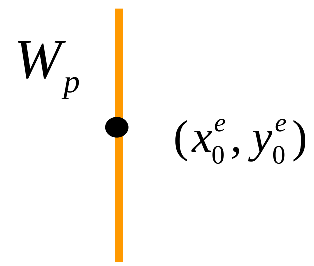
For the rectangular patch we have, from Notes 28:

$$\left\langle \psi_{mn}, J_z^i \right\rangle = I_{in} \cos\left(\frac{m\pi x_0^e}{L_e}\right) \cos\left(\frac{n\pi y_0^e}{W_e}\right) \operatorname{sinc}\left(\frac{n\pi W_p}{2W_e}\right)$$

$$\left\langle \psi_{mn}, \psi_{mn} \right\rangle = \left(\frac{W_e}{2}\right)\left(\frac{L_e}{2}\right)\left(1+\delta_{m0}\right)\left(1+\delta_{n0}\right)$$

$$\left\langle \psi_{10}, J_z^i \right\rangle = I_{in} \cos\left(\frac{\pi x_0^e}{L_e}\right)$$

$$\left\langle \psi_{10}, \psi_{10} \right\rangle = \left(\frac{W_e}{2}\right)\left(\frac{L_e}{2}\right)(1)(2)$$



Strip model of probe

CAD Formula for R_{in} (cont.)

Hence, we have

$$P_{10} = \mu h \frac{1}{\left|I_{in}\right|^2} \frac{\left|\left\langle \psi_{10}, J_z^i \right\rangle\right|^2}{\left\langle \psi_{10}, \psi_{10} \right\rangle}$$

$$= \mu h \frac{\cos^2\left(\dfrac{\pi x_0^e}{L_e}\right)}{\left(\dfrac{W_e}{2}\right)\left(\dfrac{L_e}{2}\right)(1)(2)}$$

$$= 2\left(\frac{\mu h}{W_e L_e}\right)\cos^2\left(\frac{\pi x_0^e}{L_e}\right)$$

CAD Formula for R_{in} (cont.)

Recall: $$R_{10} = \left(\frac{\omega_{10}}{k_{10}^2 \, l_{eff}} \right) P_{10}$$

Note: $$l_{eff} = 1/Q$$

We then have

$$R_{10} = \left(\frac{\omega_{10}}{k_{10}^2} Q \right) \left[\left(\frac{2\mu h}{W_e L_e} \right) \cos^2 \left(\frac{\pi x_0^e}{L_e} \right) \right]$$

or

$$R_{10} = R_{edge} \cos^2 \left(\frac{\pi x_0^e}{L_e} \right)$$

where

$$R_{edge} = \left(\frac{2}{W_e L_e} \right) \left(\frac{\mu \omega_{10}}{k_{10}^2} \right) (hQ)$$

CAD Formula for R_{in} (cont.)

To put this into a nicer form, use

$$\mu\omega_{10} = \frac{\mu\omega_{10}\sqrt{\dfrac{\mu}{\varepsilon'}}}{\sqrt{\dfrac{\mu}{\varepsilon'}}} = \eta'\left(\omega_{10}\sqrt{\mu\varepsilon'}\right) = \eta' k_{10}$$

and

$$k_{10} L_e = \pi \quad \Rightarrow \quad k_{10} = \pi / L_e$$

so that

$$R_{edge} = \left(\frac{2\eta' k_{10}}{W_e L_e}\right)\left(\frac{1}{k_{10}^2}\right)(hQ) = \left(\frac{2\eta' k_{10}}{W_e L_e}\right)\left(\frac{1}{\left(\dfrac{\pi}{L_e}\right)^2}\right)(hQ)$$

Next, use

$$\eta' k_{10} \approx \eta' k = \eta' k_0 \sqrt{\mu_r \varepsilon_r'} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r'}} k_0 \sqrt{\mu_r \varepsilon_r'} = \eta_0 k_0 \mu_r$$

CAD Formula for R_{in} (cont.)

so that

$$R_{edge} = \frac{2\eta_0 k_0 \mu_r}{W_e L_e}\left(\frac{L_e}{\pi}\right)^2 (hQ)$$

or

$$R_{edge} = \eta_0 \mu_r \left(\frac{4}{\pi}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{h}{\lambda_0} Q\right)$$

where

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}$$

CAD Formula for R_{in} (cont.)

CAD formulas for the Q terms:

$$Q_d = \frac{1}{\tan\delta}$$

$$Q_c = \frac{\eta_0}{2}\mu_r\left[\frac{(k_0 h)}{R_s^{ave}}\right]$$

$$Q_{sp} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{pc_1}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{1}{h/\lambda_0}\right)$$

$$Q_{sw} = Q_{sp}\left(\frac{e_r^{sw}}{1-e_r^{sw}}\right)$$

$$e_r^{sw} = \frac{1}{1+\frac{3\pi}{4}(k_0 h)\left(\frac{\mu_r}{c_1}\right)\left(1-\frac{1}{n_1^2}\right)^3}$$

CAD Formula for R_{in} (cont.)

Note: If we ignore Q_d, Q_c, Q_{sw} (assume a lossless patch)

then $$Q = Q_{sp} \propto \frac{1}{h/\lambda_0}$$

And therefore

$$R_{edge} \to \text{constant} \qquad \text{as} \quad h/\lambda_0 \to 0$$

If losses are present, then $R_{edge} \to 0$ as $h/\lambda_0 \to 0$