ECE 6345

Spring 2015

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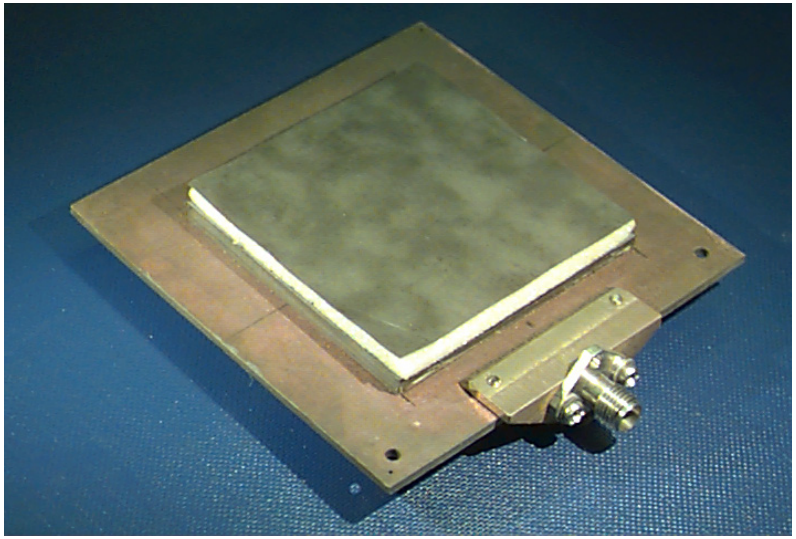


Notes 31

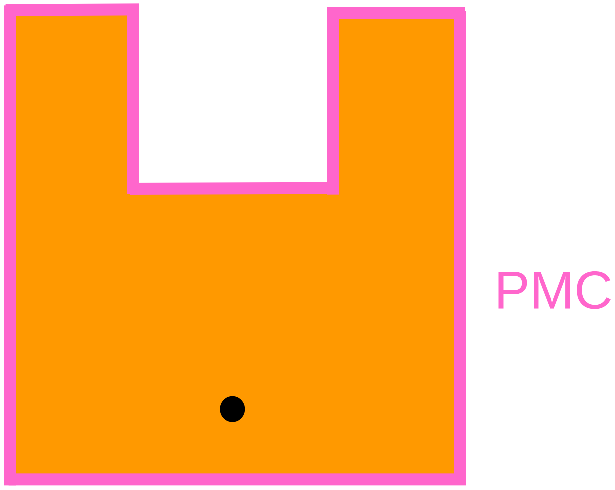
Overview

In this set of notes we overview the segmentation method for obtaining the input impedance of an arbitrary shaped patch.

This is an extension of the cavity model.

Segmentation Method

The perimeter has been extend to account for fringing.

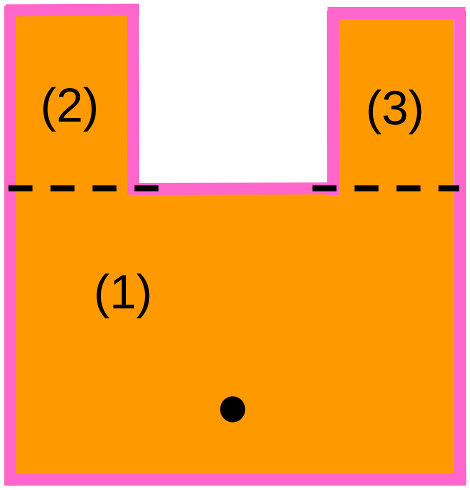


$$k_e = k_0 \sqrt{\varepsilon_{rl}^{eff}}$$

(arbitrary shaped patch)

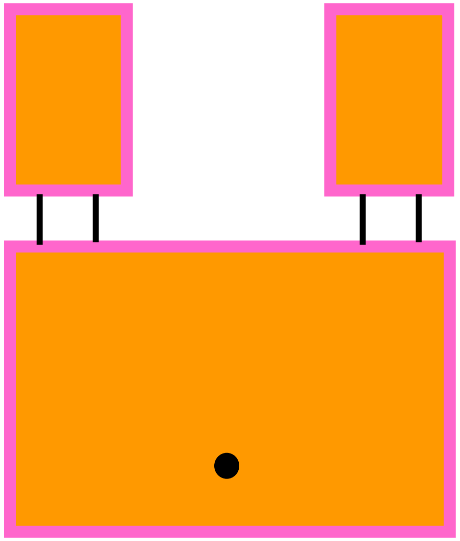
Note: The effective complex wavenumber may be determined iteratively – once the field is known inside the patch, the stored energy and the radiated power may be computed, and from this the Q may be determined.

Segmentation Method (cont.)



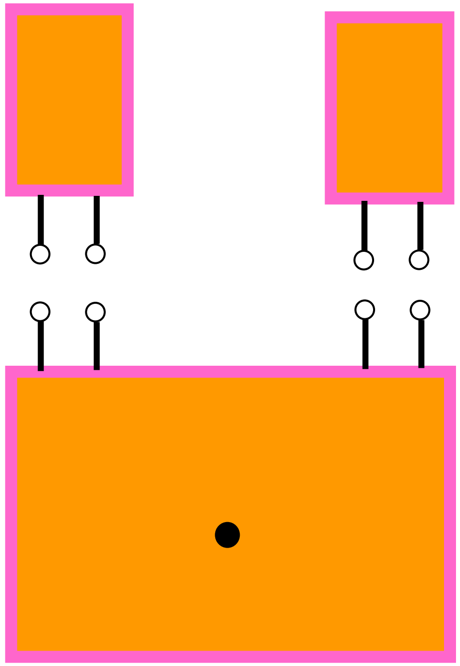
The patch is divided into a number of rectangular-shaped patches.

Segmentation Method (cont.)



Along the edges where the rectangular patches connect, we introduce ports. The ports connect the patches together.

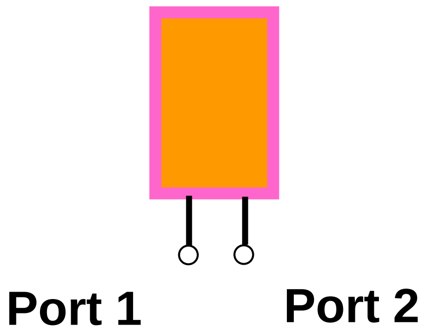
Segmentation Method (cont.)



For each rectangular patch, we calculate the Z matrix for the ports that connect to it.

Segmentation Method (cont.)

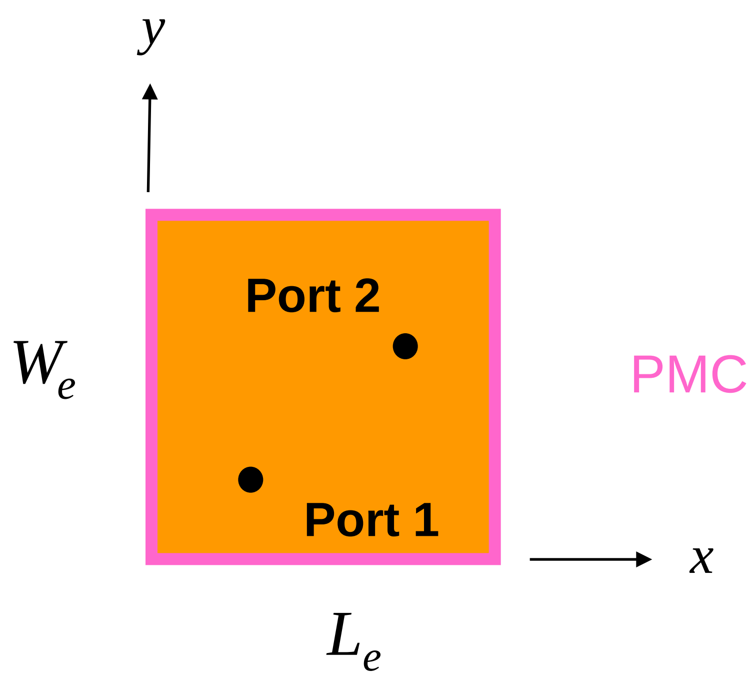
To illustrate, consider one patch:



$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

We can calculate Z_{mn} by using the cavity model.

Segmentation Method (cont.)

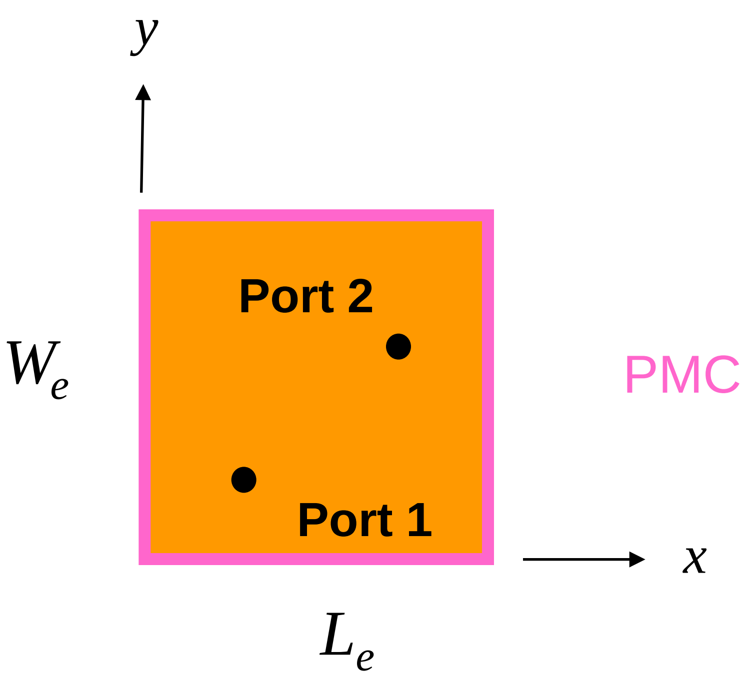


This figure shows a general two-port system for a rectangular patch.

Port 1: (x_1^e, y_1^e) $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ (the input impedance at port 1)

Port 2: (x_2^e, y_2^e) $$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$ (the input impedance at port 2)

Segmentation Method (cont.)



$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

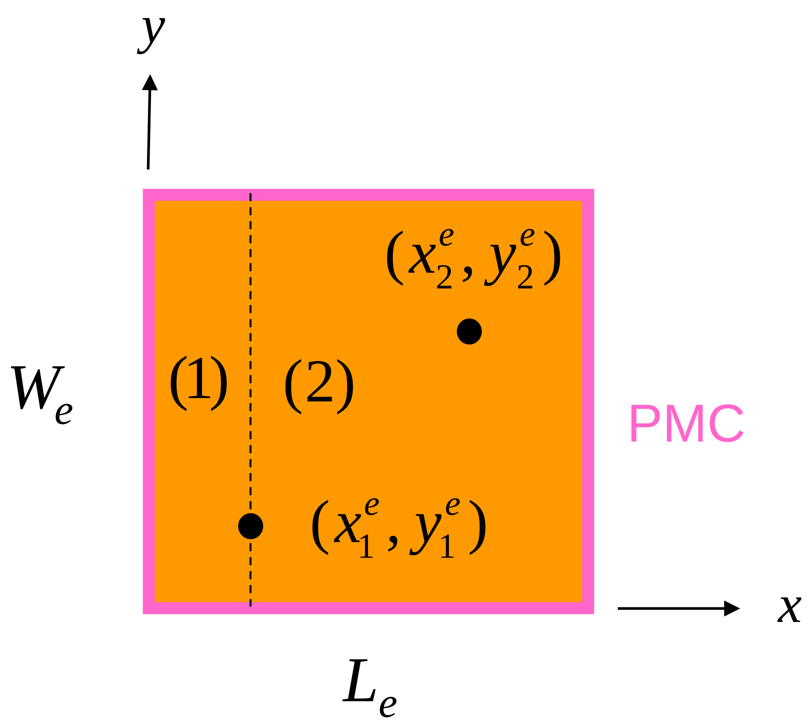
Assume

$$I_1 = 1$$

$$I_2 = 0$$

$$Z_{21} = -hE_z\left(x_2^e, y_2^e\right)$$

Segmentation Method (cont.)



Use the mode-matching method:

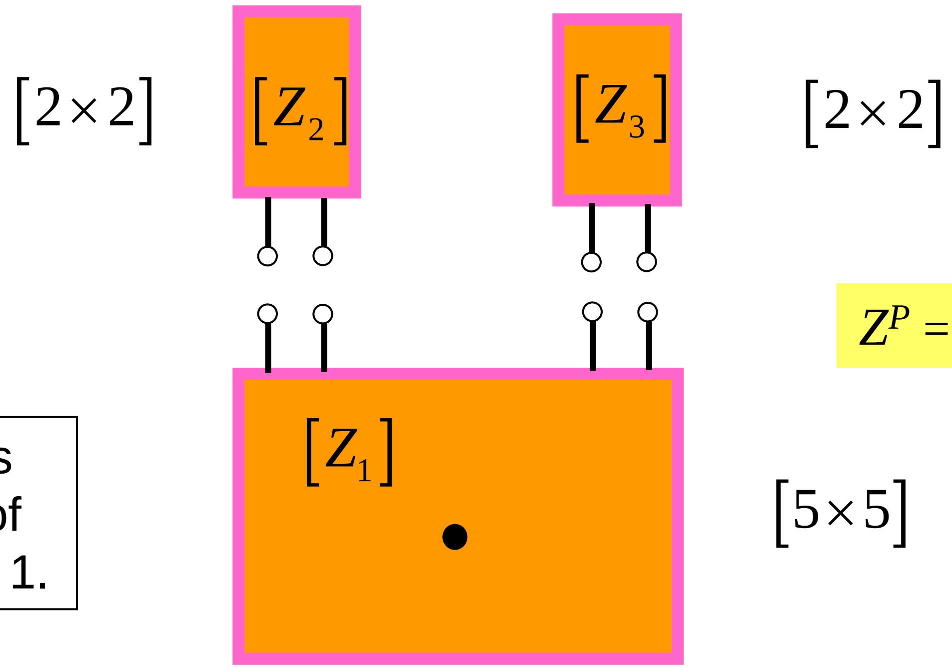
$$E_{z2} = \sum_{m=0}^{\infty} B_m \cos\left(\frac{m\pi y}{W_e}\right) \cos\left(k_{xm}\left[x - L_e\right]\right)$$

Note: B_m is a function of the location of port 1.

so that

$$Z_{21} = -h \sum_{m=0}^{\infty} B_m \cos\left(\frac{m\pi y_2^e}{W_e}\right) \cos\left(k_{xm}\left[x_2^e - L_e\right]\right)$$

Segmentation Method (cont.)



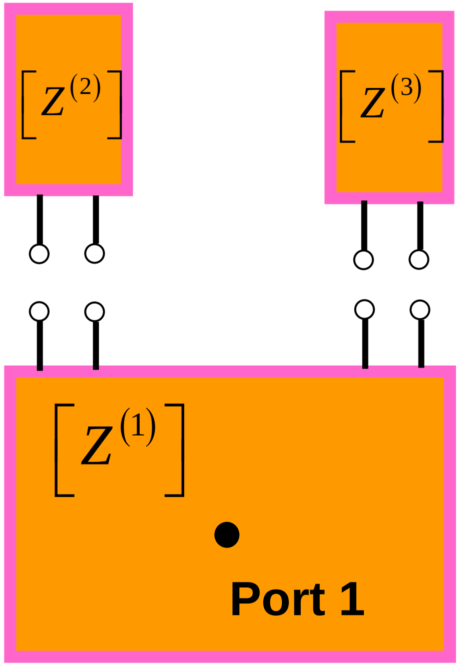
Z^P = final port matrix

Note: The feed is considered one of the ports on patch 1.

Once the Z matrices of all the patches have been computed, an overall Z matrix of the entire system Z^P is calculated, by connecting the individual Z matrices and using circuit theory.

If there is a single feed, then Z^P is a 1×1 matrix: $\left[Z^P\right] = \left[Z_{11}^P\right]_{1\times 1}$

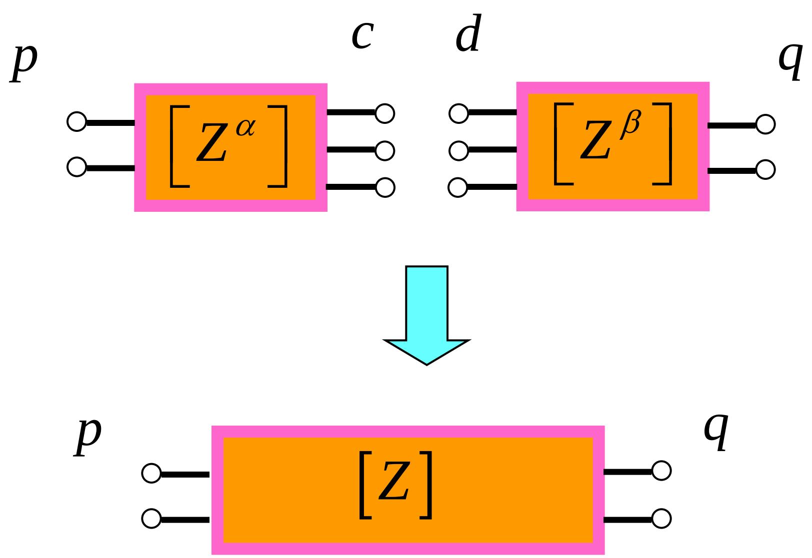
Segmentation Method (cont.)



The input impedance is then given by

$$Z_{in} = Z_{11}^{P}$$

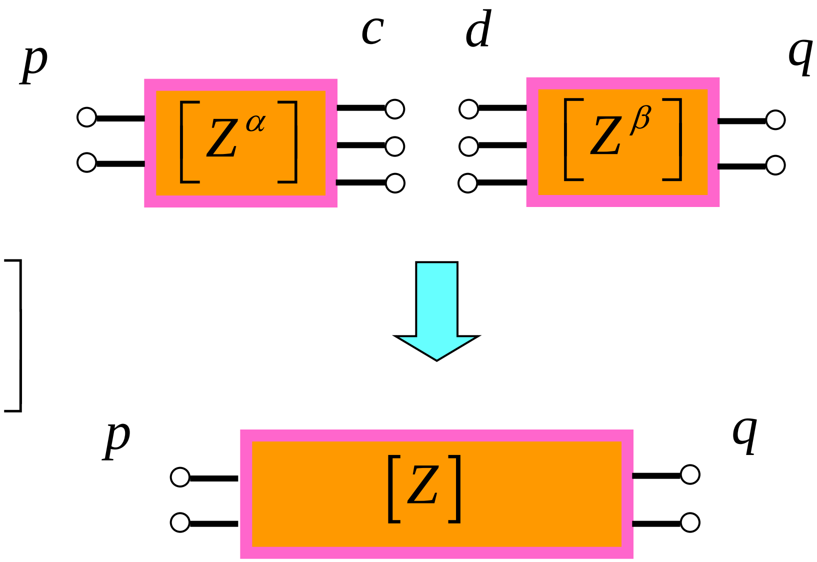
Segmentation Method (cont.)



We next show how to combine two Z matrices into a single overall Z^{P} matrix.

The ports p and q are the external ports, while c and d represent the internal (connected) ports.

Segmentation Method (cont.)



$$[Z^{\alpha}] = \begin{bmatrix} Z^{\alpha}_{pp} & Z^{\alpha}_{pc} \\ Z^{\alpha}_{cp} & Z^{\alpha}_{cc} \end{bmatrix}$$

$$[Z^{\beta}] = \begin{bmatrix} Z^{\beta}_{qq} & Z^{\beta}_{qd} \\ Z^{\beta}_{dq} & Z^{\beta}_{dd} \end{bmatrix}$$

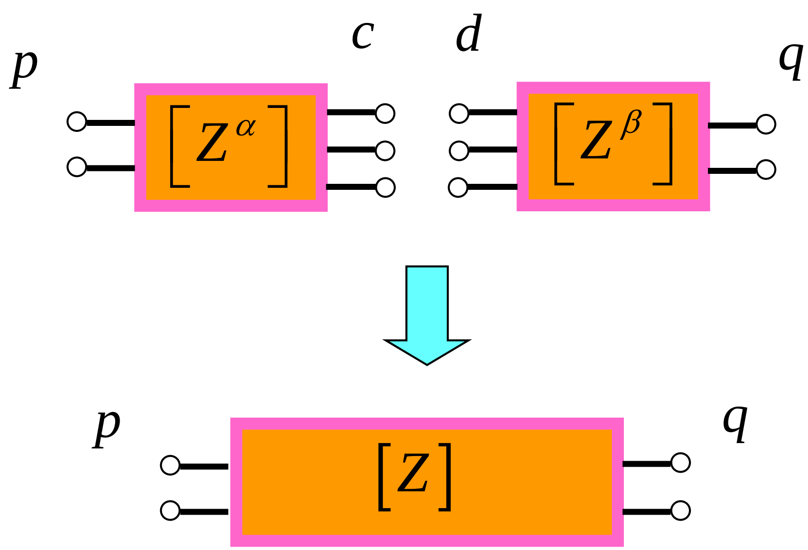
$$[V_p] = [Z^{\alpha}_{pp}][I_p] + [Z^{\alpha}_{pc}][I_c] \quad (1)$$

$$[V_c] = [Z^{\alpha}_{cc}][I_c] + [Z^{\alpha}_{cp}][I_p] \quad (3)$$

$$[V_q] = [Z^{\beta}_{qq}][I_q] + [Z^{\beta}_{qd}][I_d] \quad (2)$$

$$[V_d] = [Z^{\beta}_{dd}][I_d] + [Z^{\beta}_{dq}][I_q] \quad (4)$$

Segmentation Method (cont.)



Also, from Kirchhoff's laws we have

$$[I_c] = -[I_d] \quad (5)$$

$$[V_c] = [V_d] \quad (6)$$

Segmentation Method (cont.)

Substituting (3) and (4) into (6) and using (5) yields

$$\left[Z_{cc}^{\alpha}\right]\left[I_c\right]+\left[Z_{cp}^{\alpha}\right]\left[I_p\right]=\left[Z_{dd}^{\beta}\right]\left[-I_c\right]+\left[Z_{dq}^{\beta}\right]\left[I_q\right]$$

Hence

$$\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]\left[I_c\right]=-\left[Z_{cp}^{\alpha}\right]\left[I_p\right]+\left[Z_{dq}^{\beta}\right]\left[I_q\right]$$

so that

$$\left[I_c\right]=\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left(-\left[Z_{cp}^{\alpha}\right]\left[I_p\right]+\left[Z_{dq}^{\beta}\right]\left[I_q\right]\right) \qquad (7)$$

Segmentation Method (cont.)

Next, insert (7) into (1):

$$\left[V_p\right]=\left[Z_{pp}^{\alpha}\right]\left[I_p\right]+\left[Z_{pc}^{\alpha}\right]\left(\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left(-\left[Z_{cp}^{\alpha}\right]\left[I_p\right]+\left[Z_{dq}^{\beta}\right]\left[I_q\right]\right)\right) \quad (8)$$

Similarly, from (2) we have

$$\left[V_q\right]=\left[Z_{qq}^{\beta}\right]\left[I_q\right]+\left[Z_{qd}^{\beta}\right]\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left(-\left[Z_{dq}^{\beta}\right]\left[I_q\right]+\left[Z_{cp}^{\alpha}\right]\left[I_p\right]\right) \quad (9)$$

From the definition of the Z matrix of the overall system, we can also write

$$\left[V_p\right]=\left[Z_{pp}\right]\left[I_p\right]+\left[Z_{pq}\right]\left[I_q\right] \quad (10)$$

$$\left[V_q\right]=\left[Z_{qp}\right]\left[I_p\right]+\left[Z_{qq}\right]\left[I_q\right] \quad (11)$$

Comparing (8) and (9) with (10) and (11), we have the following results:

Segmentation Method (cont.)

$$\left[Z_{pp}\right]=\left[Z_{pp}^{\alpha}\right]-\left[Z_{pc}^{\alpha}\right]\left(\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left[Z_{cp}^{\alpha}\right]\right)$$

$$\left[Z_{pq}\right]=\left[Z_{pc}^{\alpha}\right]\left(\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left[Z_{dq}^{\beta}\right]\right)$$

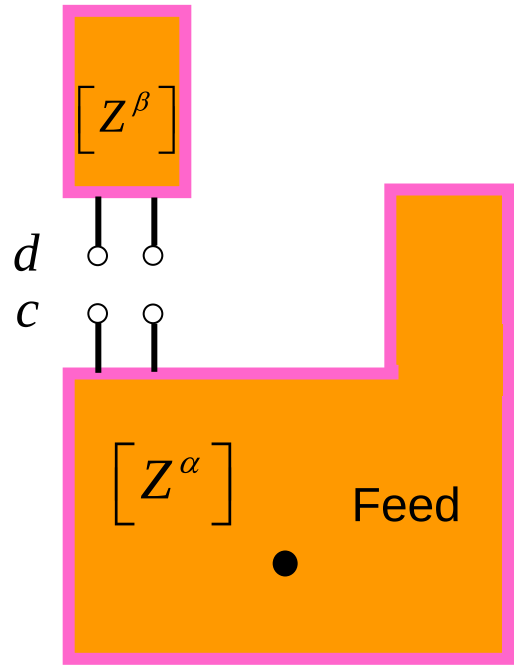
$$\left[Z_{qp}\right]=\left[Z_{qd}^{\beta}\right]\left(\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left[Z_{cp}^{\alpha}\right]\right)$$

$$\left[Z_{qq}\right]=\left[Z_{qq}^{\beta}\right]-\left[Z_{qd}^{\beta}\right]\left(\left[Z_{cc}^{\alpha}+Z_{dd}^{\beta}\right]^{-1}\left[Z_{dq}^{\beta}\right]\right)$$

Segmentation Method (cont.)

After the final patch is connected, we are left with a final port matrix Z^P.

For a single feed, we have $$\left[Z^P\right] = \left[Z_{11}^P\right]_{1\times 1}$$



Desegmentation Method

This is a variation of the segmentation method, in which we have a rectangular patch shape in which a set of smaller rectangular patches have been removed.