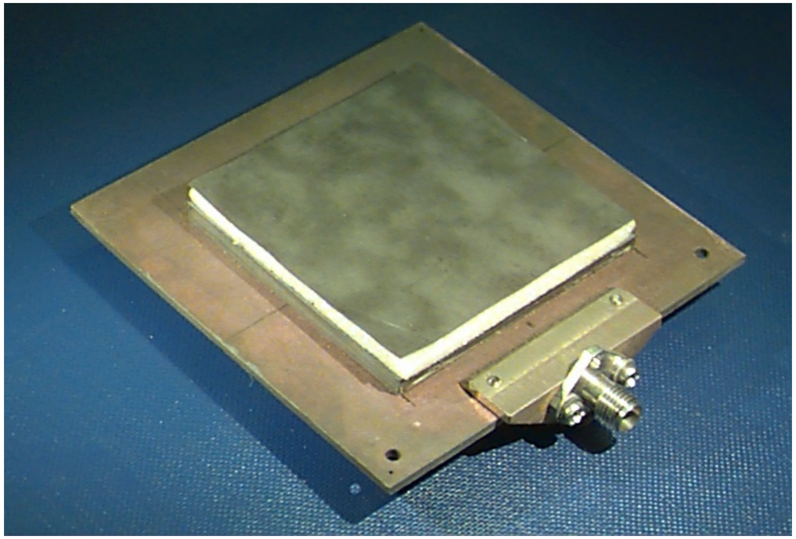
ECE 6345

Spring 2015

Prof. David R. Jackson
ECE Dept.



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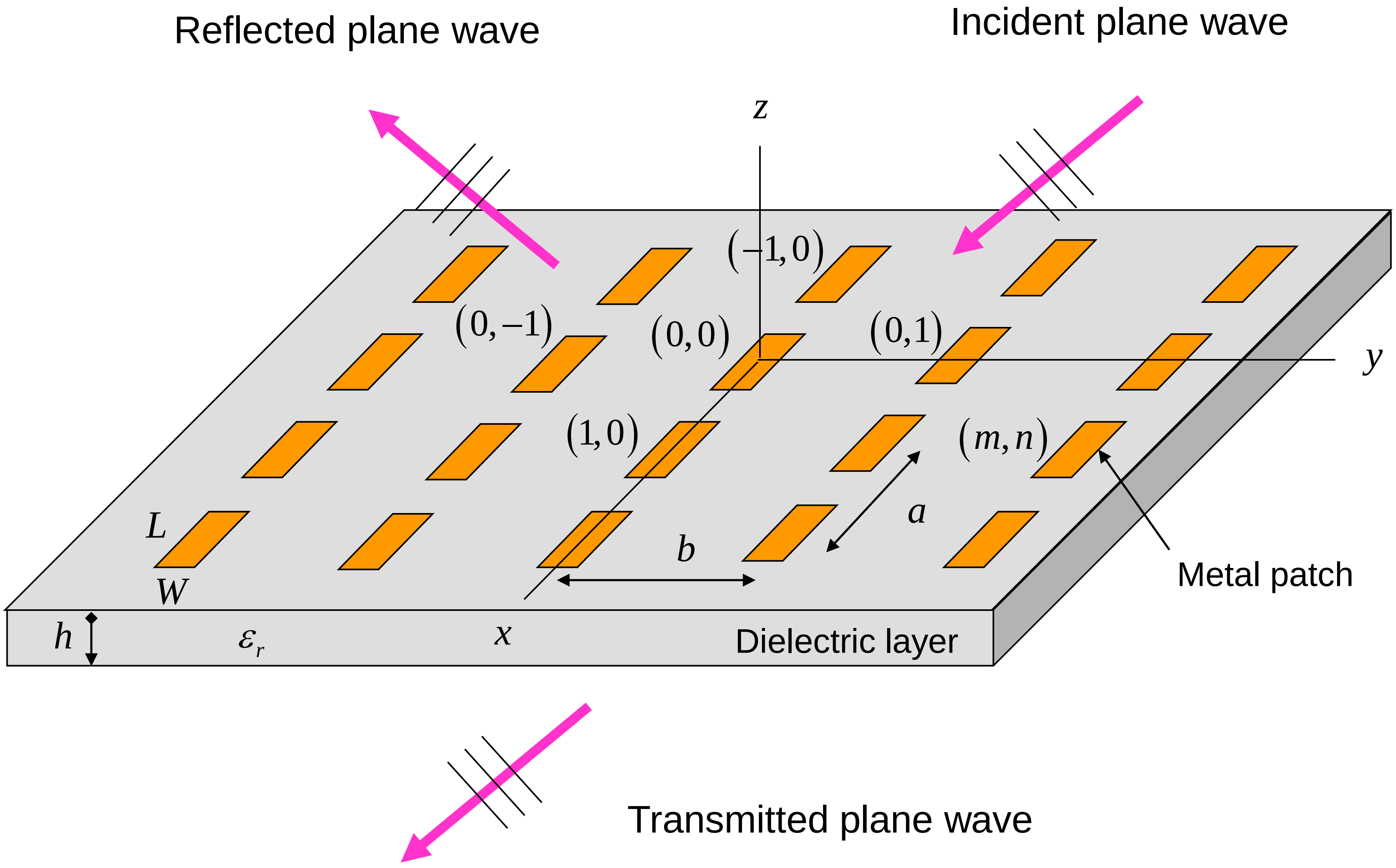
Overview

In this set of notes we examine the FSS problem in more detail, using the periodic spectral-domain Green's function.

FSS Geometry



FSS Geometry (cont.)

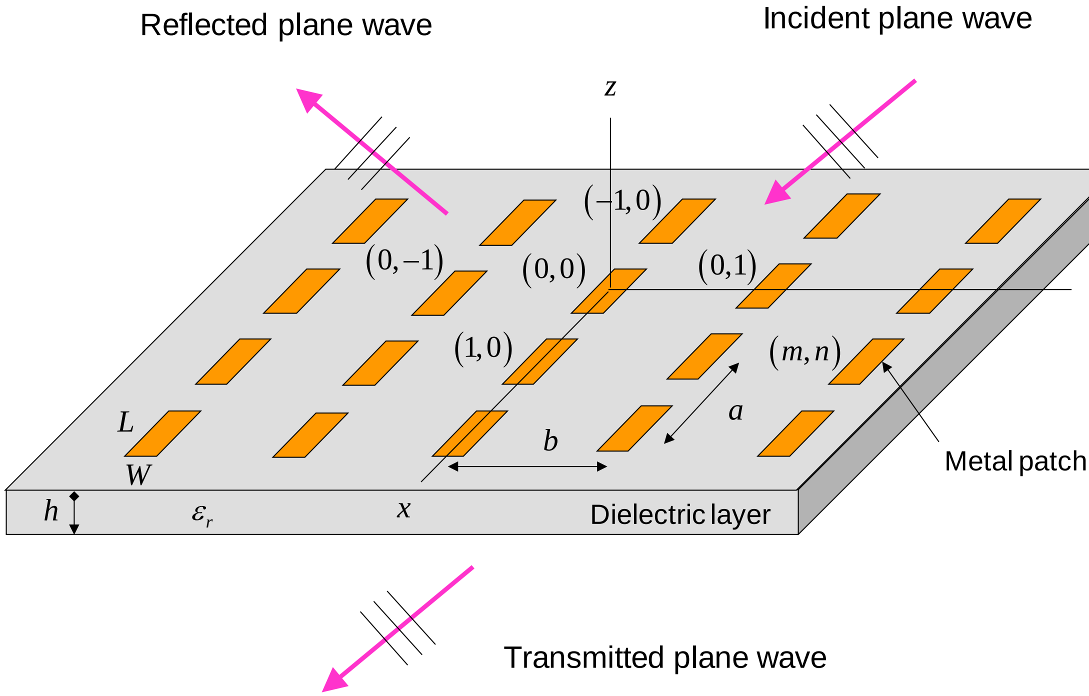
$$\psi^{inc} = A_{00}\, e^{-j\left(k_{x0}x + k_{y0}y\right)}\, e^{+jk_{z0}z}$$

$\left(\theta_0, \phi_0\right) =$ arrival angles

$$k_{x0} = -k_0 \sin\theta_0 \cos\phi_0$$

$$k_{y0} = -k_0 \sin\theta_0 \sin\phi_0$$

$$k_{z0} = k_0 \cos\theta_0$$



FSS Analysis

Assume that the unknown current on the $(0,0)$ patch is of the following form:

$$J_{sx}^{00}(x,y) = A_x^{00} B_x^{00}(x,y)$$

$$B_x^{00}(x,y) = \cos\left(\frac{\pi x}{L}\right), \quad |x| < L/2, \; |y| < W/2$$

The EFIE is then

$$A_x^{00} E_x^{\infty}\left[B_x^{00}\right] + E_x^{imp} = 0, \; |x| < L/2, \; |y| < W/2$$

Note that the "∞" superscript stands for "infinite periodic" (i.e., the fields due to the infinite periodic array of patch currents).

The superscript "imp" denotes the impressed field (seen by the patches) that exists in the absence of the metal patches. That is, the incident plane-wave field plus that which reflects from the dielectric layer.

The EFIE is enforced on the $(0,0)$ patch; it is then automatically enforced *on all patches*.

FSS Analysis (cont.)

We have, using Galerkin's method,

$$A_x^{00} \int_{S_0} B_x^{00}(x,y) E_x^{\infty}\left[B_x^{00}\right] dS + \int_{S_0} B_x^{00}\left(x,y\right) E_x^{imp}\, dS = 0$$

Define

$$Z_{xx}^{\infty} = -\int_{S_0} B_x^{00}(x,y) E_x^{\infty}\left[B_x^{00}\right] dS$$

$$R^{00} = \int_{S_0} B_x^{00}\left(x,y\right) E_x^{imp}\, dS$$

We then have

$$A_x^{00} Z_{xx}^{\infty} = R^{00}$$

FSS Analysis (cont.)

The $(0,0)$ patch current amplitude is then

$$A_x^{00} = \frac{R^{00}}{Z_{xx}^{\infty}}$$

For the patch self reaction we have

$$Z_{xx} = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}_{xx}\left(k_x, k_y\right)\left[\tilde{B}_x^{00}\left(k_x, k_y\right)\right]^2 dk_x dk_y$$

Single patch

$$Z_{xx}^{\infty} = -\frac{1}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \tilde{G}_{xx}\left(k_{xp}, k_{yq}\right)\left[\tilde{B}_x^{00}\left(k_{xp}, k_{yq}\right)\right]^2$$

Periodic patches

FSS Analysis (cont.)

For the RHS term we have

$$R^{00} = \int_{S_0} B_x^{00}(x,y)\, E_x^{imp}\, dS$$

The impressed field as a function of (x,y) can be written as

$$E_x^{imp}(x,y,0) = e^{-j\left(k_{x0}x + k_{y0}y\right)} E_x^{inc}(0,0,0)\left[1+\Gamma\right]$$

where $\Gamma = \Gamma^{TE}$ TE incidence

$\Gamma = \Gamma^{TM}$ TM incidence

This gives us

$$R^{00} = E_x^{inc}(0,0,0)\left[1+\Gamma\right] \int_{S_0} B_x^{00}(x,y)\, e^{-j\left(k_{x0}x + k_{y0}y\right)} dS$$

FSS Analysis (cont.)

Hence, we have

$$R^{00} = E_x^{inc}\left(0,0,0\right)\left[1+\Gamma\right]\tilde{B}_x^{00}\left(-k_{x0},-k_{y0}\right)$$

where

$$\tilde{B}_x^{00}\left(k_x,k_y\right) = \left(\frac{\pi}{2}LW\right)\operatorname{sinc}\left(k_y\frac{W}{2}\right)\left[\frac{\cos\left(k_x\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2-\left(k_x\frac{L}{2}\right)^2}\right]$$

$\Gamma = \Gamma^{TE}$ TE incidence

$\Gamma = \Gamma^{TM}$ TM incidence

FSS Analysis (cont.)

For the incident field we have

$$E_x^{inc}(0,0,0) = E_\theta^{inc}(0,0,0)\cos\theta\cos\phi \qquad \text{TM incidence}$$

$$E_x^{inc}(0,0,0) = E_\phi^{inc}(0,0,0)(-\sin\phi) \qquad \text{TE incidence}$$

In a typical scattering problem we would usually choose

$$E_\theta^{inc}(0,0,0) = 1 \qquad \text{TM incidence}$$

$$E_\phi^{inc}(0,0,0) = 1 \qquad \text{TE incidence}$$

FSS Analysis (cont.)

We now calculate the FSS reflection coefficient.

The field *radiated by the patch currents* for $z > 0$ is

$$E_x(x,y,z) = A_x \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}_{xx}(k_x,k_y;0,0)\, \tilde{B}_x(k_x,k_y)\, e^{-j(k_x x + k_y y)} e^{-jk_{z0} z}$$

Single patch

$\Downarrow$

$$E_x^{\infty}(x,y,z) = A_x^{00} \frac{1}{ab} \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \tilde{G}_{xx}(k_{xp},k_{yq};0,0)\, \tilde{B}_x^{00}(k_{xp},k_{yq})\, e^{-j(k_{xp} x + k_{yq} y)} e^{-jk_{z0}^{pq} z}$$

Periodic patches

where $$k_{z0}^{pq} = \left(k_0^2 - k_{xp}^2 - k_{yq}^2\right)^{1/2}$$

FSS Analysis (cont.)

The field of the specular-reflected $(0,0)$ wave *radiated by the patches* for $z > 0$ is

$$E_x^{ref,patch}(x,y,z) = A_x^{00}\frac{1}{ab}\tilde{G}_{xx}\left(k_{x0},k_{y0};0,0\right)\tilde{B}_x^{00}\left(k_{x0},k_{y0}\right)e^{-j\left(k_{x0}x+k_{y0}y\right)}e^{-jk_{z0}^{00}z}$$

The *total* specular reflected field is

$$E_x^{ref,tot}(x,y,z) = E_x^{inc}(0,0,0)\,\Gamma e^{-j\left(k_{x0}x+k_{y0}y\right)}e^{-jk_{z0}^{00}z} + A_x^{00}\frac{1}{ab}\tilde{G}_{xx}\left(k_{x0},k_{y0};0,0\right)\tilde{B}_x^{00}\left(k_{x0},k_{y0}\right)e^{-j\left(k_{x0}x+k_{y0}y\right)}e^{-jk_{z0}^{00}z}$$

where

$\Gamma =$ reflection coefficient from the layer (without the patches)

FSS Analysis (cont.)

The total FSS reflection coefficient is defined from the $(0,0)$ fields as

$$\Gamma^{FSS} \equiv \frac{E_x^{ref,tot}(0,0,0)}{E_x^{inc}(0,0,0)}$$

Note: The total reflected wave will not in general be perfectly TM_z or TE_z (unless we are in the principal planes).

The total FSS reflection coefficient is then

$$\Gamma^{FSS} = \Gamma + \frac{1}{E_x^{inc}(0,0,0)}\left[A_x^{00}\,\frac{1}{ab}\,\tilde{G}_{xx}\left(k_{x0},k_{y0};0,0\right)\,\tilde{B}_x^{00}\left(k_{x0},k_{y0}\right)\right]$$

$\Gamma = \Gamma^{TE}$ TE incidence

$\Gamma = \Gamma^{TM}$ TM incidence

$E_x^{inc}(0,0,0) = E_\theta^{inc}(0,0,0)\cos\theta\cos\phi$ TM incidence

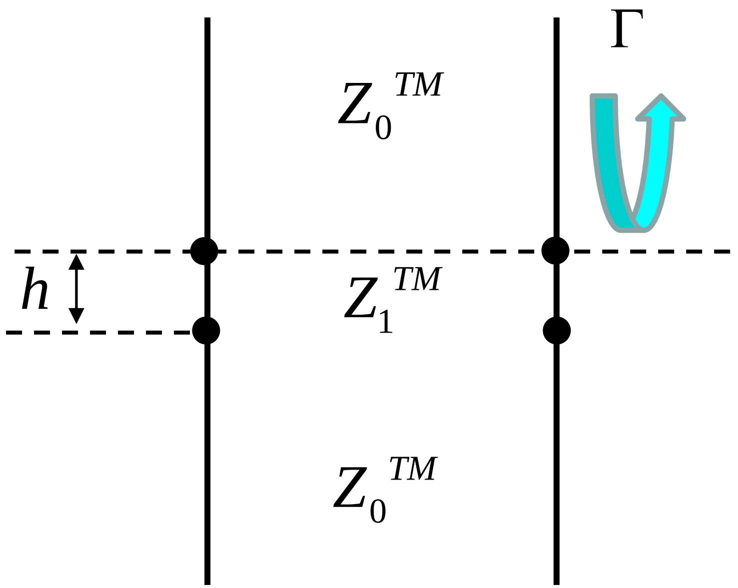
$E_x^{inc}(0,0,0) = E_\phi^{inc}(0,0,0)(-\sin\phi)$ TE incidence

FSS Analysis (cont.)

For *the layer reflection coefficient* we have

$\Gamma = \Gamma^{TE}$ TE incidence

$\Gamma = \Gamma^{TM}$ TM incidence



$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} - Z_0}$$

$$Z_{in} = Z_1 \left[\frac{Z_L + jZ_1 \tan\left(k_{z1}^{00} h\right)}{Z_1 + jZ_L \tan\left(k_{z1}^{00} h\right)} \right]$$

$$Z_L = Z_0$$

$$Z_0^{TM} = \frac{k_{z0}^{00}}{\omega \varepsilon_0} \qquad Z_0^{TE} = \frac{\omega \mu_0}{k_{z0}^{00}} \qquad Z_1^{TM} = \frac{k_{z1}^{00}}{\omega \varepsilon_0 \varepsilon_r} \qquad Z_1^{TE} = \frac{\omega \mu_0}{k_{z1}^{00}}$$

$$k_{z0}^{00} = \sqrt{k_0^2 - k_{x0}^2 - k_{y0}^2} \qquad k_{z1}^{00} = \sqrt{k_1^2 - k_{x0}^2 - k_{y0}^2} \qquad k_1 = k_0 \sqrt{\varepsilon_r}$$

FSS Analysis (cont.)

We also have

$$A_x^{00} = \frac{R^{00}}{Z_{xx}^{\infty}}$$

$$R^{00} = E_x^{inc}(0,0,0)\left[1+\Gamma\right]\tilde{B}_x^{00}\left(-k_{x0},-k_{y0}\right)$$

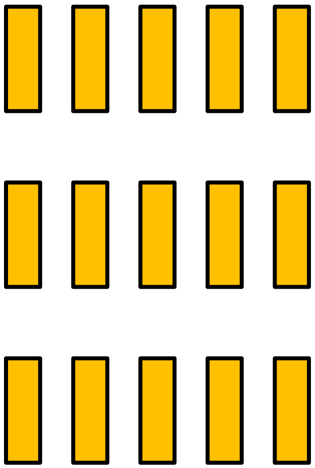
$$Z_{xx}^{\infty} = -\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}\left(k_{xp},k_{yq}\right)\left[\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)\right]^2$$

$$E_x^{inc}(0,0,0) = E_\theta^{inc}(0,0,0)\cos\theta\cos\phi$$ TM incidence

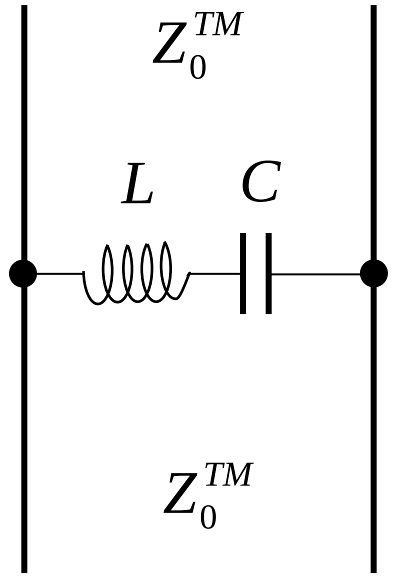
$$E_x^{inc}(0,0,0) = E_\phi^{inc}(0,0,0)\left(-\sin\phi\right)$$ TE incidence

FSS Equivalent Circuit

Approximate equivalent circuit of patch FSS



A periodic array of metal patches forms a "capacitive FSS".

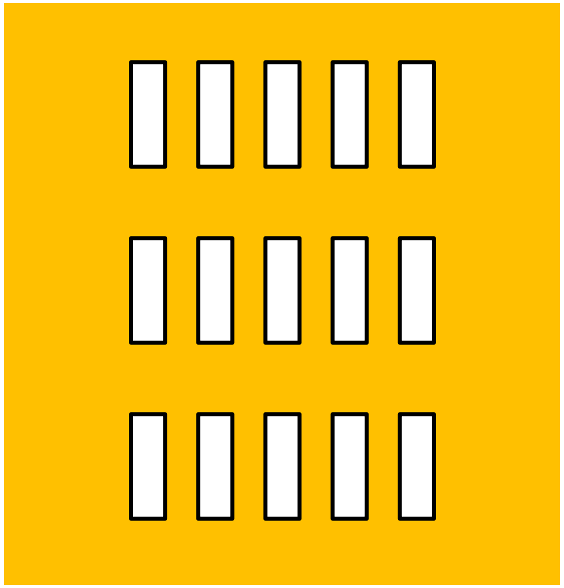


The periodic patch array is modeled as a shunt susceptance in the TEN, consisting of an L and a C.

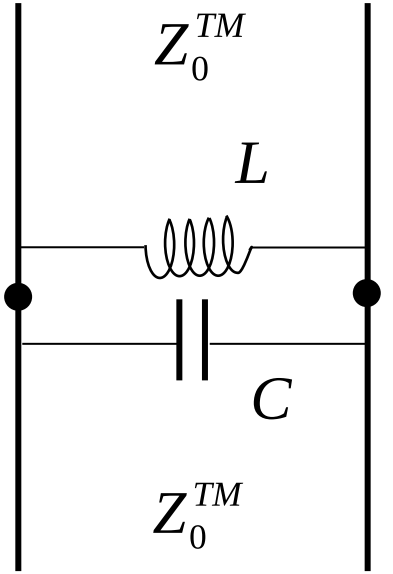
Note: There could also be a layer present.

FSS Equivalent Circuit

Approximate equivalent circuit of slot FSS



A periodic array of slots forms an "inductive FSS".



The periodic slot array is modeled as a shunt susceptance in the TEN, consisting of an L and a C.

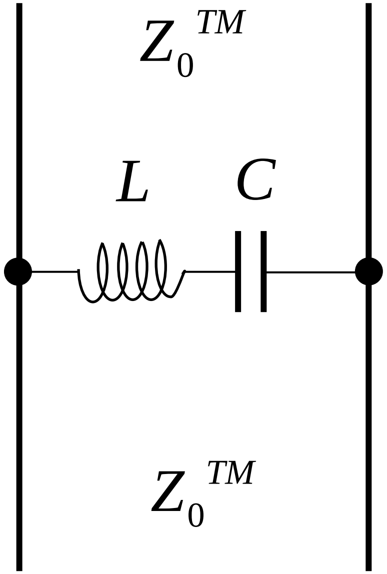
Note: There could also be a layer present.

FSS Equivalent Circuit

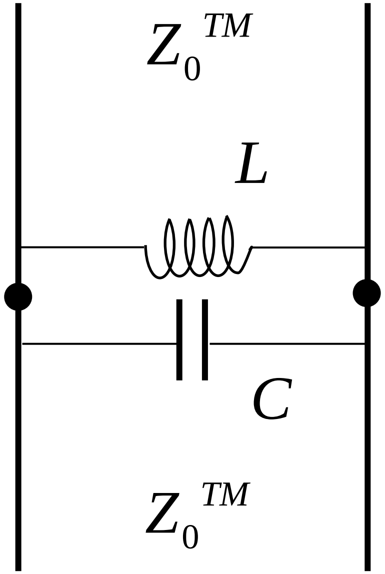
At circuit resonance:

- A patch (capacitive) FSS reflects all of the wave.
- A slot (inductive) FSS transmits all of the wave.

Patch FSS



Slot FSS



Results: FSS Patch Array in Free Space

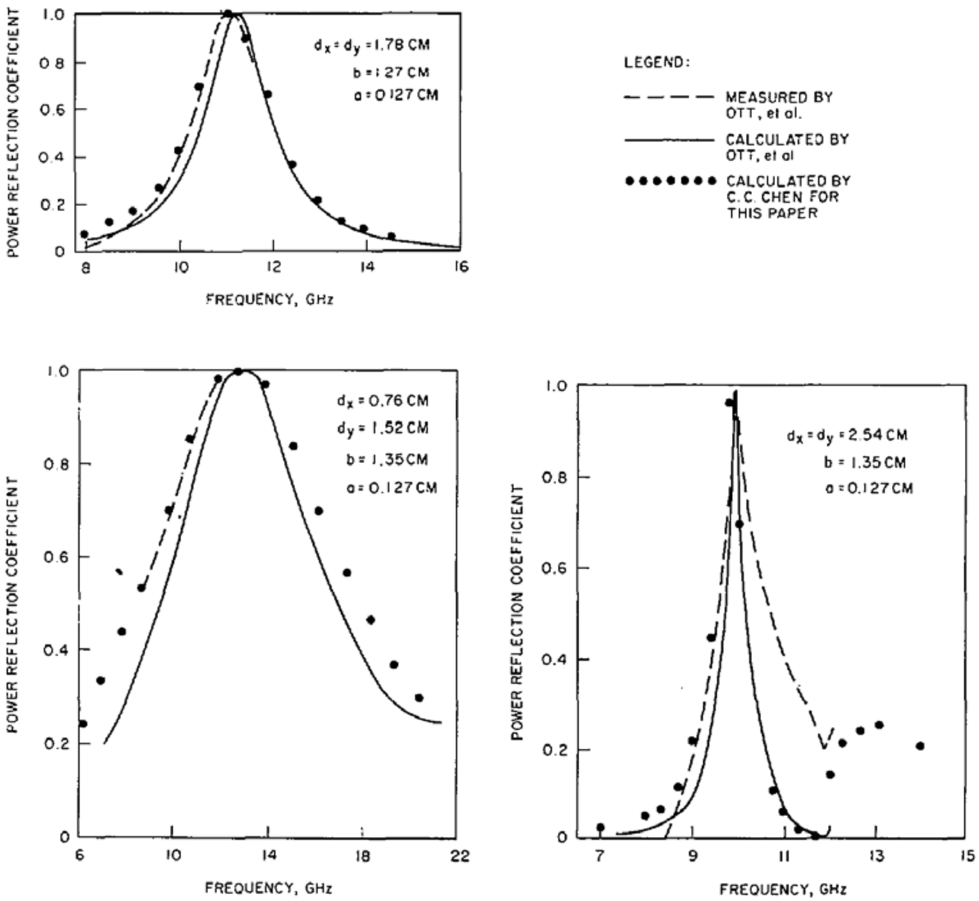


Fig. 2. Calculated and measured power reflection coefficients (after [1]) of narrow plates with rectangular lattice arrangement.

C. C. Chen, "Scattering by a two-dimensional array of conducting patches," IEEE Trans. Antennas and Propagation, pp. 660-665, Sept. 1970.

Note the total reflection at the resonance frequency!