ECE 6345

Spring 2015

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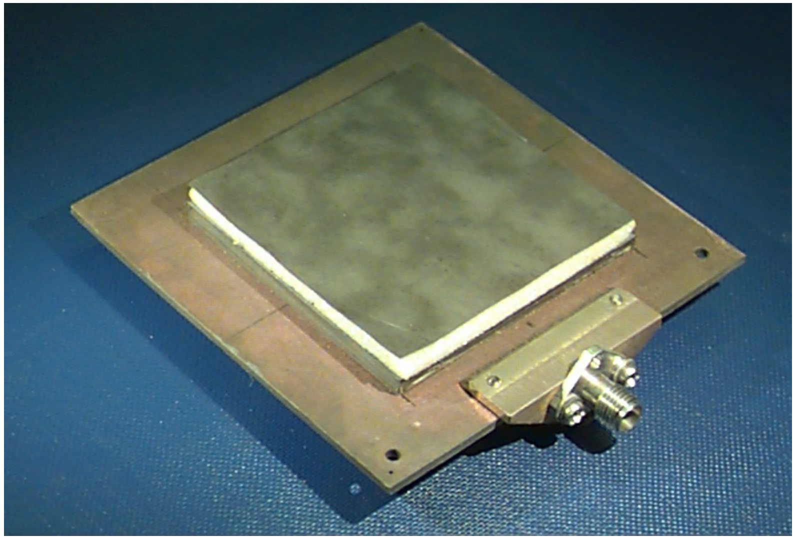

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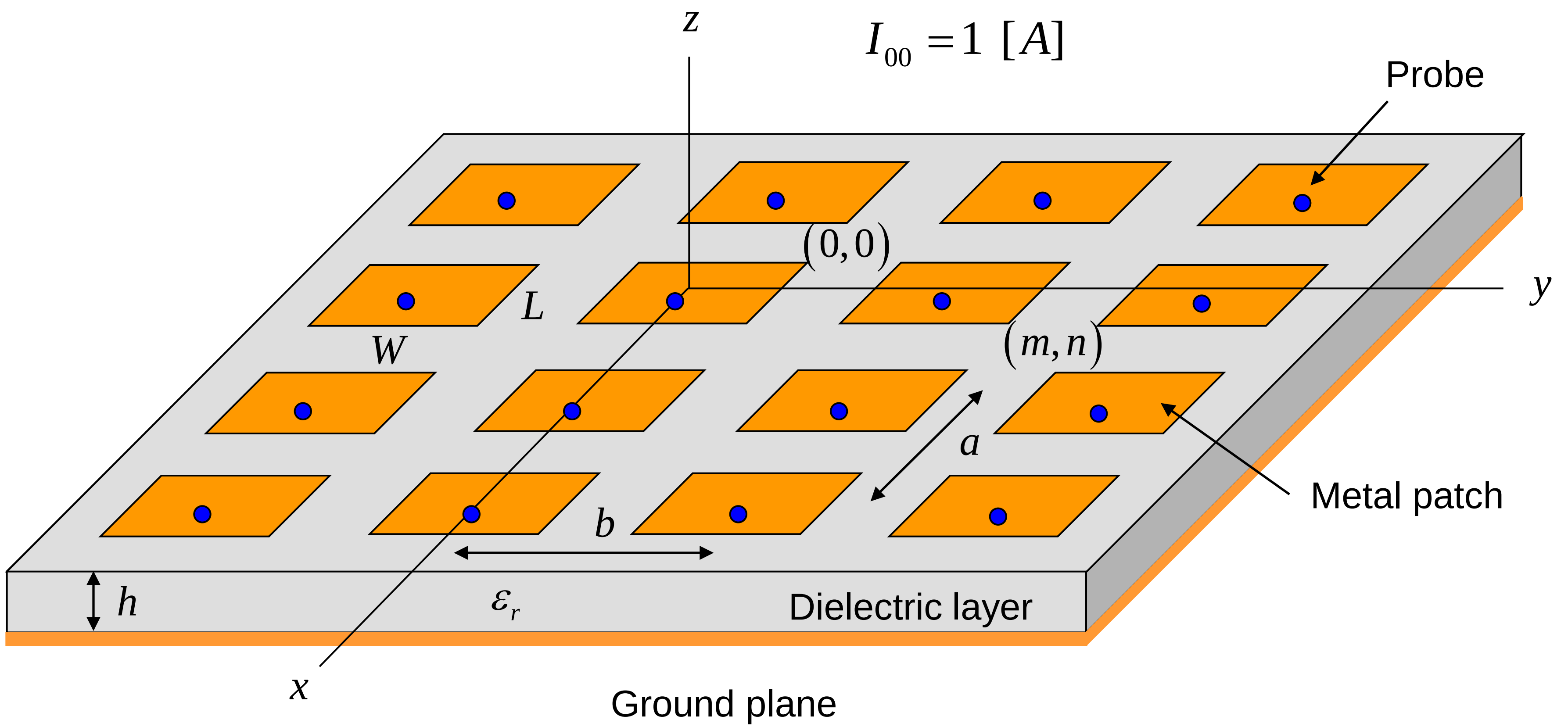
Phased Array Geometry

$$I_{mn} = I_{00}\, e^{-j\left(k_{x0} ma + k_{y0} nb\right)} \quad \text{(probe currents)}$$



Phased Array Analysis

Assume that the unknown current on the (0,0) patch is of the following form:

$$J_{sx}^{00}\left(x,y\right)=A_x^{00}B_x^{00}\left(x,y\right)$$

$$B_x^{00}\left(x,y\right)=\cos\left(\frac{\pi x}{L}\right),\quad |x|<L/2,\ |y|<W/2$$

The EFIE (after Galerkin testing) is

$$A_x^{00}\int_{S_0}B_x^{00}(x,y)\,E_x^{\infty}\left[B_x^{00}\right]dS+\int_{S_0}B_x^{00}(x,y)\,E_x^{\infty}\left[J_z^{i}\right]dS=0$$

Hence, we can write

$$A_x^{00}Z_{xx}^{\infty}+Z_{xz}^{\infty}=0$$

The infinity superscript indicates the field from a periodic array of sources (patches or probes).

so

$$A_x^{00}=-\frac{Z_{xz}^{\infty}}{Z_{xx}^{\infty}}$$

Phased Array Analysis (cont.)

The "active" or "scan" input impedance is (following exactly the same procedure as done previously for the single patch):

$$Z_{in}^{A} = Z_{probe} - A_x^{00} \left\langle B_x^{\infty}, J_z^{i} \right\rangle$$
$$= Z_{probe} + A_x^{00} Z_{zx}^{\infty}$$

This is the input impedance for a given patch when all patches are fed.

Hence we can write

$$Z_{in} = Z_{probe} - \frac{Z_{zx}^{\infty} Z_{xz}^{\infty}}{Z_{xx}^{\infty}}$$

Z_{zx}^{∞} = field of infinite periodic patches, integrated over single probe.

Z_{xz}^{∞} = field of infinite periodic probes, integrated over single patch.

Phased Array Analysis (cont.)

From reciprocity, it follows that

$$Z_{zx}^{\infty} = Z_{xz}^{\infty}$$

(though this is not obvious!)

LHS = field of periodic patches integrated over a single vertical probe.

RHS = field of periodic probes integrated over a single patch.

The proof is given on the next slide.

Phased Array Analysis (cont.)

To see this, consider the following:

$$\left\langle B_x, J_z^i \right\rangle = \left\langle J_z^i, B_x \right\rangle$$ (single patch element)

Expressing the two reactions in the general form of spectral integrals, we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_1\left(k_x, k_y\right) e^{-j\left(k_x \Delta_x + k_y \Delta_y\right)}\, dk_x dk_y = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2\left(k_x, k_y\right) e^{-j\left(k_x \Delta_x + k_y \Delta_y\right)}\, dk_x dk_y$$

where $\left(\Delta_x, \Delta_y\right) =$ arbitrary offset between elements

Since this holds for an arbitrary offset, we have

$$F_1\left(k_x, k_y\right) = F_2\left(k_x, k_y\right)$$

Phased Array Analysis (cont.)

Hence

$$\frac{(2\pi)^2}{A}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}F_1\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}\Delta_x+k_{yq}\Delta_y\right)}$$

$$=\frac{(2\pi)^2}{A}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}F_2\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}\Delta_x+k_{yq}\Delta_y\right)}$$

so $\left\langle B_x^{\infty}, J_z^{i}\right\rangle = \left\langle J_z^{i\infty}, B_x\right\rangle$

or $Z_{zx}^{\infty} = Z_{xz}^{\infty}$

Phased Array Analysis (cont.)

The “active” or “scan” input impedance can thus be written as

$$Z_{in}^{A} = Z_{probe} - \frac{\left(Z_{zx}^{\infty}\right)^2}{Z_{xx}^{\infty}}$$

where

$$Z_{xx}^{\infty} = -\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}\left(k_{xp}, k_{yq}\right)\left[\tilde{B}_{x}^{00}\left(k_{xp}, k_{yq}\right)\right]^2$$

$$Z_{zx}^{\infty} = -\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{j\omega\varepsilon_1}\left(jk_{tpq}\right)\left(I_i^{TM}(-h)\right)\tilde{B}_x\left(k_{xp}, k_{yq}\right)\left(\frac{k_{xp}}{k_{tpq}}\right) h\,\mathrm{sinc}\left(k_{z1pq}h\right)e^{-j\left(k_{xp}x_0 + k_{yq}y_0\right)}$$

Phased Array Analysis (cont.)

At a scan blindness point:

$$Z_{zx}^{\infty} \to \infty$$

$$Z_{xx}^{\infty} \to \infty$$

$$Z_{in} = Z_{probe} - \frac{\left(Z_{zx}^{\infty}\right)^2}{Z_{xx}^{\infty}}$$

so $$Z_{in} \to \infty$$

Also,

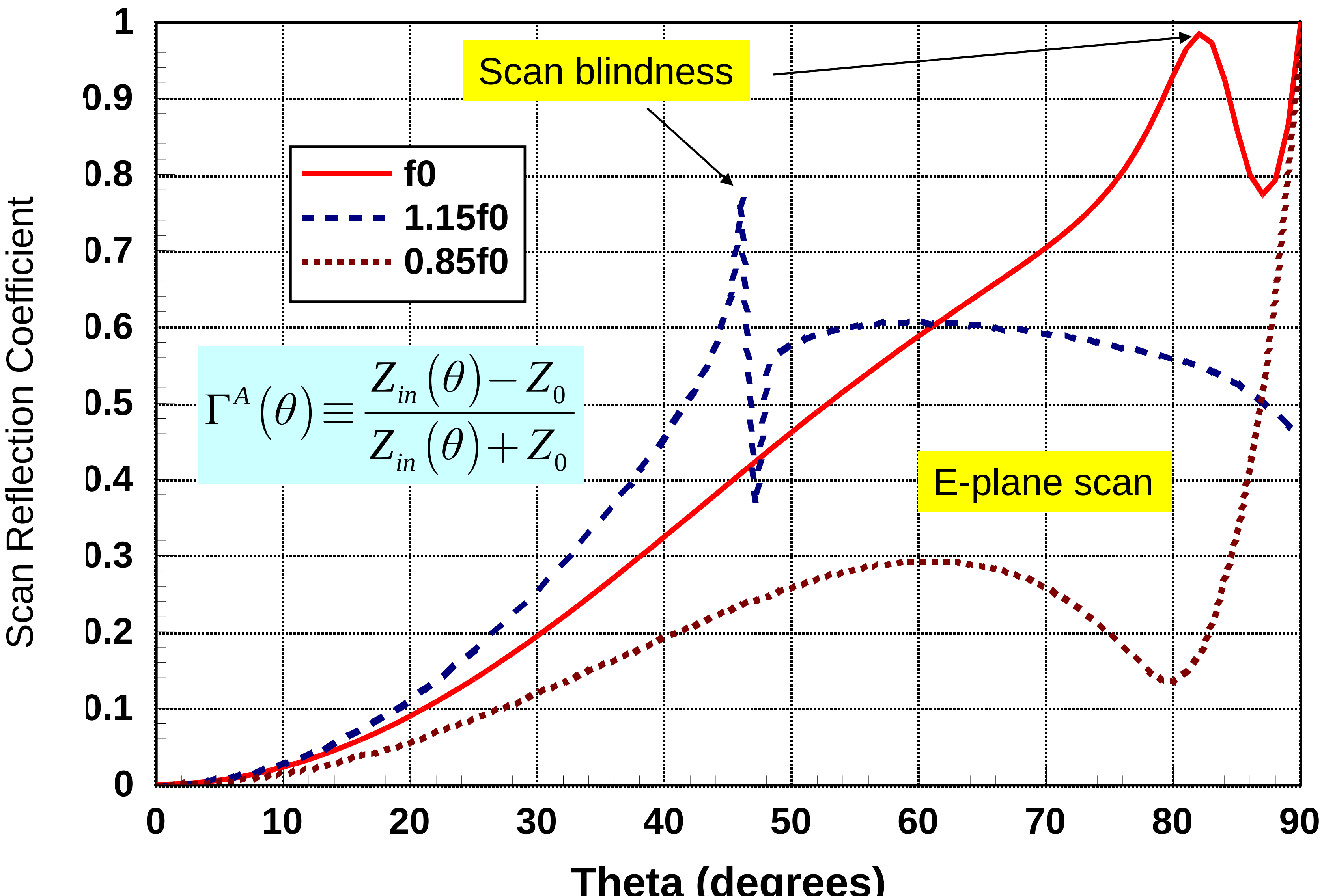
$$A_x^{00} \to 0$$ (no current on the patches)

since $$A_x^{00} = -\frac{Z_{xz}^{\infty}}{Z_{xx}^{\infty}}$$

(The denominator has a stronger singularity than the numerator, since the patches radiate much more of a surface-wave field than do the probes.)

Phased Array Analysis (cont.)

From Waterhouse's short-course slides



$$\Gamma^A(\theta) \equiv \frac{Z_{in}(\theta) - Z_0}{Z_{in}(\theta) + Z_0}$$