ECE 6345

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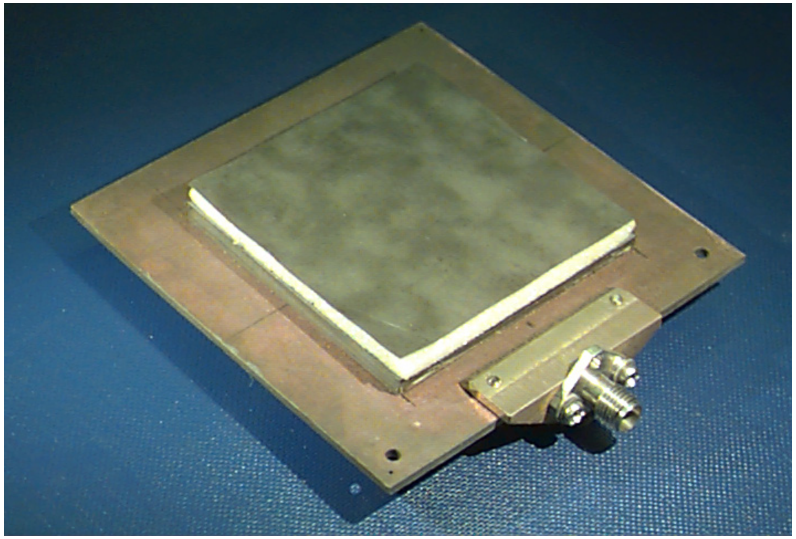
ECE Dept.



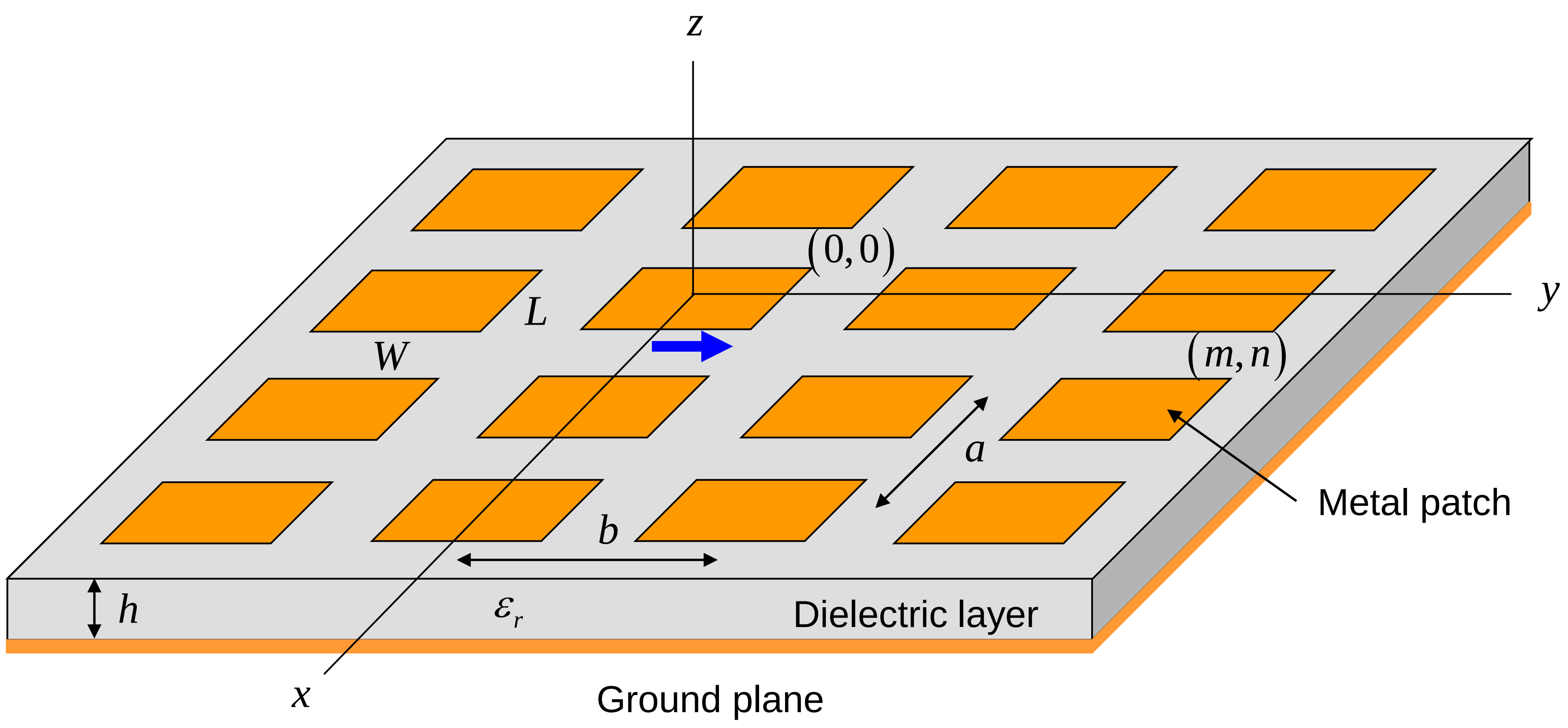
Notes 35

Overview

In this set of notes we examine the Array Scanning Method (ASM) for calculating the field of a single source near an infinite periodic structure.

ASM Geometry

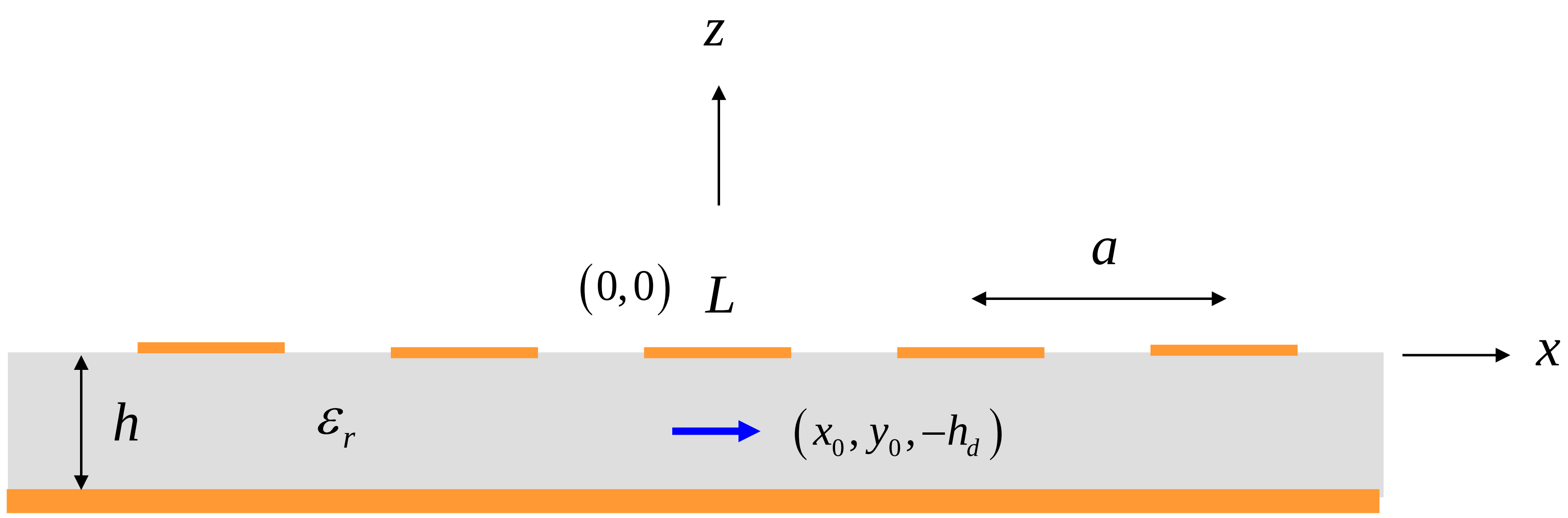
Consider an infinite 2D periodic array of metal patches excited by a *single* (nonperiodic) dipole source.



ASM Geometry

The dipole source is shown below one of the patches, but it can be anywhere.

Note that the field produced by the dipole is NOT periodic.

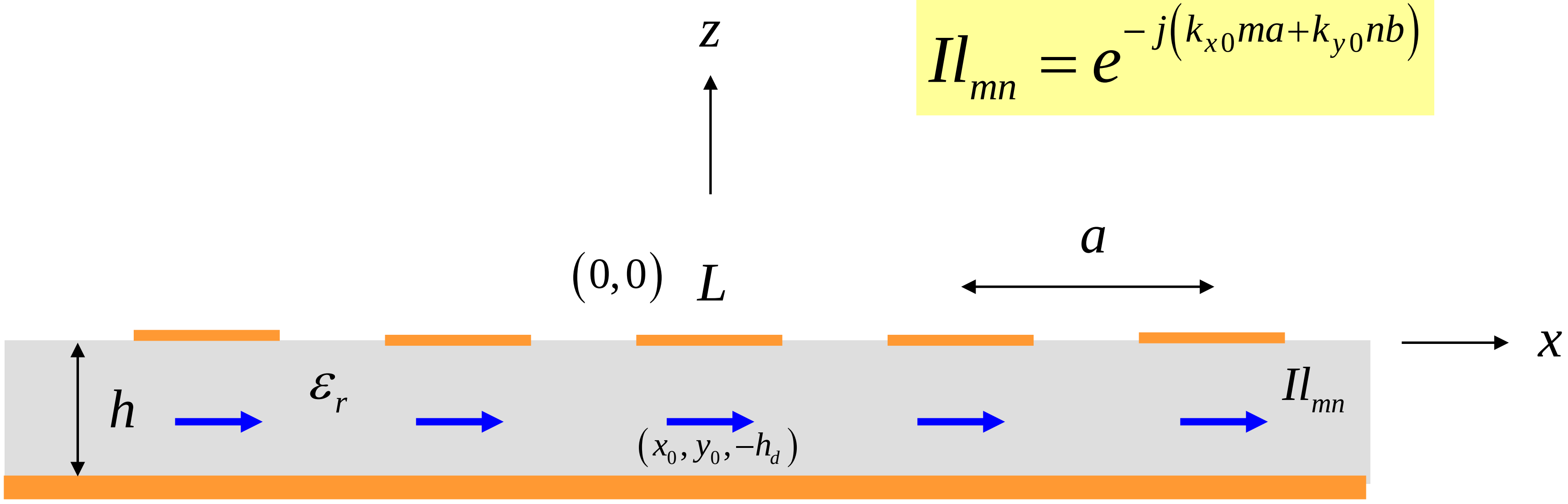


Side view

ASM Analysis

We first consider an infinite 2D periodic array of metal patches excited by an infinite periodic array of dipole sources.

$$Il_{mn} = e^{-j\left(k_{x0}ma + k_{y0}nb\right)}$$



This is an infinite periodic "phased array" problem.

We think of (k_{x0}, k_{y0}) as *variables*.

ASM Analysis (cont.)

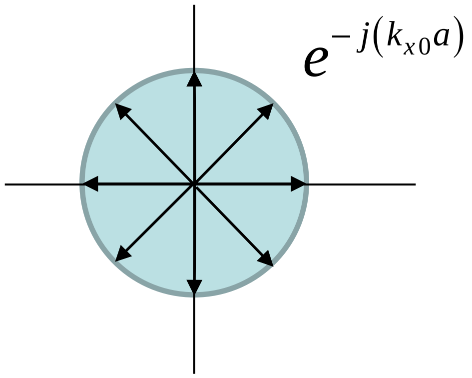
Note that

$$\int_{-\pi/a}^{\pi/a} e^{-j(k_{x0} m a)}\, dk_{x0} = \left. \frac{e^{-j(k_{x0} m a)}}{-jma} \right|_{-\pi/a}^{\pi/a} = \frac{e^{-j(m\pi)} - e^{+j(m\pi)}}{-jma} = 0$$

$$m \neq 0$$

Picture for $m = 1$

$$-\pi < k_{x0} a < \pi$$



Complex plane

Hence we can say that

$$\int_{-\pi/a}^{\pi/a} e^{-j(k_{x0} m a)}\, dk_{x0} = \begin{cases} 0, & m \neq 0 \\ \dfrac{2\pi}{a}, & m = 0 \end{cases}$$

ASM Analysis (cont.)

Denote

$$E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right) = \text{field produced by infinite periodic array problem with phasing } \left(k_{x0},k_{y0}\right)$$

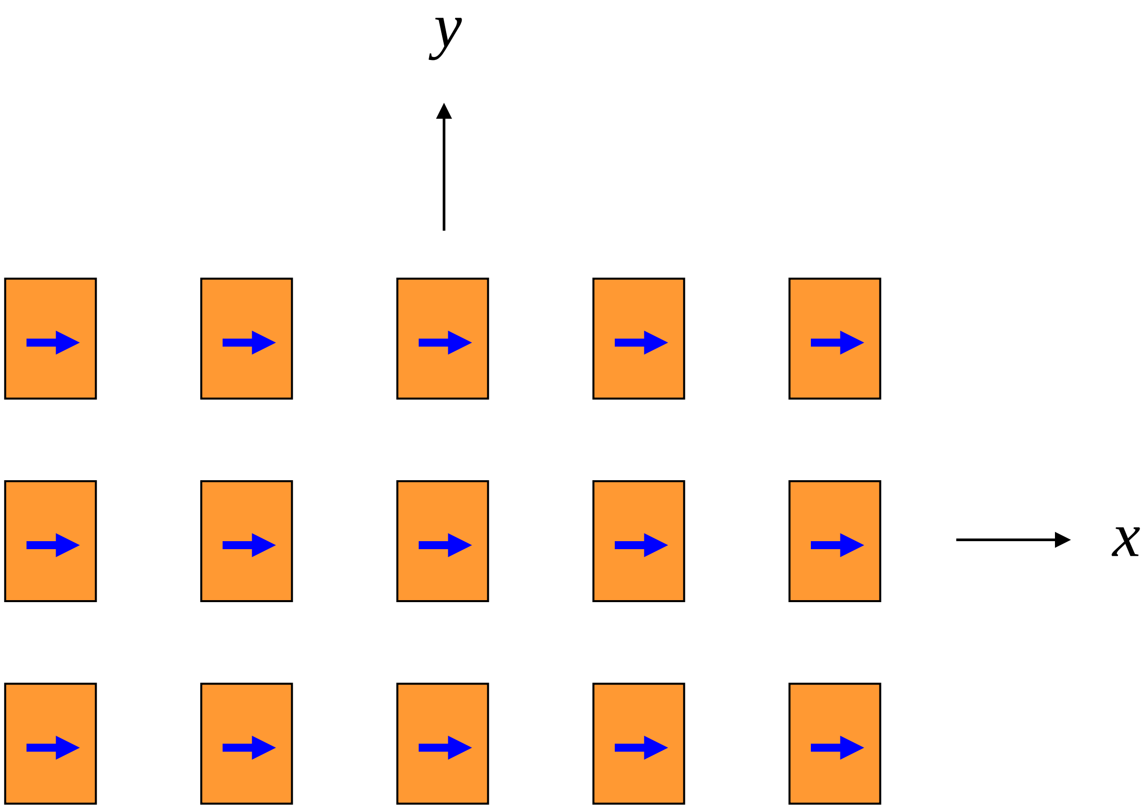
Then

$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)dk_{x0} = \text{field produced by a single column of dipole sources}$$

ASM Analysis (cont.)

Denote

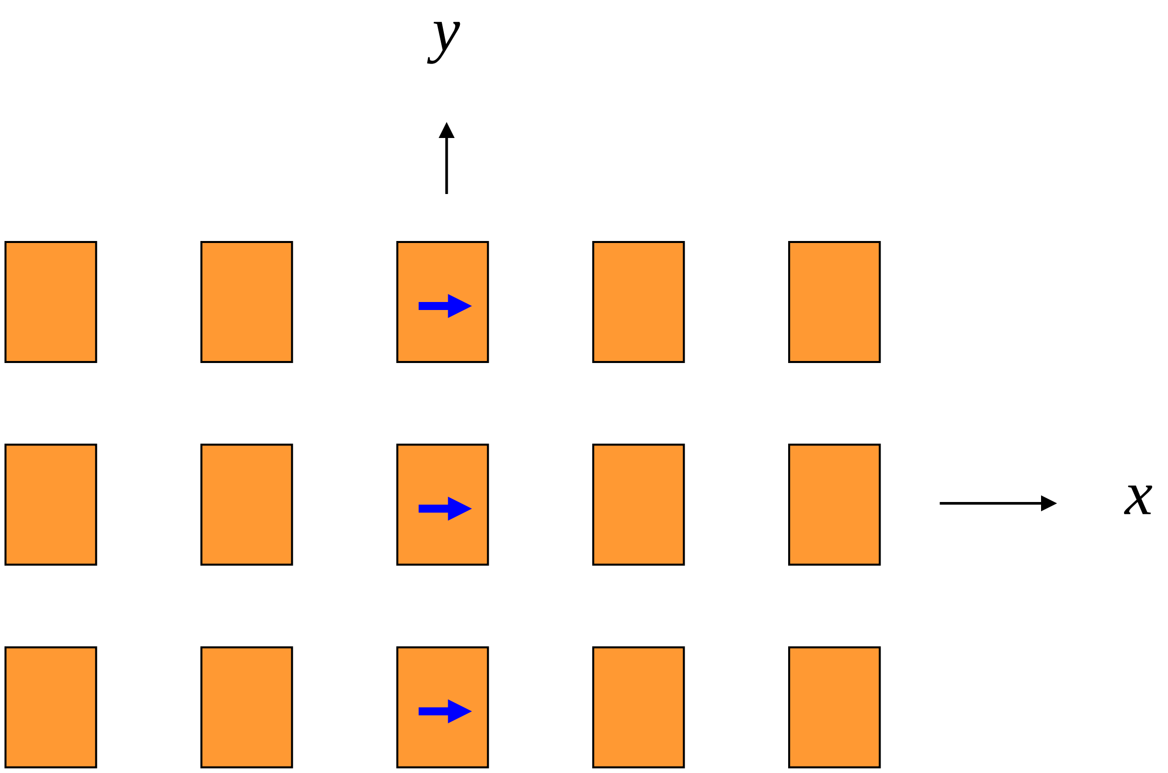
$$E_x^{\infty}\left(x, y, z; k_{x0}, k_{y0}\right) = \text{field produced by infinite periodic array problem with phasing} \left(k_{x0}, k_{y0}\right)$$



ASM Analysis (cont.)

$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)dk_{x0} = \text{field produced by}$$

a single column of dipole sources



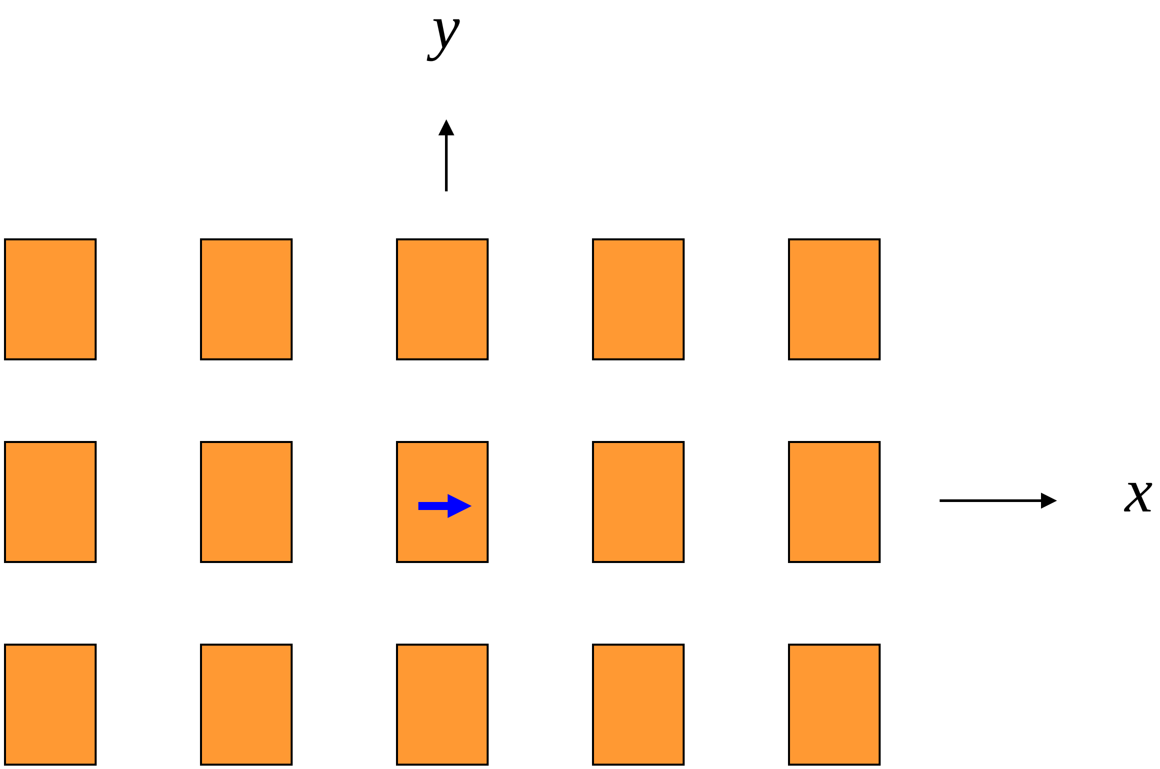
ASM Analysis (cont.)

Next, we apply the same procedure to the phasing in the y direction:

$$\int_{-\pi/b}^{\pi/b} e^{-j\left(k_{y0} nb\right)}\, dk_{y0} = \begin{cases} 0, & n \neq 0 \\ \dfrac{2\pi}{b}, & n = 0 \end{cases}$$

ASM Analysis (cont.)

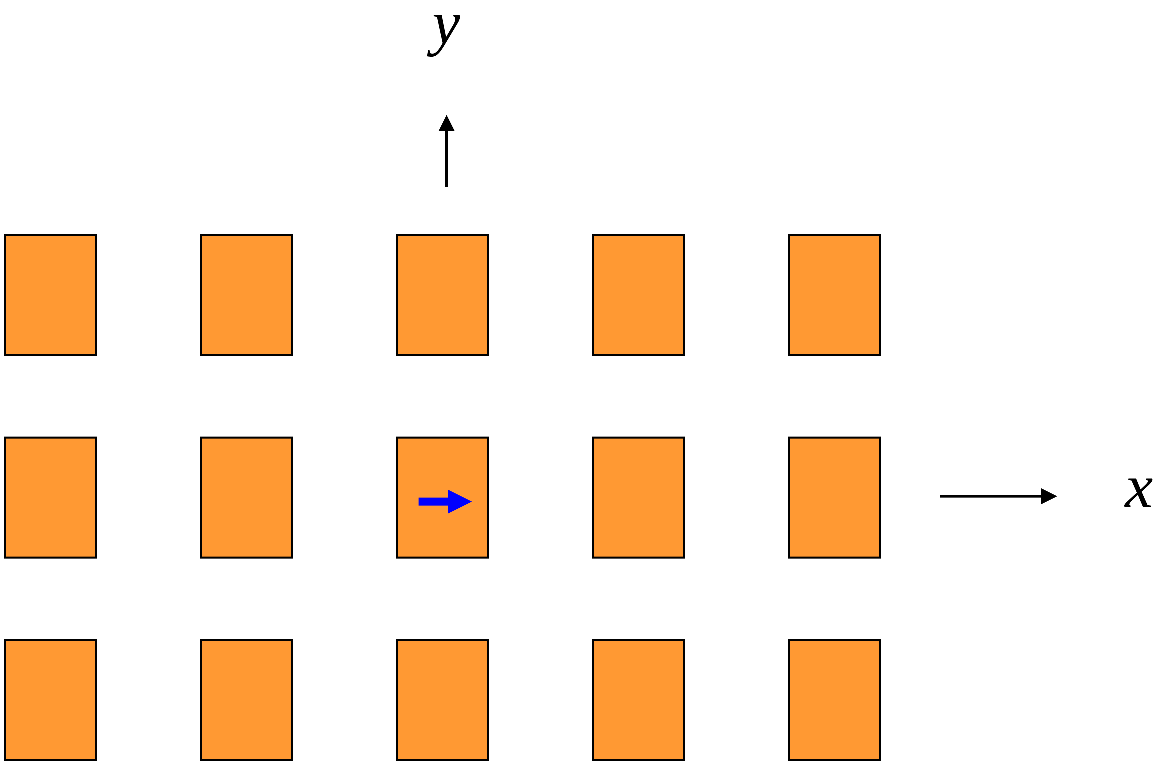
$$\frac{ab}{(2\pi)^2}\int_{-\pi/b}^{\pi/b}\int_{-\pi/a}^{\pi/a} E_x^{\infty}\left(x,y,z;k_{x0},k_{y0}\right)dk_{x0}\,dk_{y0} = \text{field from single dipole}$$



ASM Analysis (cont.)

Conclusion:

$$E_x(x,y,z) = \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} E_x^{\infty}(x,y,z;k_{x0},k_{y0})\, dk_{x0}\, dk_{y0}$$



ASM Analysis (cont.)

Assume that unknown current on the $(0,0)$ patch in the periodic 2D phased array problem is of the following form:

$$J_{sx}^{00}(x,y) = A_x^{00} B_x^{00}(x,y)$$

$$B_x^{00}(x,y) = \cos\left(\frac{\pi x}{L}\right), \quad |x| < L/2, \; |y| < W/2$$

The EFIE is then

$$A_x^{00} E_x^{\infty}\left[B_x^{00}\right] + E_x^{\infty}\left[J_{sx}^{dip\,00}\right] = 0, \; |x| < L/2, \; |y| < W/2$$

Note that the "∞" superscript stands for "infinite periodic" (i.e., the fields due to the infinite periodic array of patch currents).

The EFIE is enforced on the $(0,0)$ patch; it is then automatically enforced on all patches.

FSS Analysis (cont.)

We have, using Galerkin's method,

$$A_x^{00}\int_{S_0} B_x^{00}(x,y)E_x^{\infty}\left[B_x^{00}\right]dS+\int_{S_0} B_x^{00}\left(x,y\right)E_x^{\infty}\left[J_{sx}^{dip\,00}\right]dS=0$$

Define:

$$Z_{xx}^{\infty}=-\int_{S_0} B_x^{00}(x,y)E_x^{\infty}\left[B_x^{00}\right]dS$$

$$R^{00}=\int_{S_0} B_x^{00}\left(x,y\right)E_x^{\infty}\left[J_{sx}^{dip\,00}\right]dS$$

We then have

$$A_x^{00}Z_{xx}^{\infty}=R^{00}$$

FSS Analysis (cont.)

The (0,0) patch current amplitude is then

$$A_x^{00}\left(k_{x0},k_{y0}\right)=\frac{R^{00}\left(k_{x0},k_{y0}\right)}{Z_{xx}^{\infty}\left(k_{x0},k_{y0}\right)}$$

We have, for the denominator:

$$Z_{xx}=-\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{G}_{xx}\left(k_x,k_y\right)\left[\tilde{B}_x^{00}\left(k_x,k_y\right)\right]^2 dk_x dk_y$$

⇓

$$Z_{xx}^{\infty}=-\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}\left(k_{xp},k_{yq}\right)\left[\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)\right]^2$$

FSS Analysis (cont.)

For the numerator term we have

$$R^{00} = \int_{S_0} B_x^{00}(x,y)\, E_x^{\infty}\left[J_{sx}^{dip\,00}\right] dS$$

$$= \int_{S_0} J_{sx}^{dip\,00}(x,y)\, E_x^{\infty}\left[B_x^{00}\right] dS$$

(from "periodic" reciprocity, as discussed for the phased array)

$$= E_x^{\infty}\left[B_x^{00}\right]\left(x_0, y_0, -h_d\right)$$

The general form of the field from the periodic array of patch basis functions is

$$E_x^{\infty}(x,y,z)\left[B_x^{\infty}\right] = \frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}\left(k_{xp},k_{yq};z,0\right)\,\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}x+k_{yq}y\right)}$$

FSS Analysis (cont.)

Hence, we have

$$R^{00}\left(k_{x0},k_{y0}\right)=\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}\left(k_{xp},k_{yq};-h_d,0\right)\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)e^{-j\left(k_{xp}x_0+k_{yq}y_0\right)}$$

where

$$\tilde{B}_x^{00}\left(k_{xp},k_{yq}\right)=\left(\frac{\pi}{2}LW\right)\operatorname{sinc}\left(k_{yq}\frac{W}{2}\right)\left[\frac{\cos\left(k_{xp}\frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2-\left(k_{xp}\frac{L}{2}\right)^2}\right]$$

FSS Analysis (cont.)

The total field is given by

$$E_x(x,y,z) = E_x^{dipole}(x,y,z) + E_x^{patches}(x,y,z)$$

so that

$$E_x(x,y,z) = E_x^{dipole}(x,y,z) + \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} A_x^{00}(k_{x0},k_{y0}) \left[\frac{1}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \tilde{G}_{xx}(k_{xp},k_{yq};z,0)\, \tilde{B}_x^{00}(k_{xp},k_{yq}) e^{-j(k_{xp}x + k_{yq}y)} \right] dk_{x0}\, dk_{y0}$$

The field is the sum of the fields due to the original (single) dipole and the currents on the infinite 2D array, produced by the original (single) dipole.

Note: There is no need to use the ASM to find the fields from the original (single) dipole; it is simpler to find this directly using the (non-periodic) SDI method.

FSS Analysis (cont.)

We can also "unfold" the paths.

$$E_x^{patches}(x,y,z)=\frac{ab}{(2\pi)^2}\int_{-\pi/b}^{\pi/b}\int_{-\pi/a}^{\pi/a}A_x^{00}(k_{x0},k_{y0})\left[\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{G}_{xx}(k_{xp},k_{yq};z,0)\;\tilde{B}_x^{00}(k_{xp},k_{yq})e^{-j(k_{xp}x+k_{yq}y)}\right]dk_{x0}\,dk_{y0}$$

$$A_x^{00}(k_{x0},k_{y0})=A_x^{00}(k_{xp},k_{yq})$$

(same current on dipole sources,

and hence the same patch currents)

$$E_x^{patches}(x,y,z)=\frac{ab}{(2\pi)^2}\int_{-\pi/b}^{\pi/b}\int_{-\pi/a}^{\pi/a}\left[\frac{1}{ab}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}A_x^{00}(k_{xp},k_{yq})\tilde{G}_{xx}(k_{xp},k_{yq};z,0)\;\tilde{B}_x^{00}(k_{xp},k_{yq})e^{-j(k_{xp}x+k_{yq}y)}\right]dk_{x0}\,dk_{y0}$$

FSS Analysis (cont.)

$$E_x^{patches}(x,y,z) = \frac{ab}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} \left[\frac{1}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} A_x^{00}(k_{xp},k_{yq}) \tilde{G}_{xx}(k_{xp},k_{yq};z,0)\, \tilde{B}_x^{00}(k_{xp},k_{yq}) e^{-j(k_{xp}x + k_{yq}y)} \right] dk_{x0}\, dk_{y0}$$

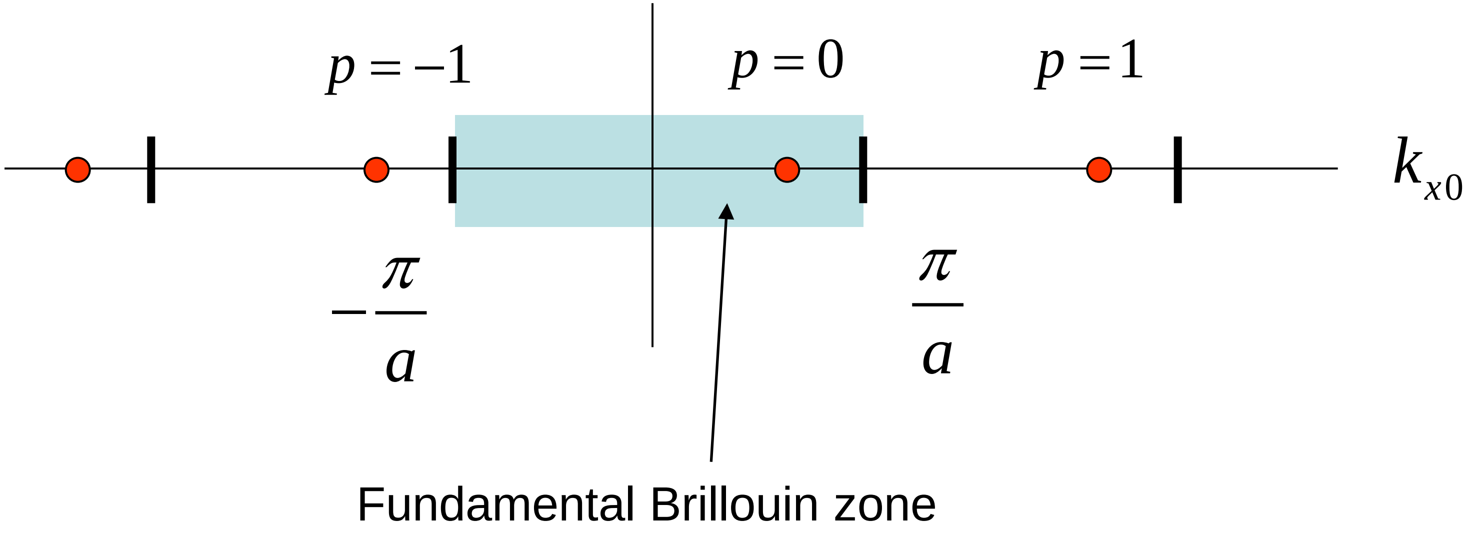
$$E_x^{patches}(x,y,z) = \frac{ab}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{ab} A_x^{00}(k_{x0},k_{y0}) \left[\tilde{G}_{xx}(k_{x0},k_{y0};z,0) \tilde{B}_x^{00}(k_{x0},k_{y0}) e^{-j(k_{x0}x + k_{y0}y)} dk_{x0}\, dk_{y0} \right]$$

Note $p = 0$ and $q = 0$ here.

FSS Analysis (cont.)

Physical explanation of the path unfolding (illustrated for the k_{x0} integral):

$$k_{xp} = k_{x0} + \frac{2\pi p}{a}$$



FSS Analysis (cont.)

Final result:

$$E_x^{patches}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\pi/b}^{\pi/b} \int_{-\pi/a}^{\pi/a} A_x^{00}(k_{x0},k_{y0}) \left[\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \tilde{G}_{xx}(k_{xp},k_{yq};z,0)\, \tilde{B}_x^{00}(k_{xp},k_{yq}) e^{-j\left(k_{xp}x+k_{yq}y\right)} \right] dk_{x0}\, dk_{y0}$$

or

$$E_x^{patches}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_x^{00}(k_{x0},k_{y0}) \left[\tilde{G}_{xx}(k_{x0},k_{y0};z,0) \tilde{B}_x^{00}(k_{x0},k_{y0}) e^{-j\left(k_{x0}x+k_{y0}y\right)} dk_{x0}\, dk_{y0} \right]$$

where

$$A_x^{00}(k_{x0},k_{y0}) = \frac{R^{00}(k_{x0},k_{y0})}{Z_{xx}^{\infty}(k_{x0},k_{y0})}$$

$$R^{00}(k_{x0},k_{y0}) = \frac{1}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \tilde{G}_{xx}(k_{xp},k_{yq};-h_d,0)\, \tilde{B}_x^{00}(k_{xp},k_{yq}) e^{-j\left(k_{xp}x_0+k_{yq}y_0\right)}$$

$$Z_{xx}^{\infty} = -\frac{1}{ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \tilde{G}_{xx}(k_{xp},k_{yq}) \left[\tilde{B}_x^{00}(k_{xp},k_{yq}) \right]^2$$