ECE 6345

Spring 2015

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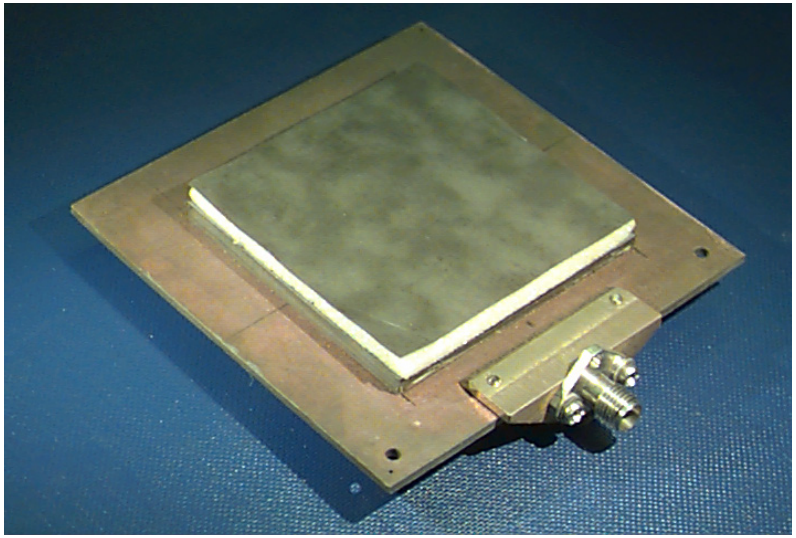
ECE Dept.



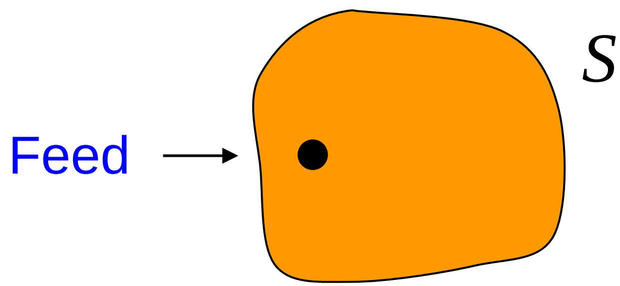
Notes 3

Overview

This set of notes discusses the Q of a patch, and its different components, as well as the radiation efficiency of the patch.

- Define Q and its components: Q_d, Q_c, Q_{sp}, Q_{sw}
- Calculate Q_d, Q_c, Q_{sp}, Q_{sw}
- Calculate the radiation efficiency

Q of the Patch



The patch is allowed to have an arbitrary shape.

$$Q = \omega_0 \frac{U_S}{P_D^{ave}} \quad \text{so} \quad \frac{1}{Q} = \frac{P_D^{ave}}{\omega_0 U_S}$$

Note: Here ω_0 denotes the real part of the complex resonance frequency (instead of ω_0').

$$P_D^{ave} = P_D^{sp} + P_D^{sw} + P_D^{c} + P_D^{d}$$

Q of the Patch (cont.)

Hence

$$\frac{1}{Q} = \frac{P_D^{sp}}{\omega_0 U_s} + \frac{P_D^{sw}}{\omega_0 U_s} + \frac{P_D^{c}}{\omega_0 U_s} + \frac{P_D^{d}}{\omega_0 U_s}$$

or

$$\frac{1}{Q} = \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} + \frac{1}{Q_c} + \frac{1}{Q_d}$$

Note: Combining Q terms is like combining resistors in parallel.

Note: A smaller Q is a more dominant one!

Calculation of Q_d

$$U_S = \langle U_S \rangle = \langle U_E \rangle + \langle U_H \rangle = 2\langle U_E \rangle$$

$$= 2\int_V \frac{1}{4}\varepsilon' \left|\underline{E}\right|^2 dV$$

(We have equal time-average stored energies at resonance.)

$$= \frac{1}{2}\varepsilon' h \int_S \left|\underline{E}\right|^2 dS$$

$$P_D^d = \int_V \frac{1}{2}\left(\omega_0 \varepsilon''\right)\left|\underline{E}\right|^2 dV$$

$$= \frac{1}{2}\omega_0 \varepsilon'' h \int_S \left|\underline{E}\right|^2 dS$$

Calculation of Q_d (cont.)

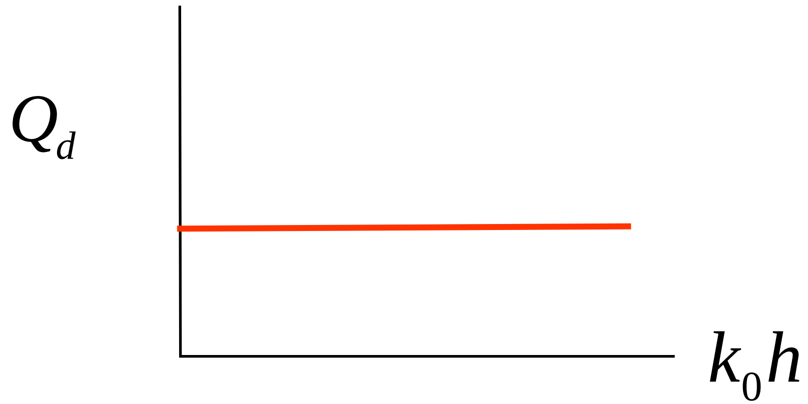
Hence

$$Q^d = \omega_0 \frac{\frac{1}{2}\varepsilon' h \int_S \left|\underline{E}\right|^2 dS}{\frac{1}{2}\omega_0 \varepsilon'' h \int_S \left|\underline{E}\right|^2 dS} = \frac{\varepsilon'}{\varepsilon''}$$

Therefore

$$Q^d = \frac{1}{\tan\delta}$$

The dielectric Q factor is independent of substrate thickness.



Calculation of Q_c

$$P_D^c = \int_{S_P} \frac{1}{2} R_s^P \left|\underline{H}\right|^2 dS + \int_{S_G} \frac{1}{2} R_s^G \left|\underline{H}\right|^2 dS$$

Hence

$$P_D^{ave} = \frac{1}{2}\left(R_s^P + R_s^G\right)\int_S \left|\underline{H}\right|^2 dS$$

$$= R_s^{ave} \int_S \left|\underline{H}\right|^2 dS$$

Calculation of Q_c (cont.)

For the stored energy,

$$U_S = \langle U_S \rangle = \langle U_E \rangle + \langle U_H \rangle = 2\langle U_H \rangle$$

where

$$\langle U_H \rangle = \int_V \frac{1}{4}\mu \left|\underline{H}\right|^2 dV$$

$$= \frac{1}{4}\mu_0\mu_r h \int_S \left|\underline{H}\right|^2 dS$$

Calculation of Q_c (cont.)

Hence,

$$Q_c = \omega_0 \frac{2\left(\frac{1}{4}\mu_0\mu_r h \int_S \left|\underline{H}\right|^2 dS\right)}{R_s^{ave} \int_S \left|\underline{H}\right|^2 dS}$$

Use $$\omega_0 = \frac{k_0}{\sqrt{\mu_0\varepsilon_0}}$$

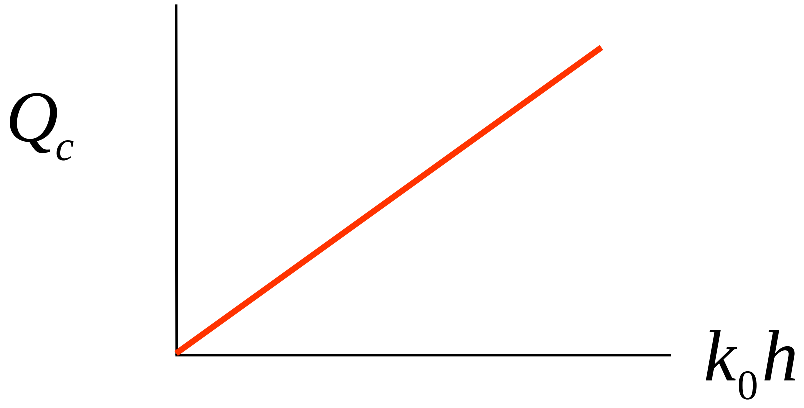
so

$$Q_c = \frac{\frac{1}{2}\mu_0\mu_r (k_0 h)}{R_s^{ave}\sqrt{\mu_0\varepsilon_0}}$$

Calculation of Q_c (cont.)

Hence, we have

$$Q_c = \left(\frac{\eta_0}{2}\right)\mu_r\left[\frac{(k_0 h)}{R_s^{ave}}\right]$$



The conducting Q factor becomes more important as the substrate gets thinner.

Calculation of Q_{sp}

Rectangular Patch: (The derivation is given later.)

$$Q_{sp} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{pc_1}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{1}{h/\lambda_0}\right)$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4} \qquad \left(n_1 = \sqrt{\varepsilon_r \mu_r}\right)$$

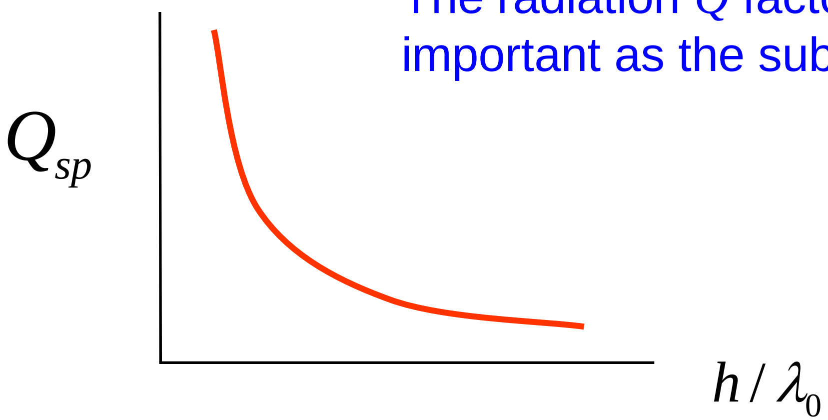
$$p = 1 + \frac{a_2}{10}\left(k_0 W_e\right)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)\left(k_0 W_e\right)^4$$
$$+ c_2\left(\frac{1}{5}\right)\left(k_0 L_e\right)^2 + a_2 c_2\left(\frac{1}{70}\right)\left(k_0 W_e\right)^2\left(k_0 L_e\right)^2$$

Calculation of Q_{sp} (cont.)

where

$$a_2 = -0.16605$$
$$a_4 = \ \ 0.00761$$
$$c_2 = -0.0914153$$

The radiation Q factor becomes more important as the substrate gets thicker.



$$Q_{sp} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{pc_1}\right)\left(\frac{L_e}{W_e}\right)\left(\frac{1}{h/\lambda_0}\right)$$

Calculation of Q_{SW}

Surface-wave radiation efficiency:

$$e_r^{sw} \equiv \frac{P_{sp}}{P_{sp} + P_{sw}}$$

so

$$\frac{1}{e_r^{sw}} = \frac{P_{sp} + P_{sw}}{P_{sp}} = 1 + \frac{P_{sw}}{P_{sp}}$$

Hence

$$P_{sw} = P_{sp}\left(\frac{1}{e_r^{sw}} - 1\right)$$

Calculation of Q_{SW} (cont.)

Now look at the Q's :

$$Q_{sw} = \omega_0 \frac{U_S}{P_{sw}}$$

$$Q_{sp} = \omega_0 \frac{U_S}{P_{sp}}$$

Hence, we have

$$\frac{Q_{sw}}{Q_{sp}} = \frac{P_{sp}}{P_{sw}}$$

$$= \frac{1}{\frac{1}{e_r^{sw}} - 1}$$

$$= \frac{e_r^{sw}}{1 - e_r^{sw}}$$

Calculation of Q_{sw} (cont.)

Hence, we have

$$Q_{sw} = Q_{sp}\left(\frac{e_r^{sw}}{1-e_r^{sw}}\right)$$

For the radiation efficiency, the patch can be approximated by a horizontal electric dipole at the center of the patch.

$$e_r^{sw} \approx e_r^{hed}$$

Calculation of Q_{SW} (cont.)

$$e_r^{hed} = \frac{P_{sp}^{hed}}{P_{sp}^{hed} + P_{sw}^{hed}} = \frac{1}{1 + \left(P_{sw}^{hed} / P_{sp}^{hed} \right)}$$

$$p_{sp}^{hed} \approx \frac{1}{\lambda_0^2} (k_0 h)^2 \left(80\pi^2 \mu_r^2 c_1 \right)$$

$$p_{sw}^{hed} \approx \frac{1}{\lambda_0^2} (k_0 h)^3 \left(60\pi^3 \mu_r^3 \left(1 - \frac{1}{n_1^2} \right)^3 \right)$$

where

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

$$n_1 = \sqrt{\varepsilon_r \mu_r}$$

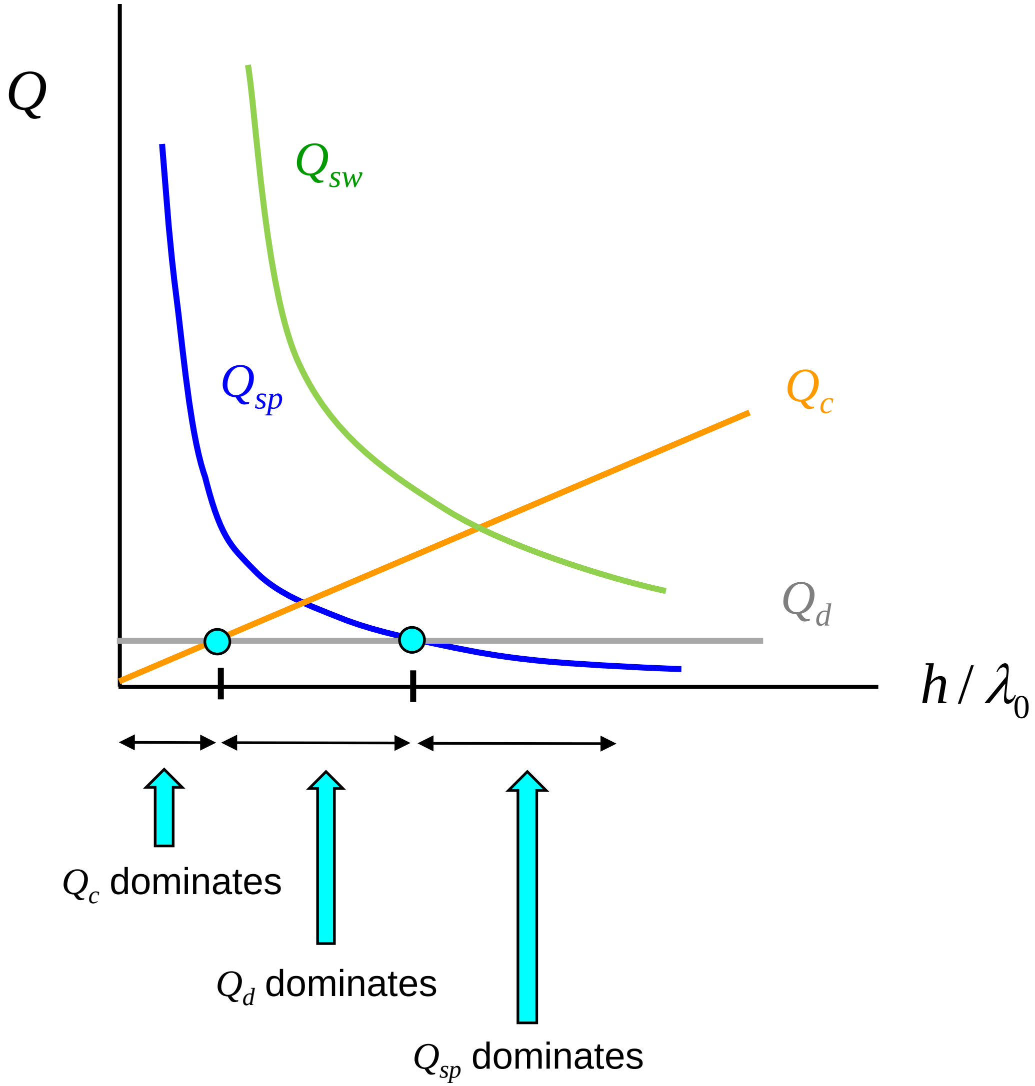
Calculation of Q_{SW} (cont.)

Hence, we have

$$e_r^{sw} \approx e_r^{hed} = \frac{1}{1 + (k_0 h)\left(\frac{3\pi}{4}\right)\mu_r \frac{1}{c_1}\left(1 - \frac{1}{n_1^2}\right)^3}$$

This holds for any shape patch
(assuming the HED approximation is accurate).

Comparison of Qs



Radiation Efficiency

$$e_r \equiv \frac{P_{sp}}{P_{tot}} = \frac{P_{sp}}{P_{sp} + P_{sw} + P_c + P_d}$$

Note that $Q_i = \omega_0 \frac{U_s}{P_i}$

Hence $P_i \propto \frac{1}{Q_i}$

Radiation Efficiency (cont.)

Hence
$$e_r = \frac{\frac{1}{Q_{sp}}}{\frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} + \frac{1}{Q_c} + \frac{1}{Q_d}}$$

or
$$e_r = \frac{\frac{1}{Q_{sp}}}{\frac{1}{Q}}$$

or
$$e_r = \frac{Q}{Q_{sp}}$$

Radiation Efficiency (cont.)

Also, we can write

$$e_r = \frac{P_{sp}}{P_{tot}} = \left(\frac{P_{sp}}{P_r}\right)\left(\frac{P_r}{P_{tot}}\right)$$

where $P_r = P_{sp} + P_{sw}$

Define:

$$e_r^{sw} = \frac{P_{sp}}{P_r}$$

$$e_r^{diss} = \frac{P_r}{P_{tot}}$$

Radiation Efficiency (cont.)

Then

$$e_r = e_r^{sw} e_r^{diss}$$

where

$$e_r^{sw} = \frac{Q_r}{Q_{sp}}$$

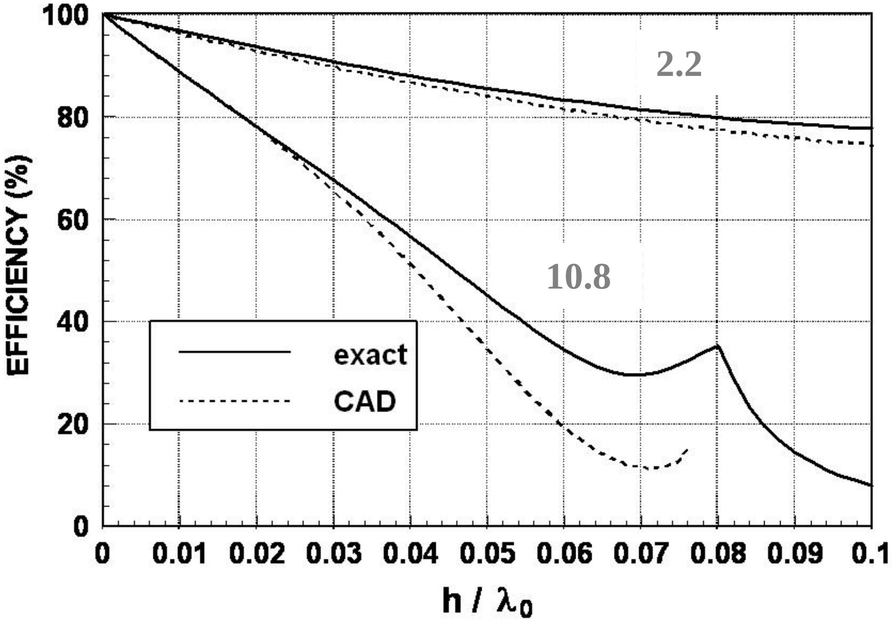
$$e_r^{diss} = \frac{Q}{Q_r}$$

and

$$\frac{1}{Q_r} = \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}$$

Radiation Efficiency (cont.)

e_r^{sw}



e_r