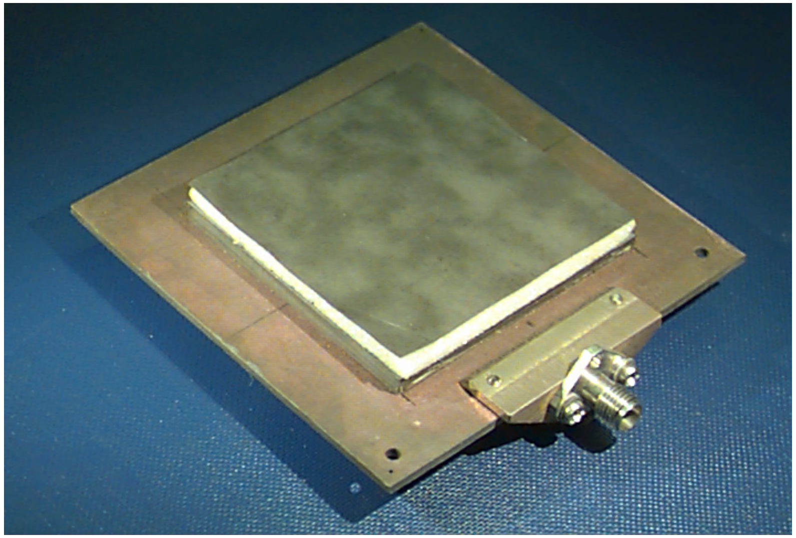
ECE 6345

Spring 2015

Prof. David R. Jackson
ECE Dept.



Notes 4

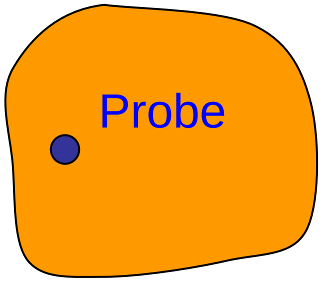
Overview

This set of notes discusses the probe inductance of a coax-fed patch.

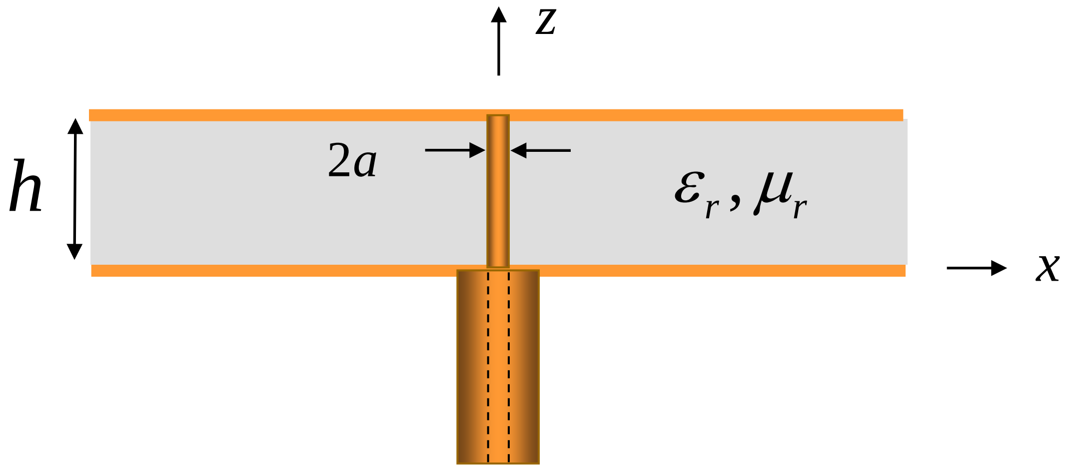
- Introduce probe model for a parallel-plate waveguide
- Use this model to calculate the probe inductance

Probe Inductance

Patch



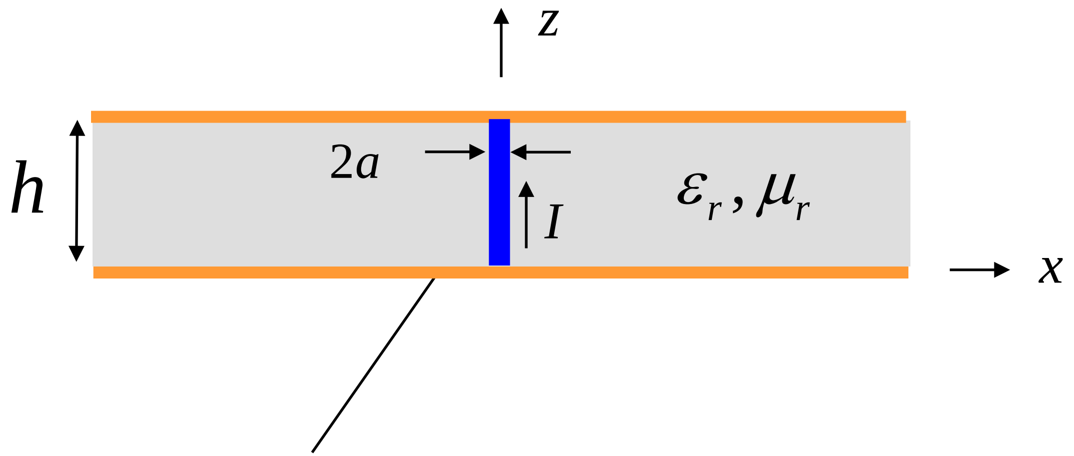
Parallel-Plate Waveguide Model



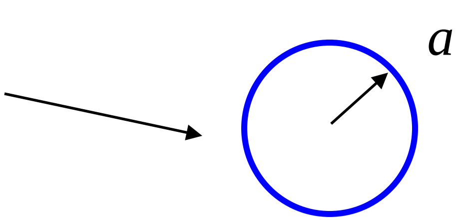
Probe Inductance

Assume that $\dfrac{\partial}{\partial z} = 0$

The probe current is assumed to be uniform in the z direction, and the metal is removed by the equivalence principle. Radiation from the coax "frill" is neglected.



Hollow tube of uniform surface current



$$J_{sz} = \frac{I}{2\pi a}$$

Probe Inductance (cont.)

Assume $\underline{E} = \underline{\hat{z}}\, E_z(\rho)$ since $\dfrac{\partial}{\partial \phi} = 0 \quad \dfrac{\partial}{\partial z} = 0$

$$\nabla^2 E_z + k^2 E_z = 0$$

General solution:

$$E_z = \begin{pmatrix} \cos(n\phi) \\ \sin(n\phi) \end{pmatrix} \begin{pmatrix} J_n(k_\rho \rho) \\ Y_n(k_\rho \rho) \end{pmatrix} \begin{pmatrix} \cos(k_z z) \\ \sin(k_z z) \end{pmatrix}$$

$$k_\rho = \left(k^2 - k_z^2\right)^{1/2}$$

$$k_z = \frac{m\pi}{h}$$

Choose

$m = 0, n = 0$

Probe Inductance (cont.)

Hence

$$E_z = \begin{pmatrix} J_0(k\rho) \\ Y_0(k\rho) \end{pmatrix}$$

← finite at the origin

← infinite at the origin

or

$$E_z = \begin{pmatrix} H_0^{(1)}(k\rho) \\ H_0^{(2)}(k\rho) \end{pmatrix}$$

← incoming wave

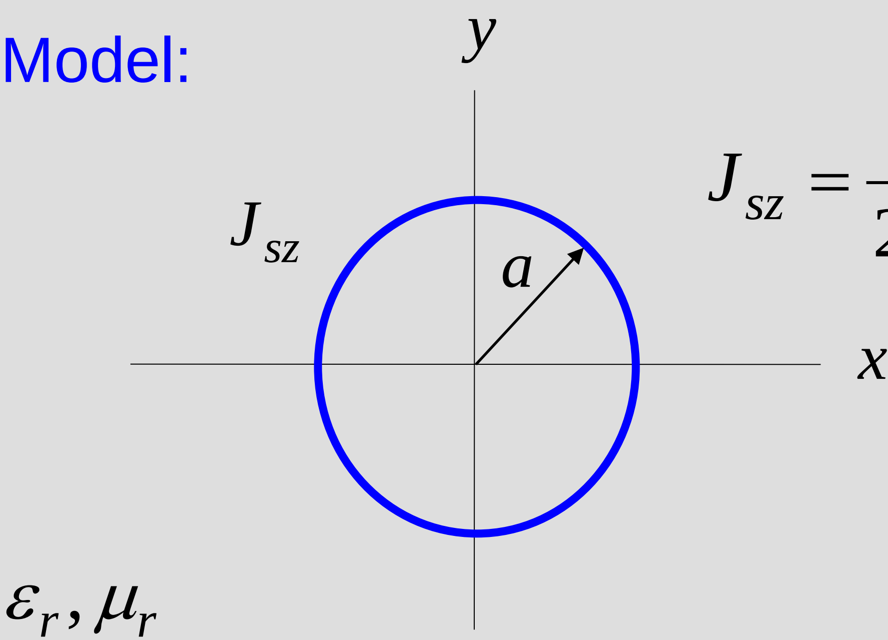
← outgoing wave

$$H_n^{(1)}(z) \equiv J_n(z) + jY_n(z)$$

$$H_n^{(2)}(z) \equiv J_n(z) - jY_n(z)$$

Probe Inductance (cont.)

Model:



$$J_{sz} = \frac{I}{2\pi a} \quad [\text{A/m}]$$

Hollow tube of current

Note: The tube may be thought of as being infinite in the z direction.

Probe Inductance (cont.)

$\rho < a$

$$E_z^- = A^- J_0(k\rho)$$

$\rho > a$

$$E_z^+ = A^+ H_0^{(2)}(k\rho)$$

$\rho = a$

$$E_z^- = E_z^+ \quad \text{(BC1)}$$

$$H_\phi^+ - H_\phi^- = J_{sz} \quad \text{(BC2)}$$

Probe Inductance (cont.)

BC 1: $$A^{-} J_0(ka) = A^{+} H_0^{(2)}(ka)$$

BC 2: $$H_\phi = \frac{-1}{j\omega\mu}\left(\nabla \times \underline{E}\right) \cdot \underline{\hat{\phi}}$$

$$= \frac{-1}{j\omega\mu}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right)$$

$$= \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial \rho}$$

Probe Inductance (cont.)

so $$\frac{k}{j\omega\mu}\left[A^{+}H_0^{(2)\prime}(ka) - A^{-}J_0{}'(ka)\right] = \frac{I}{2\pi a}$$ BC 2

Hence, eliminating A^- using BC 1, we have:

$$A^{+}H_0^{(2)\prime}(ka) - \left[A^{+}\frac{H_0^{(2)}(ka)}{J_0(ka)}\right]J_0{}'(ka) = \left(\frac{I}{2\pi a}\right)\left(\frac{j\omega\mu}{k}\right)$$

or

$$A^{+}\left[J_0(ka)H_0^{(2)\prime}(ka) - J_0{}'(ka)\,H_0^{(2)}(ka)\right] = \frac{I}{2\pi a}\left(\frac{j\omega\mu}{k}\right)J_0(ka)$$

Probe Inductance (cont.)

Wronskian Identity:

$$J_n(x)\,H_n^{(2)\prime}(x) - J_n{}'(x)\,H_n^{(2)}(x) = -j\left(\frac{2}{\pi x}\right)$$

Hence $$A^{+}\left(-j\frac{2}{\pi(ka)}\right) = \left(\frac{I}{2\pi a}\right)\left(\frac{j\omega\mu}{k}\right)J_0(ka)$$

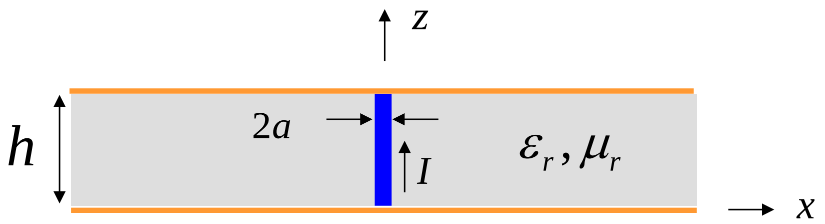
or $$A^{+} = -\left(\frac{I}{4}\right)(\omega\mu)\,J_0(ka)$$

Next, use $\omega\mu = k\eta$ so $A^{+} = -\eta k\left(\frac{I}{4}\right)J_0(ka)$

Probe Inductance (cont.)

Hence

$$E_z^{+} = -\eta k \left(\frac{I}{4} \right) J_0(ka) H_0^{(2)}(k\rho)$$



Next, we use $Z_{in} = \dfrac{V}{I} = \dfrac{-h \left. E_z \right|_{\rho=a}}{I}$

so

$$Z_{in} = \eta \frac{(kh)}{4} J_0(ka) H_0^{(2)}(ka)$$

Note: The imaginary part should be fairly accurate for the probe feed of a patch, but not the real part (the radiation effects are very different).

Probe Inductance (cont.)

Taking the imaginary part: $$X_{in} = -\eta \frac{(kh)}{4} J_0\left(ka\right) Y_0(ka)$$

$$\left(\text{Recall}: H_0^{(2)}(ka) = J_0(ka) - jY_0(ka)\right)$$

For $ka << 1$

$$J_0(ka) \approx 1 \qquad Y_0(ka) \approx \frac{2}{\pi}\left(\gamma + \ln\left(\frac{ka}{2}\right)\right)$$

where $\gamma \doteq 0.57722$ (Euler's constant)

Probe Inductance (cont.)

Approximating the Y_0 Bessel function, we have

$$X_{in} = -\eta \frac{(kh)}{4}\left[\frac{2}{\pi}\left(\gamma + \ln\left(\frac{ka}{2}\right)\right)\right]$$

$$= \eta\left(\frac{kh}{2\pi}\right)\left[-\gamma + \ln\left(\frac{2}{ka}\right)\right]$$

or
$$X_{in} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\mu_r \varepsilon_r} \left(\frac{h}{\lambda_0}\right)\left[-\gamma + \ln\left(\frac{2}{\sqrt{\mu_r \varepsilon_r}\, k_0 a}\right)\right]$$

so
$$X_p = \eta_0 \mu_r \left(\frac{h}{\lambda_0}\right)\left[-\gamma + \ln\left(\frac{2}{\sqrt{\mu_r \varepsilon_r}\, k_0 a}\right)\right]$$

Probe Inductance (cont.)

Or, we can write

$$X_p = \eta_0 \, \mu_r \left(\frac{h}{\lambda_0} \right) \left[-\gamma + \ln\left(\frac{1}{a/\lambda_0} \right) - \ln(\pi \sqrt{\mu_r \varepsilon_r}) \right]$$

or

$$X_p = \eta_0 \mu_r \left(\frac{h}{\lambda_0} \right) \left[\ln\left(\frac{1}{a/\lambda_0} \right) - \gamma - \ln(\pi) - \ln\left(\sqrt{\mu_r \varepsilon_r} \right) \right]$$

Probe Inductance (cont.)

We can solve for the probe inductance as

$$L_p = \frac{X_p}{\omega} = \frac{X_p}{k_0}\sqrt{\mu_0\varepsilon_0} = \left[\frac{1}{2\pi}\sqrt{\mu_0\varepsilon_0}\,\lambda_0\right]X_p$$

$$= \left[\frac{1}{2\pi}\sqrt{\mu_0\varepsilon_0}\,\lambda_0\right]\eta_0\mu_r\left(\frac{h}{\lambda_0}\right)\left[\ln\left(\frac{1}{a/\lambda_0}\right) - \gamma - \ln(\pi) - \ln\left(\sqrt{\mu_r\varepsilon_r}\right)\right]$$

Hence

$$L_p = \frac{1}{2\pi}\mu_0\mu_r h\left[\ln\left(\frac{1}{a/\lambda_0}\right) - \gamma - \ln(\pi) - \ln\left(\sqrt{\mu_r\varepsilon_r}\right)\right]$$

The probe inductance is *relatively constant* with frequency.

Example

$$\varepsilon_r = 2.94$$

$$h = 0.1524 \ [\text{cm}] \qquad (60 \text{ mils})$$

$$a = 0.0635 \ [\text{cm}] \qquad (50[\Omega] \ \text{SMA Connector})$$

$$f = 2.0 \quad [\text{GHz}] \quad (\lambda_0 = 14.9896 \ [\text{cm}])$$

$$X_p = (376.73)(0.01017)\left[\ln(236.06) - 0.57722 - \ln(\pi) - \ln\left(\sqrt{2.94}\right)\right]$$

$$= (376.73)(0.01017)(3.2029)$$

$$X_p = 12.3 \ [\Omega]$$