ECE 6345

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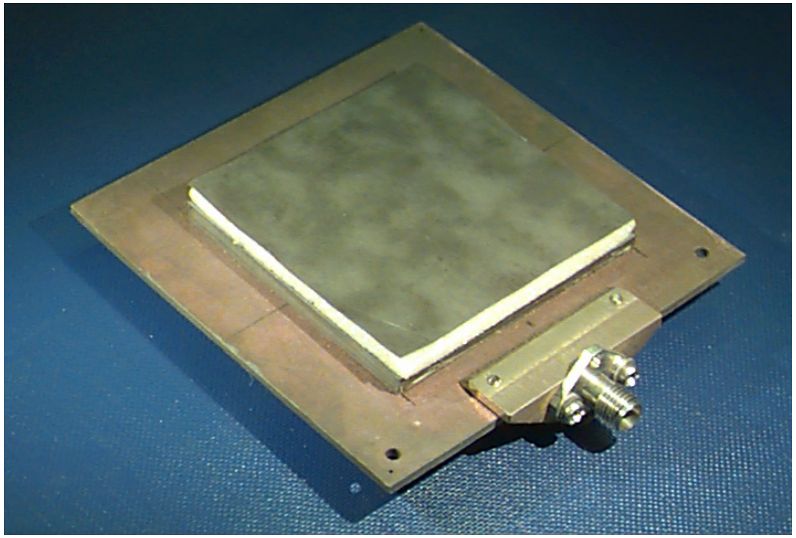


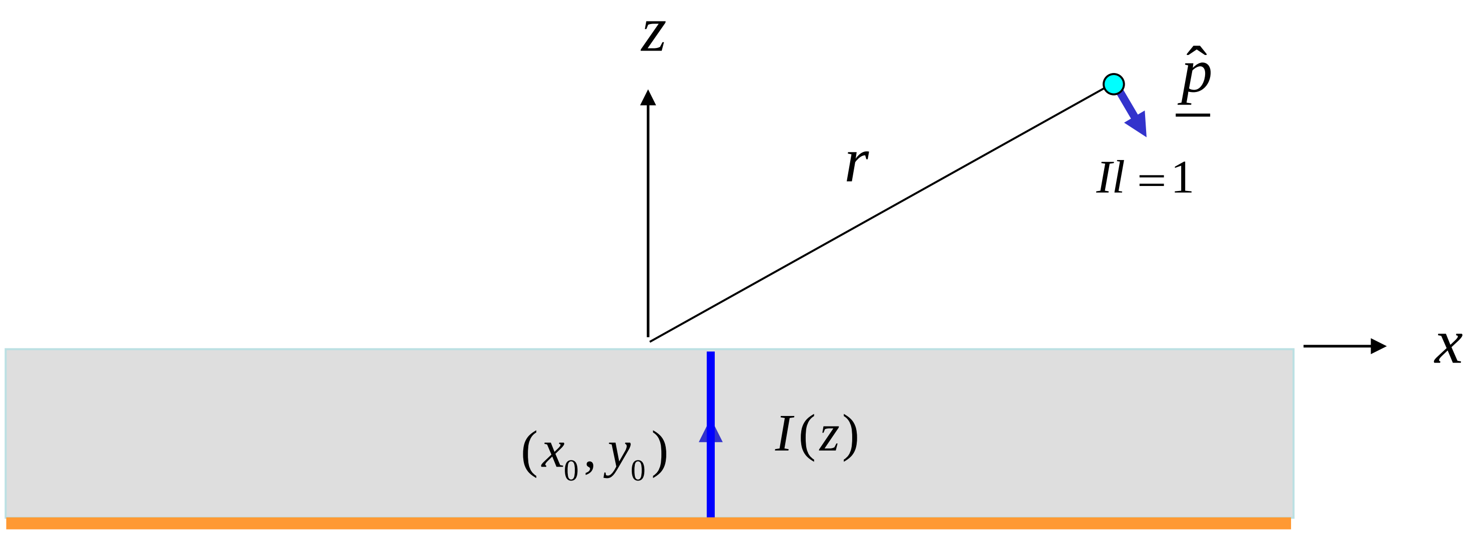
Notes 8

Overview

In this set of notes we calculate the far field of a vertical probe feed.

- We first formulate the far-field radiation from a uniform probe current of fixed (arbitrary) amplitude.
- The probe current is then related to patch input impedance.

Radiation from Probe Current



The origin is at the center of the patch (which has been removed).

$$E_{\phi}^{FF} = 0$$

This follows from reciprocity, since

$$\hat{\underline{\phi}} \cdot \hat{\underline{z}} = 0$$

$$E_{\theta}^{FF}(r,\theta,\varphi) = \langle b, a \rangle$$

$$= \int_{-h}^{0} I(z)\, E_z^{pw}(z)\, dz$$

Radiation from Probe Current (cont.)

Inside the substrate we have

$$E_z = \frac{1}{j\omega\varepsilon_1}\left(\nabla \times \underline{H}\right)\cdot \underline{\hat{z}}$$

$$= \frac{1}{j\omega\varepsilon_1}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

Transverse variation:

$$e^{j\left(k_x x + k_y y\right)}$$

Hence $E_z^b = \frac{1}{j\omega\varepsilon_1}\left[jk_x H_y^{pw} - jk_y H_x^{pw} \right]$

Radiation from Probe Current (cont.)

$$\underline{H}^{ipw} = \underline{\hat{\phi}} \left(-\frac{E_{\theta}^{ipw}}{\eta_0} \right) = \underline{\hat{\phi}} \left(-\frac{E_0}{\eta_0} \right) e^{j\left(k_x x + k_y y + k_z z\right)}$$

$$H_x^{ipw} = -\frac{E_0}{\eta_0} e^{j\left(k_x x + k_y y + k_z z\right)} \left(-\sin\phi\right)$$

$$H_y^{ipw} = -\frac{E_0}{\eta_0} e^{j\left(k_x x + k_y y + k_z z\right)} \cos\phi$$

Radiation from Probe Current (cont.)

For the total (incident plus reflected) plane-wave field we have (for the x component):

$$H_x^{pw}(x_0, y_0, 0) = H_x^{ipw}(x_0, y_0, 0)\left(1 - \Gamma^{TM}\right)$$

$$= \frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \sin\phi \left(1 - \Gamma^{TM}\right)$$

Denote $H_x^{pw}(x_0, y_0, z) = f(z)$

$$= A\cos\left(k_{z1}(z+h)\right)$$

where

$$k_{z1} = \left(k_1^2 - k_x^2 - k_y^2\right)^{1/2}$$

$$= \left(k_1^2 - k_0^2 \sin^2\theta\right)^{1/2}$$

$$= k_0\sqrt{\varepsilon_r - \sin^2\theta}$$

$$= k_0 N_1(\theta)$$

Setting $z = 0$ we have $A = \dfrac{H_x^{pw}(x_0, y_0, 0)}{\cos\left(k_{z1}h\right)}$

Hence $H_x^{pw}(x_0, y_0, z) = H_x^{pw}(x_0, y_0, 0)\left[\dfrac{\cos\left(k_{z1}(z+h)\right)}{\cos\left(k_{z1}h\right)}\right]$

Radiation from Probe Current (cont.)

We then have

$$H_x^{pw}(x_0, y_0, z) = \frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1 - \Gamma^{TM}\right) \sin\phi \, \sec\left(k_{z1} h\right) \cos k_{z1}(z+h)$$

Similarly,

$$H_y^{pw}(x_0, y_0, z) = \frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1 - \Gamma^{TM}\right) \left(-\cos\phi\right) \sec\left(k_{z1} h\right) \cos k_{z1}(z+h)$$

Next, use

$$k_x = k_0 \sin\theta \cos\phi$$
$$k_y = k_0 \sin\theta \sin\phi$$

Radiation from Probe Current (cont.)

Recall: $E_z^{pw} = \dfrac{1}{\omega\varepsilon_1}\left[k_x H_y^{pw} - k_y H_x^{pw}\right]$

so

$$E_z^{pw} = \frac{1}{\omega\varepsilon_1}\frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{TM}\right)\sec\left(k_{z1}h\right)\cos k_{z1}(z+h)$$
$$\cdot\left[k_0 \sin\theta\left(-\cos^2\phi - \sin^2\phi\right)\right]$$

or

$$E_z^{pw} = -\frac{1}{\omega\varepsilon_1}\frac{E_0}{\eta_0} e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{TM}\right)\sec\left(k_{z1}h\right)\cos k_{z1}(z+h)\; k_0 \sin\theta$$
$$= -\frac{1}{\varepsilon_r} E_0\, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1-\Gamma^{TM}\right)\sec\left(k_{z1}h\right)\cos k_{z1}(z+h)\; \sin\theta$$

Radiation from Probe Current (cont.)

The far field pattern is then

$$E_\theta^{FF}(r,\theta) = \frac{-1}{\varepsilon_r} E_0 \, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1 - \Gamma^{TM}\right) \sec\left(k_{z1} h\right) \sin\theta \int_{-h}^{0} I(z) \cos\left(k_{z1}(z+h)\right) dz$$

Assume $I(z) = I_0$

$$\begin{aligned}
\int_{-h}^{0} I(z)\cos\left(k_{z1}(z+h)\right)dz &= I_0 \int_{-h}^{0} \cos\left(k_{z1}(z+h)\right)dz \\
&= I_0 \int_{0}^{h} \cos\left(k_{z1} u\right) du \\
&= \frac{I_0}{k_{z1}} \sin\left(k_{z1} h\right) \\
&= I_0 h \,\mathrm{sinc}\left(k_{z1} h\right)
\end{aligned}$$

Radiation from Probe Current (cont.)

Hence

$$E_\theta^{FF}(r,\theta) = \frac{-1}{\varepsilon_r} E_0 \, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1 - \Gamma^{TM}\right) \sec\left(k_{z1} h\right) \left[\left(I_0 h\right) \operatorname{sinc}\left(k_{z1} h\right) \sin\theta\right]$$

or

$$E_\theta^{FF}(r,\theta) = \left(I_0 h\right) \left(\frac{-1}{\varepsilon_r}\right) E_0 \, e^{j\left(k_x x_0 + k_y y_0\right)} \left(1 - \Gamma^{TM}\right) \operatorname{tanc}\left(k_0 h N_1(\theta)\right) \sin\theta$$

where

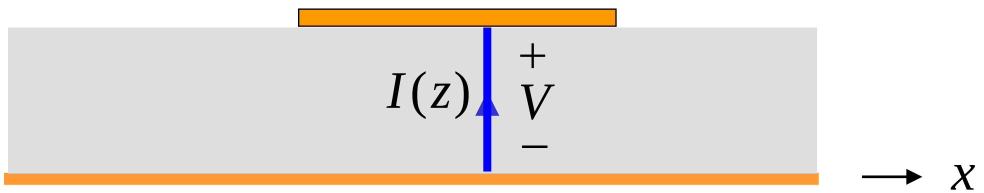
$$\operatorname{tanc}(x) = \frac{\tan(x)}{x}$$

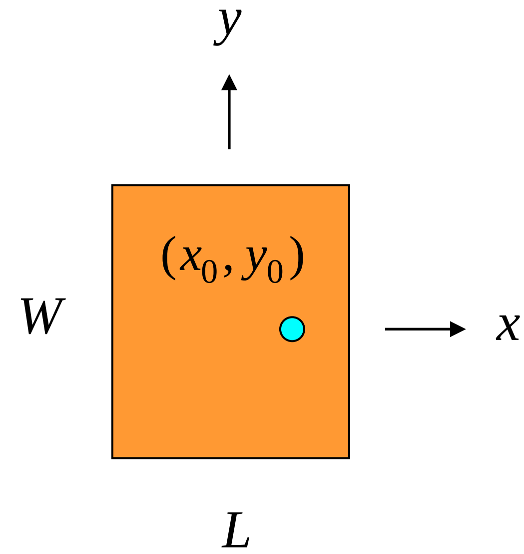
Calculation of Probe Current

We want to find the relation between these two.

$$J_{sx}(x,y) = A_{10}^{J} \cos\left(\frac{\pi x}{L}\right)$$

$$I(z) = I_0$$





$$V = Z_{in} I_0 = -h\, E_z(x_0, y_0)$$

Z_{in} = input impedance of dominant mode.

Hence

$$E_z(x_0, y_0) = -\frac{Z_{in} I_0}{h}$$

Calculation of Probe Current (cont.)

Assume $$E_z^{10}(x,y) = A_{10}^E \sin\left(\frac{\pi x}{L}\right)$$

Then $$A_{10}^E = \frac{E_z^{10}(x_0, y_0)}{\sin\left(\frac{\pi x_0}{L}\right)}$$

$$= \frac{-Z_{in} I_0}{h \sin\left(\frac{\pi x_0}{L}\right)}$$

Therefore,

$$E_z^{10}(x,y) = \left[\frac{-Z_{in} I_0}{h \sin\left(\frac{\pi x_0}{L}\right)}\right] \sin\left(\frac{\pi x}{L}\right)$$

Calculation of Probe Current (cont.)

From previous calculations,

$$\underline{J}_s = \frac{1}{j\omega\mu} \nabla E_z$$

Hence

$$J_{sx}^{10} = \frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{-Z_{in} I_0}{h \sin\left(\frac{\pi x_0}{L}\right)}\right]\cos\left(\frac{\pi x}{L}\right) = A_{10}^{J} \cos\left(\frac{\pi x}{L}\right)$$

so that

$$A_{10}^{J} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{in} I_0}{h \sin\left(\frac{\pi x_0}{L}\right)}\right]$$

or

$$\frac{A_{10}^{J}}{I_0} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{Z_{in}}{h \sin\left(\frac{\pi x_0}{L}\right)}\right]$$

Calculation of Probe Current (cont.)

At resonance,

$$Z_{in} = R_{edge} \sin^2\left(\frac{\pi x_0}{L}\right)$$

(The origin is at the center of the patch.)

Hence,

$$\frac{A_{10}^J}{I_0} = -\frac{1}{j\omega\mu}\left(\frac{\pi}{L}\right)\left[\frac{R_{edge}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right)$$

Calculation of Probe Current (cont.)

We can also write this as

$$\frac{I_{patch}}{I_0} = \frac{W A_{10}^J}{I_0} = -\frac{\pi}{j\omega\mu}\left(\frac{W}{L}\right)\left[\frac{R_{edge}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right)$$

$$= j\frac{\pi}{k_0\eta_0}\left(\frac{W}{L}\right)\left[\frac{R_{edge}}{h}\right]\sin\left(\frac{\pi x_0}{L}\right)$$

I_{patch} is the total current flowing across the center of the patch.

so

$$\frac{I_{patch}}{I_0} = j\frac{\pi}{\eta_0}\left(\frac{1}{k_0 h}\right)\left(\frac{W}{L}\right)R_{edge}\sin\left(\frac{\pi x_0}{L}\right)$$

Calculation of Probe Current (cont.)

Hence,

$$\frac{I_{patch}}{I_0} = j\pi \frac{R_{edge}}{\eta_0} \left(\frac{1}{k_0 h} \right) \left(\frac{W}{L} \right) \sin\left(\frac{\pi x_0}{L} \right)$$

Note: For a thin substrate, the patch current is much larger than the probe current.

Note: $I_0 h \propto (k_0 h)^2$ (This is a measure of how strong the probe radiates.)

The probe radiation gets small quickly as the substrate gets thinner.