ECE 6345

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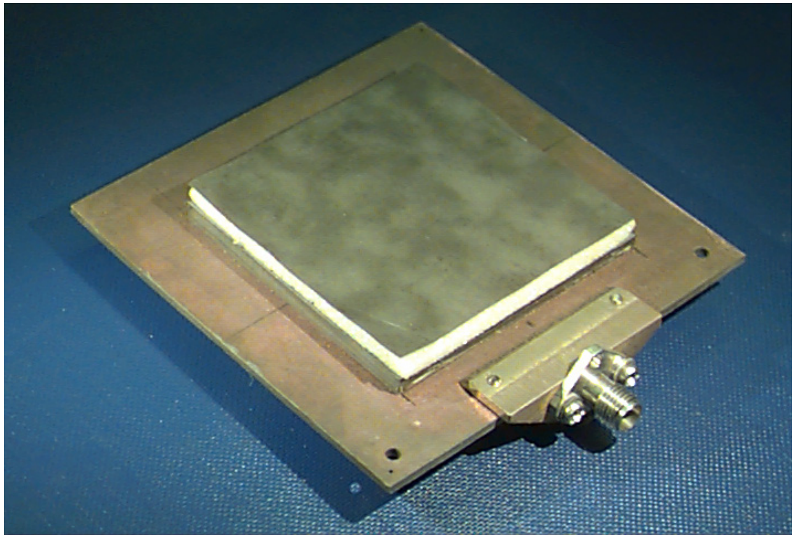


Notes 9

Overview

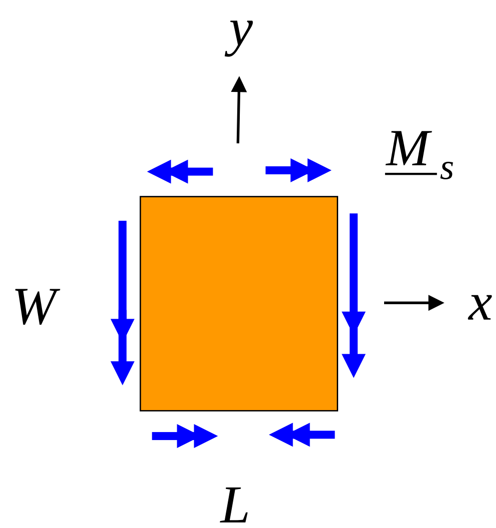
In this set of notes we calculate the far field of a rectangular patch using the magnetic current model.

The analysis assumes an infinite substrate, but for a truncated substrate we can use the same final result, setting the substrate permittivity to that of air (please see the discussion in Notes 6).

Magnetic Current Model

Assume:

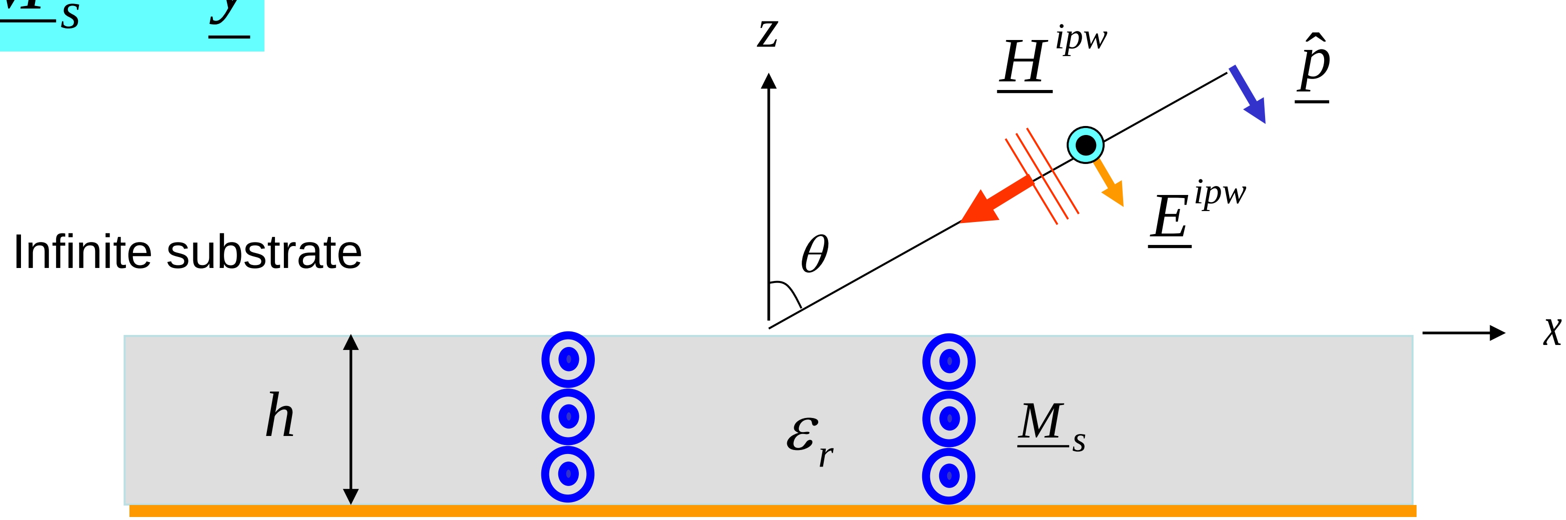
$$E_z^{1,0} = -\sin\left(\frac{\pi x}{L}\right)$$



Only the radiating edges contribute to the E- and H-plane patterns, so we will ignore the non-radiating edges.

Radiating edges:

$$\underline{M}_s = -\underline{\hat{y}}$$



Magnetic Current Model (cont.)

$$E_p^{FF}\left(r,\theta,\phi\right) = <a,b>$$

$$= <b,a>$$

$$= -\int_S \left(\underline{H}^{pw} \cdot \underline{M}_s^{a}\right) dS$$

$$= -\int_S H_y^{pw} M_{sy}^{a} \, dS$$

$$S = S_L + S_R$$ (left+right edges)

$$M_{sy} = -1$$

Magnetic Current Model (cont.)

TEN modeling equation:

$$H_y^{pw}\left(x,y,z\right)=\psi_t(x,y)\,I(z)$$

$$\psi_t(x,y)=e^{j\left(k_x x+k_y y\right)}$$

$$k_x=k_0\sin\theta\cos\phi$$

$$k_y=k_0\sin\theta\sin\phi$$

Magnetic Current Model (cont.)

$$H_y^{pw}(x,y,z) = \psi_t(x,y)\, I(z)$$

$$= \psi_t(x,y)\, I(0)\left[\frac{\cos\left(k_{z1}(z+h)\right)}{\cos\left(k_{z1}h\right)}\right]$$

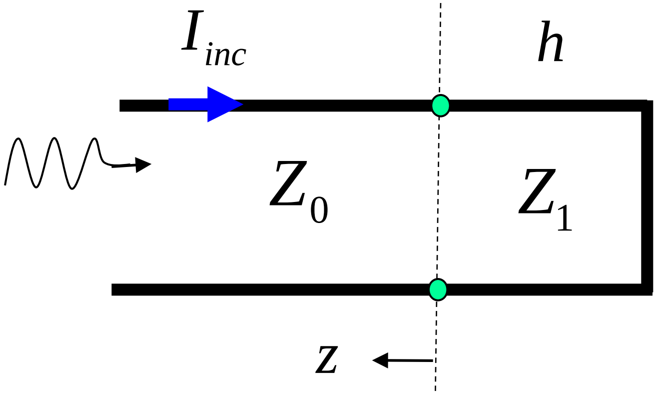
$$= H_y^{pw}(x,y,0)\left[\frac{\cos\left(k_{z1}(z+h)\right)}{\cos\left(k_{z1}h\right)}\right]$$

Also,

$$H_y^{pw}(x,y,0) = H_y^{ipw}(x,y,0)\left(1-\Gamma^{TM/TE}\right)$$

$$TM_z: \; \underline{\hat{p}} = \underline{\hat{\theta}}$$

$$TE_z: \; \underline{\hat{p}} = \underline{\hat{\phi}}$$



Magnetic Current Model (cont.)

$$\underline{\hat{p}} = \underline{\hat{\theta}}:$$

$$\underline{H}^{ipw} = -\underline{\hat{\phi}}\frac{E_\theta^{ipw}}{\eta_0} = -\underline{\hat{\phi}}\left(\frac{E_0}{\eta_0}\right)\psi\left(x,y,z\right) = -\underline{\hat{\phi}}\left(\frac{E_0}{\eta_0}\right)\psi_t(x,y)\, e^{jk_z z}$$

$$H_y^{ipw}\left(x,y,0\right) = \left(\frac{E_0}{\eta_0}\right)\left(-\cos\phi\right)\psi_t(x,y)$$

$$\underline{\hat{p}} = \underline{\hat{\phi}}:$$

$$\underline{H}^{ipw} = \underline{\hat{\theta}}\frac{E_\phi^{ipw}}{\eta_0} = \underline{\hat{\theta}}\left(\frac{E_0}{\eta_0}\right)\psi\left(x,y,z\right) = \underline{\hat{\theta}}\left(\frac{E_0}{\eta_0}\right)\psi_t(x,y)\, e^{jk_z z}$$

$$H_y^{ipw}\left(x,y,0\right) = \left(\frac{E_0}{\eta_0}\right)\left(\cos\theta\sin\phi\right)\psi_t(x,y)$$

Magnetic Current Model (cont.)

Therefore we have

$$H_y^{pw}(x,y,z) = e^{j(k_x x + k_y y)} I^+ \left(1 - \Gamma^{TM/TE}\right) \sec(k_{z1} h) \cos k_{z1}(z+h)$$

where

$$I^+ = \left(\frac{E_0}{\eta_0}\right)(-\cos\phi), \;\; E_\theta \;\; (TM_z)$$

$$I^+ = \left(\frac{E_0}{\eta_0}\right)(\cos\theta \sin\phi), \;\; E_\phi \;\; (TE_z)$$

We can thus now consider

$M_{sy} = -1$

$$E_p^{FF}(r,\theta,\phi) = -\int_{S_L + S_R} H_y^{pw}(x,y,z)(-1)\, dS$$

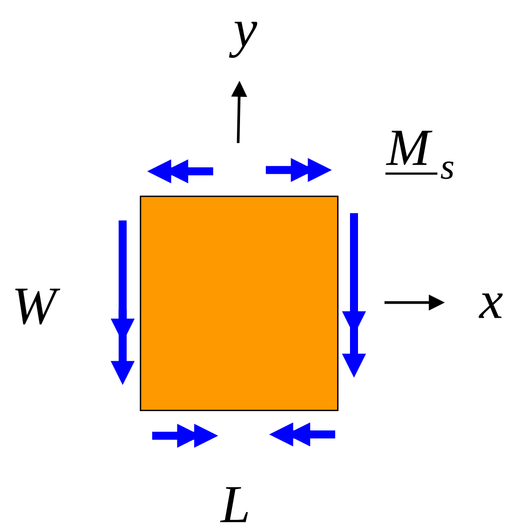
Magnetic Current Model (cont.)

Consider S_R :

$$x = \frac{L}{2}$$

$$y \in \left(-\frac{W}{2}, \frac{W}{2}\right)$$

$$z \in (-h, 0)$$



Therefore we have

$$E_p^{right}(r,\theta,\phi) = \int_{-h}^{0} \int_{-\frac{W}{2}}^{+\frac{W}{2}} e^{j\left(k_x \frac{L}{2}\right)} e^{jk_y y} I^+ \left(1 - \Gamma^{TM/TE}\right) \sec\left(k_{z1} h\right) \cos k_{z1}\left(z+h\right) \, dy \, dz$$

Magnetic Current Model (cont.)

$$E_p^{right}(r,\theta,\phi) = I^{+}(1-\Gamma)\sec(k_{z1}h)\,e^{j\left(k_x\frac{L}{2}\right)} \underbrace{\int_{-\frac{W}{2}}^{+\frac{W}{2}} e^{jk_y y}\,dy}_{\text{integral1}} \underbrace{\int_{-h}^{0} \cos k_{z1}(z+h)\,dz}_{\text{integral2}}$$

Integral 1: $W\,\mathrm{sinc}\left(\dfrac{k_y W}{2}\right)$

Integral 2: $h\,\mathrm{sinc}(k_{z1}h)$

Hence we have

$$E_p^{right}(r,\theta,\phi) = I^{+}\left(1-\Gamma^{TM/TE}\right)e^{jk_x\frac{L}{2}}(Wh)\,\mathrm{sinc}\left(\frac{k_y W}{2}\right)\mathrm{tanc}(k_{z1}h)$$

where $\mathrm{tanc}(x) \equiv \dfrac{\tan x}{x}$

Magnetic Current Model (cont.)

For S_L : Replace $\dfrac{L}{2} \rightarrow -\dfrac{L}{2}$

For $S_L + S_R$: $e^{jk_x \frac{L}{2}} + e^{-jk_x \frac{L}{2}}$

which yields $2\cos\left(k_x \dfrac{L}{2}\right)$

Hence we have

$$E_p^{FF}(r,\theta,\phi) = I^+\left(1-\Gamma^{TM/TE}\right)2\cos\left(k_x \frac{L}{2}\right)(Wh)\operatorname{sinc}\left(k_y \frac{W}{2}\right)\operatorname{tanc}\left(k_{z1}h\right)$$

Magnetic Current Model (cont.)

Substituting for I^{+} in the TM$_z$ and TE$_z$ cases, we have

$$E_{\theta}^{FF}(r,\theta,\phi) = -2Wh\left(\frac{E_0}{\eta_0}\right)\cos\phi\left(1-\Gamma^{TM}(\theta)\right)\cos\left(k_x\frac{L}{2}\right)\operatorname{sinc}\left(k_y\frac{W}{2}\right)\operatorname{tanc}\left(k_{z1}h\right)$$

$$E_{\phi}^{FF}(r,\theta,\phi) = 2Wh\left(\frac{E_0}{\eta_0}\right)(\cos\theta\sin\phi)\left(1-\Gamma^{TE}(\theta)\right)\cos\left(k_x\frac{L}{2}\right)\operatorname{sinc}\left(k_y\frac{W}{2}\right)\operatorname{tanc}\left(k_{z1}h\right)$$

$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi r}\right)e^{-jk_0 r}$$

$$k_x = k_0\sin\theta\cos\phi$$
$$k_y = k_0\sin\theta\sin\phi$$

Magnetic Current Model (cont.)

Examine the reflection coefficient term:

$$1-\Gamma = 1-\left(\frac{Z_{in}-Z_0}{Z_{in}+Z_0}\right)$$

$$= \frac{2Z_0}{Z_{in}+Z_0}$$

$$\Gamma = \Gamma^{TM} \quad \text{for} \quad \underline{\hat{p}} = \underline{\hat{\theta}}$$

$$\Gamma = \Gamma^{TE} \quad \text{for} \quad \underline{\hat{p}} = \underline{\hat{\phi}}$$

$$Z_{in}^{TM/TE} = jZ_1^{TM/TE}\tan\left(k_{z1}h\right)$$

$$= jZ_1^{TM/TE}\tan\left(k_0 N_1(\theta)h\right)$$

$$Z_1^{TM} = \frac{\eta_0}{\varepsilon_r} N_1(\theta)$$

$$Z_1^{TE} = \frac{\eta_0 \mu_r}{N_1(\theta)}$$

$$Z_0^{TM} = \eta_0 \cos\theta$$

$$Z_0^{TE} = \frac{\eta_0}{\cos\theta}$$

Magnetic Current Model (cont.)

Substituting in for the impedances and simplifying the results, we have

$$1-\Gamma^{TM}(\theta)=\frac{2}{1+j\left(\dfrac{N_1(\theta)\sec\theta}{\varepsilon_r}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$

$$1-\Gamma^{TE}(\theta)=\frac{2}{1+j\left(\dfrac{\mu_r\cos\theta}{N_1(\theta)}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$