ECE 6345

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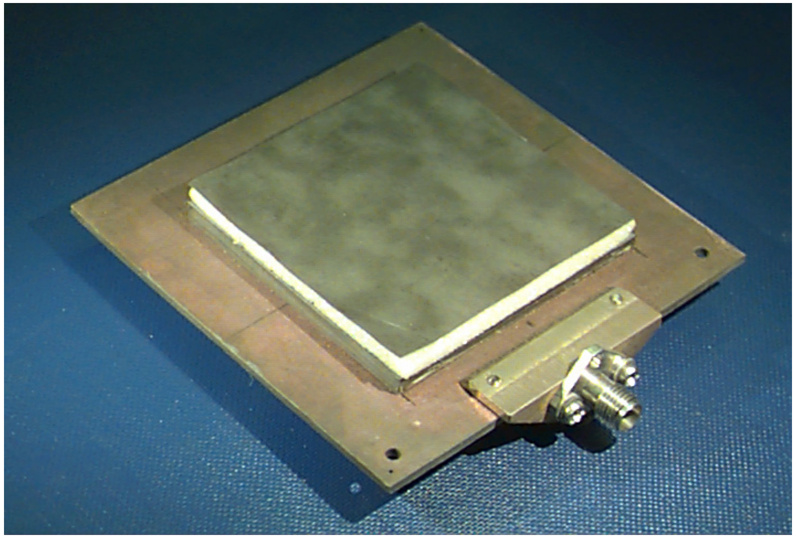
Prof. David R. Jackson
ECE Dept.



Notes 12

Overview

In this set of notes we derive a general expression for the "p factor" that is used to determine the space-wave power radiated by the rectangular patch.

In the next set of notes we will evaluate the integrals that appear and actually develop a final closed-form CAD expression for the p factor.

The p Factor

Definition of the p factor:

$$p \equiv \frac{P_{sp}}{P_{sp}^{dip}}$$

P_{sp} = power radiated by the actual rectangular patch

P_{sp}^{dip} = power radiated by a dipole that has the equivalent dipole moment

$$\left(Il\right)_{patch} = \frac{2}{\pi}\left(WL\right)$$

The *p* Factor (cont.)

We then have

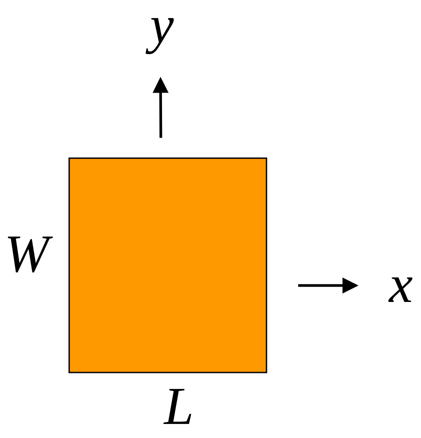
$$P_{sp} = P_{sp}^{dip}\, p$$

From Notes 11, we have

$$P_{sp}^{dip} = (Il)^2 k_0^2 \left(\frac{\eta_0}{32\pi}\right) \int_0^{\pi/2} \left[|F(\theta)|^2 + |G(\theta)|^2 \right] \sin\theta \, d\theta$$

$$P_{sp}^{dip} \approx \left(\frac{2}{\pi} WL\right)^2 (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi}\right) \mu_r^2 c_1$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$



This assumes

$$J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L}\right)$$

The p Factor (cont.)

Calculation of the space-wave radiated power:

$$P_{sp} = \int_0^{2\pi} \int_0^{\pi/2} S_r(r,\theta,\phi)\, r^2 \sin\theta \, d\theta \, d\phi$$

where

$$S_r = \frac{1}{2\eta_0}\left[\left|E_\theta\right|^2 + \left|E_\phi\right|^2 \right]$$

and

$$E_p\left(r,\theta,\phi\right) = E_p^{hex}\left(r,\theta,\phi\right) \;\; \tilde{J}_{sx}^{1,0}\left(k_x, k_y\right) \qquad \left(p = \theta \text{ or } \phi\right)$$

and E_p^{hex} = far field of unit-amplitude horizontal electric dipole in the x direction.

The *p* Factor (cont.)

Denote the array factor as $A(\theta,\phi)=\tilde{J}_{sx}^{1,0}\left(k_x,k_y\right)$

Then

$$E_p\left(r,\theta,\phi\right)=E_p^{hex}\left(r,\theta,\phi\right)A\left(\theta,\phi\right)$$

and

$$S_r\left(r,\theta,\phi\right)=S_r^{hex}\left|A\left(\theta,\phi\right)\right|^2$$

The p Factor (cont.)

Hence

$$P_{sp} = \int_0^{2\pi} \int_0^{\pi/2} S_r^{hex} \left| A(\theta, \phi) \right|^2 r^2 \sin\theta \, d\theta \, d\phi$$

We also have

$$P_{sp}^{dip} = \left| (Il)_{patch} \right|^2 \int_0^{2\pi} \int_0^{\pi/2} S_r^{hex} \left(r, \theta, \phi \right) r^2 \sin\theta \, d\theta \, d\phi$$

We can write the moment of the equivalent dipole as

$$\begin{aligned} (Il)_{patch} &= \int_{-L/2}^{+L/2} \int_{-W/2}^{+W/2} J_{sx}^{1,0}(x, y) \, dy \, dx \\ &= \tilde{J}_{sx}^{1,0}(0,0) = A\left(0,0\right) \end{aligned}$$

The *p* Factor (cont.)

Hence

$$p = \frac{\int_0^{2\pi}\int_0^{\pi/2} S_r^{hex}\left(r,\theta,\phi\right)\left|A(\theta,\phi)\right|^2 r^2 \sin\theta \, d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} S_r^{hex}\left(r,\theta,\phi\right)\left|A(0,0)\right|^2 r^2 \sin\theta \, d\theta d\phi}$$

The p Factor (cont.)

This may be written as

$$p = \frac{\int_0^{2\pi}\int_0^{\pi/2}\left[\left|F(\theta)\right|^2 \sin^2\phi + \left|G(\theta)\right|^2 \cos^2\phi\right]\left|A(\theta,\phi)\right|^2 \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi/2}\left[\left|F(\theta)\right|^2 \sin^2\phi + \left|G(\theta)\right|^2 \cos^2\phi\right]\left|A(0,0)\right|^2 \sin\theta \, d\theta \, d\phi}$$

Note: $A(\theta,\phi)$ depends on W, L but not the substrate parameters.

The p Factor (cont.)

The patch array factor is

$$A(\theta,\phi) = \left(\frac{\pi}{2}WL\right)\operatorname{sinc}\left[k_y \frac{W}{2}\right]\left[\frac{\cos\left(k_x \frac{L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(k_x \frac{L}{2}\right)^2}\right]$$

$$k_x = k_0 \sin\theta\cos\phi$$
$$k_y = k_0 \sin\theta\sin\phi$$

As $W, L \to 0$ $\quad A(\theta,\phi) \to \frac{2}{\pi}(WL)$

so $A(\theta,\phi) \to A(0,0)$ $\quad$ Hence $p \to 1$