ECE 6345

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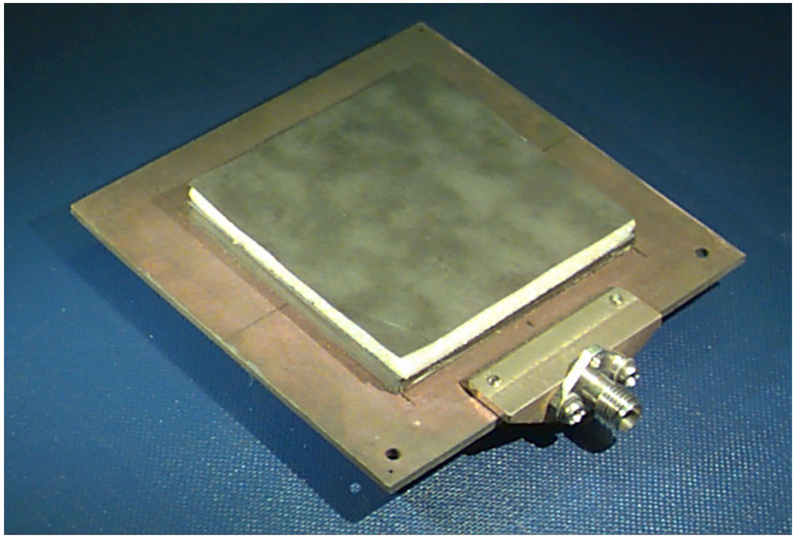

Notes 14

Overview

In this set of notes we derive a CAD formula for the space-wave Q factor (Q_{sp}) of the rectangular patch, using the previously-derived CAD formula for the space-wave radiated power.

CAD Formula for Q_{sp}

Recall $$Q_{sp} = \omega_0 \left(\frac{U_s}{P_{sp}} \right)$$

$$= \omega_0 \left(\frac{2U_H}{P_{sp}} \right)$$

We have, from Notes 12,

$$P_{sp} = p\, P_{sp}^{dip}$$

$$= p \left(\frac{2}{\pi} WL \right)^2 (k_0 h)^2 \, k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1$$

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

Where it is assumed that $$J_{sx}^{1,0} = \cos\left(\frac{\pi x}{L} \right)$$

(The origin is at the center of the patch.)

CAD Formula for Q_{sp} (cont.)

To calculate the stored magnetic energy, use $\underline{H} = \underline{\hat{y}} \cos\left(\frac{\pi x}{L}\right)$

The stored magnetic energy is then

$$
\begin{aligned}
U_H &= \frac{1}{4} \int_V \mu_0 \mu_r H_y^2 \, dV \\
&= \frac{1}{4} h \int_S \mu_0 \mu_r H_y^2 \, dS \\
&= \frac{1}{4} h W \mu_0 \mu_r \int_{-L/2}^{L/2} \cos^2\left(\frac{\pi x}{L}\right) dx \\
&= \frac{1}{4} h W \mu_0 \mu_r \left(\frac{L}{2}\right) \\
&= \frac{1}{8} (hWL) \mu_0 \mu_r
\end{aligned}
$$

CAD Formula for Q_{sp} (cont.)

Hence

$$Q_{sp} = 2\omega_0 \left(\frac{\frac{1}{8} hWL \;\; \mu_0 \mu_r}{P_{sp}} \right) = \frac{2\omega_0 \left(\frac{1}{8} hWL \right) \mu_0 \mu_r}{p \left(\frac{2}{\pi} WL \right)^2 (k_0 h)^2 \, k_0^2 \left(\frac{\eta_0}{6\pi} \right) \mu_r^2 c_1}$$

Simplify, using $k_1 L = \pi$

or $k_0 L \sqrt{\mu_r \varepsilon_r} = \pi$

so $k_0 = \dfrac{\pi}{L} \dfrac{1}{\sqrt{\mu_r \varepsilon_r}}$

CAD Formula for Q_{sp} (cont.)

Also, use

$$\omega_0 = \frac{k_0}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \left(\frac{\pi}{L} \frac{1}{\sqrt{\mu_r \varepsilon_r}} \right)$$

This eliminates k_0 and ω_0 and leaves only the patch dimensions.

The result is

$$Q_{sp} = \frac{3\pi}{8} \varepsilon_r \left(\frac{L}{W} \right) \left(\frac{1}{k_0 h} \right) \left(\frac{1}{c_1} \right) \left(\frac{1}{p} \right)$$

or

$$\frac{1}{Q_{sp}} = \frac{8}{3\pi} (k_0 h) \left(\frac{W}{L} \right) \left(\frac{1}{\varepsilon_r} \right) c_1 p$$

Recall: $BW = \frac{1}{\sqrt{2}Q}$

$$\frac{1}{Q} = \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} + \frac{1}{Q_c} + \frac{1}{Q_d}$$

CAD Formula for Q_{sp} (cont.)

Recall that

$$p \approx 1 + \frac{a_2}{10}\left(k_0 W\right)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)\left(k_0 W\right)^4 + c_2\left(\frac{1}{5}\right)\left(k_0 L\right)^2 + a_2 c_2\left(\frac{1}{70}\right)\left(k_0 W\right)^2\left(k_0 L\right)^2$$

where

$$a_2 = -0.16605$$
$$a_4 = 0.00761$$
$$c_2 = -0.0914153$$

Also, recall that

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$