ECE 6345

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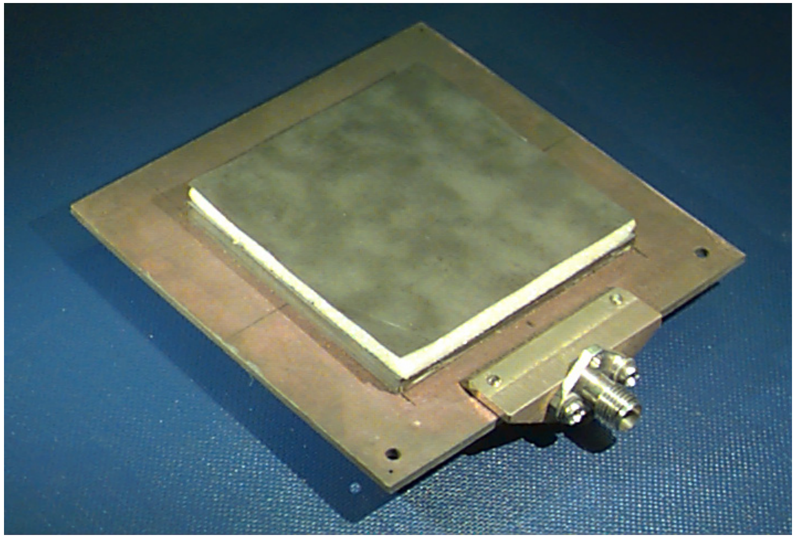
Prof. David R. Jackson
ECE Dept.



Notes 15

Overview

In this set of notes we calculate CAD formulas for the directivity and gain of the rectangular patch antenna.

Directivity

Definition of directivity:

$$D(\theta,\phi) \equiv \frac{S_r(r,\theta,\phi)}{\left(\dfrac{P_{sp}}{4\pi r^2}\right)} = \frac{4\pi r^2 S_r(r,\theta,\phi)}{P_{sp}}$$

For typical substrate thicknesses, we usually have

$$D_{max} = D(0,0)$$

Note: The angle ϕ is actually arbitrary when $\theta = 0$, but we choose $\phi = 0$.

where

$$D(0,0) = \frac{4\pi r^2 S_r(r,0,0)}{P_{sp}}$$

Directivity (cont.)

The space-wave radiated power of the patch is (from Notes 12)

$$P_{sp} = p\, P_{sp}^{dip}$$

The radiated power density from the patch in the far field is

$$\begin{aligned} S_r\left(r,0,0\right) &= S_r^{hex}\left(r,0,0\right)\left|A(0,0)\right|^2 \\ &= S_r^{hex}\left(r,0,0\right)\left|(I\,l)_{patch}\right|^2 \\ &= S_r^{dip}(r,0,0) \end{aligned}$$

$$D(0,0) = \frac{4\pi r^2 S_r\left(r,0,0\right)}{P_{sp}} = \frac{4\pi r^2 S_r^{dip}(r,0,0)}{pP_{sp}^{dip}} = \frac{1}{p}\left(\frac{4\pi r^2 S_r^{dip}(r,0,0)}{P_{sp}^{dip}}\right)$$

Directivity (cont.)

Hence

$$D(0,0) = \frac{1}{p} D^{dip}(0,0)$$

We next calculate the directivity of the dipole:

$$D^{dip}(0,0) = \frac{4\pi r^2 S_r^{dip}(r,0,0)}{P_{sp}^{dip}}$$

From previous calculations in Notes 11:

$$P_{sp}^{dip} \approx (Il)^2 (k_0 h)^2 k_0^2 \left(\frac{\eta_0}{6\pi}\right) \mu_r^2 c_1$$

Directivity (cont.)

We then need

$$S_r^{dip}\left(r,0,0\right)=\frac{1}{2\eta_0}\left[\left|E_\theta^{dip}\left(0,0\right)\right|^2+\left|E_\phi^{dip}\left(0,0\right)\right|^2\right]$$

where

$$E_\theta(r,\theta,\phi)=(I\,l)\,E_0\cos\phi\;G(\theta)$$
$$E_\phi(r,\theta,\phi)=(I\,l)\,E_0\left(-\sin\phi\right)F(\theta)$$

So that

$$E_\theta(r,0,0)=(I\,l)\,E_0\,G(0)$$
$$E_\phi(r,0,0)=0$$

Directivity (cont.)

Hence

$$S_r^{dip}(r,0,0) = \frac{1}{2\eta_0} |I\,l|^2 \, |E_0|^2 \, |G(0)|^2$$

where

$$G(\theta) = \frac{2\cos\theta}{1 - j\left(\dfrac{N_1(\theta)\sec\theta}{\mu_r}\right)\cot\left(k_0 h N_1(\theta)\right)}$$

Note:

$$k_0 h\, N_1(\theta) = k_0 h \sqrt{n_1^2 - \sin^2\theta}$$

$$k_0 h\, N_1(0) = k_0 h\, n_1 = k_1 h$$

so

$$G(0) = \frac{2}{1 - j\left(\dfrac{n_1}{\mu_r}\right)\cot\left(k_1 h\right)}$$

Directivity (cont.)

We then have

$$S_r^{dip}(r,0,0) = \frac{1}{2\eta_0}|Il|^2 \left(\frac{\omega\mu_0}{4\pi r}\right)^2 \frac{4}{1+\left(\dfrac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}$$

We can re-write this using: $\omega\mu_0 = k_0\eta_0$

$$S_r^{dip}(r,0,0) = \eta_0 |Il|^2 k_0^2 \left(\frac{1}{4\pi r}\right)^2 \left(\frac{2}{1+\left(\dfrac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}\right)$$

Directivity (cont.)

To summarize so far, we have

$$D^{dip}(0,0)=\frac{4\pi r^2 S_r^{dip}(r,0,0)}{P_{sp}^{dip}}$$

with

$$S_r^{dip}(r,0,0)=\eta_0\left|Il\right|^2 k_0^2\left(\frac{1}{4\pi r}\right)^2\left(\frac{2}{1+\left(\frac{\varepsilon_r}{\mu_r}\right)\cot^2(k_1 h)}\right)$$

$$P_{sp}^{dip}=\left|Il\right|^2(k_0 h)^2 k_0^2\left(\frac{\eta_0}{6\pi}\right)\mu_r^2 c_1$$

Directivity (cont.)

The result is

$$D^{dip}(0,0) = 3\left(\frac{1}{k_0 h}\right)^2 \left(\frac{1}{\mu_r^2 c_1}\right)\left[\frac{1}{1+\dfrac{\varepsilon_r}{\mu_r}\cot^2(k_1 h)}\right]$$

This may be re-written as

$$D^{dip}(0,0) = 3\left(\frac{\tan(k_1 h)}{k_1 h}\right)^2 \left(\frac{k_1}{k_0}\right)^2 \left(\frac{1}{\mu_r^2 c_1}\right)\left[\frac{1}{\tan^2(k_1 h)+\dfrac{\varepsilon_r}{\mu_r}}\right]$$

Directivity (cont.)

or

$$D^{dip}(0,0) = 3\left(\frac{\tan(k_1 h)}{k_1 h}\right)^2 \left(\frac{\varepsilon_r}{\mu_r c_1}\right)\left[\frac{1}{\tan^2(k_1 h) + \dfrac{\varepsilon_r}{\mu_r}}\right]$$

or

$$D^{dip}(0,0) = 3\,\mathrm{tanc}^2\left(k_1 h\right)\left(\frac{1}{c_1}\right)\left[\frac{1}{1 + \dfrac{\mu_r}{\varepsilon_r}\tan^2(k_1 h)}\right]$$

where $\mathrm{tanc}\left(x\right) \equiv \dfrac{\tan\left(x\right)}{x}$

Directivity (cont.)

Since the substrate is assumed to be thin, we can further approximate this as

$$D^{dip}(0,0) \approx \frac{3}{c_1}$$

where $$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

Note: for $n_1 >> 1,\ D^{dip}(0,0) \approx 3$

$$\left(D^{dip}(0,0) \approx 4.77 \text{ dB} \right)$$

Directivity (cont.)

For the patch we have:

$$D(0,0) = \frac{1}{p} D^{dip}(0,0) \approx \frac{1}{p} 3 \,\mathrm{tanc}^2(k_1 h) \left(\frac{1}{c_1}\right) \left[\frac{1}{1 + \frac{\mu_r}{\varepsilon_r} \tan^2(k_1 h)} \right] \approx \frac{3}{p c_1}$$

where

$$c_1 = 1 - \frac{1}{n_1^2} + \frac{2/5}{n_1^4}$$

and

$$p = 1 + \frac{a_2}{10}(k_0 W)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)(k_0 W)^4 + c_2\left(\frac{1}{5}\right)(k_0 L)^2 + a_2 c_2 \left(\frac{1}{70}\right)(k_0 W)^2 (k_0 L)^2$$

with

$$a_2 = -0.16605$$
$$a_4 = 0.00761$$
$$c_2 = -0.0914153$$

Gain

The gain of the patch is related to the directivity as

$$G\left(0,0\right) = D(0,0)\,e_r$$

where

$$e_r = \frac{Q}{Q_{sp}}$$

and

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}$$

Note: CAD formulas for all Q's have now been derived, except for Q_{sw}.

CAD formulas for all of the Q factors were presented in Notes 3.