ECE 6345

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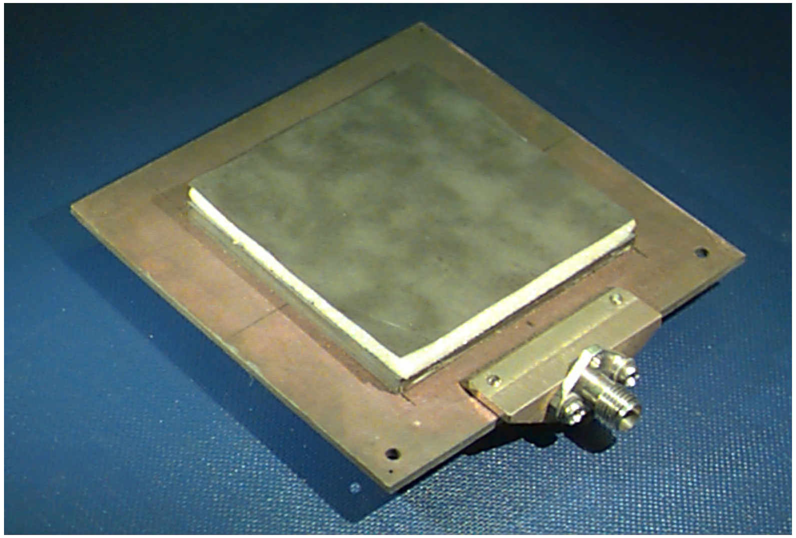

Notes 18

Overview

In this set of notes we calculate a CAD formula for the directivity of the circular patch, which is accurate for a thin substrate.

The formula is based on a CAD formula for the p factor that was discussed in Notes 18.

Directivity

We have:

$$D(0,0) \equiv \frac{4\pi r^2 S_r(r,0,0)}{P_{sp}}$$

where

$$S_r(r,\theta,\phi) = \frac{1}{2\eta_0}\left[\left|E_\theta\right|^2 + \left|E_\phi\right|^2\right]$$

$$E_\theta(r,\theta,\phi) = \frac{E_0}{\eta_0}(ah)\cos\phi\,\mathrm{tanc}(k_{z1}h)\,Q(\theta)\;2\pi\,J_1'(k_0 a\sin\theta)$$

$$E_\phi(r,\theta,\phi) = -\frac{E_0}{\eta_0}(ah)\sin\phi\,\mathrm{tanc}\left(k_{z1}h\right)P(\theta)\;2\pi J_{inc}(k_o a\sin\theta)$$

$$\left|E_0\right| = \frac{\omega\,\mu_0}{4\pi\,r} = \frac{k_0\,\eta_0}{4\pi\,r}$$

Directivity (cont.)

We thus have

$$4\pi r^2 S_r(r,\theta,\phi) = \frac{\pi}{2\eta_0}(k_0 a)^2 h^2 \,\mathrm{tanc}^2(k_{z1}h)$$
$$\cdot\left[\cos^2\phi\left|Q(\theta)\right|^2 J_1'^2(k_0 a\sin\theta) + \sin^2\phi\left|P(\theta)\right|^2 J_{inc}^2(k_0 a\sin\theta)\right]$$

We now let $\theta \to 0$ to calculate the numerator of the directivity expression.

As $x \to 0$:

$$\theta \to 0 \quad \Rightarrow \quad x \to 0$$

$$(x \equiv k_0 a \sin\theta)$$

$$J_1'(x) \to \frac{1}{2}$$

$$J_{inc}'(x) = \frac{J_1(x)}{x} \to \frac{1}{2}$$

Directivity (cont.)

Hence

$$4\pi r^2 S_r(r,0,0) = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 \,\mathrm{tanc}^2(k_{z1}h)$$

$$\cdot\left[\cos^2\phi\left|Q(0)\right|^2 + \sin^2\phi\left|P(0)\right|^2\right]$$

where

$$P(\theta) = \cos\theta\,(1-\Gamma^{TE}(\theta)) = \frac{2\cos\theta}{1+j\left(\dfrac{\mu_r\cos\theta}{N_1(\theta)}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$

$$Q(\theta) = 1-\Gamma^{TM}(\theta) = \frac{2}{1+j\left(\dfrac{N_1(\theta)\sec\theta}{\varepsilon_r}\right)\tan\left(k_0 h\, N_1(\theta)\right)}$$

$$N_1(\theta) = \sqrt{n_1^2 - \sin^2\theta}$$

$$n_1 = \sqrt{\varepsilon_r \mu_r}$$

We then see that

$$P(0) = Q(0) = \frac{2}{1+j\sqrt{\dfrac{\mu_r}{\varepsilon_r}}\tan(k_1 h)}$$

Directivity (cont.)

We then have that

$$4\pi r^2 S_r(r,0,0) = \frac{\pi}{8\eta_0}(k_0 a)^2 h^2 \operatorname{tanc}^2(k_{z1} h)$$

$$.\left[\cos^2\phi + \sin^2\phi\right] \left|P(0)\right|^2$$

where

$$\left|P(0)\right|^2 = \frac{4}{1 + \left(\dfrac{\mu_r}{\varepsilon_r}\right)\tan^2(k_1 h)}$$

Directivity (cont.)

Hence, we have

$$4\pi r^2 \, S_r\left(r,0,\theta\right) = \frac{\pi}{8\eta_0}\left(k_0 a\right)^2 h^2 \, \mathrm{tanc}^2\left(k_1 h\right)\left[\frac{4}{1+\left(\dfrac{\mu_r}{\varepsilon_r}\right)\tan^2\left(k_1 h\right)}\right]$$

For the denominator in the directivity formula, we have from Notes 16

$$P_{sp} = \frac{\pi}{8\eta_0}\left(k_0 a\right)^2 h^2 \, I_c$$

$$= \frac{\pi}{8\eta_0}\left(k_0 a\right)^2 h^2 \, I_0 p$$

Directivity (cont.)

Hence

$$D(0,0)=\frac{\dfrac{\pi}{8\eta_0}(k_0a)^2h^2\,\mathrm{tanc}^2(k_1h)\left[\dfrac{4}{1+\left(\dfrac{\mu_r}{\varepsilon_r}\right)\tan^2(k_1h)}\right]}{\dfrac{\pi}{8\eta_0}(k_0a)^2h^2\,I_0p}$$

Simplifying, we have

$$D(0,0)=\frac{4\,\mathrm{tanc}^2(k_1h)}{p\,I_0\left[1+\left(\dfrac{\mu_r}{\varepsilon_r}\right)\tan^2(k_1h)\right]}$$

Directivity (cont.)

Since we are assuming that $h / \lambda_0 << 1$

then $D(0,0) \approx \dfrac{4}{p\, I_0}$

For the I_0 term, we have from Notes 17 that $I_0 \approx \dfrac{4}{3}$

Hence $D(0,0) \approx \dfrac{3}{p}$

Summary

$$D(0,0)=\frac{4\,\tan\mathrm{c}^2(k_1 h)}{p\,I_0\left[1+\left(\dfrac{\mu_r}{\varepsilon_r}\right)\tan^2(k_1 h)\right]}$$

or

$$D(0,0)\approx\frac{3}{p}$$

where

$$I_0\approx\frac{4}{3}$$

$$p\approx\sum_{k=0}^{6}(k_0 a)^{2k}\;e_{2k}$$

with

$$\begin{aligned}
e_0 &= 1\\
e_2 &= -0.400000\\
e_4 &= 0.0785710\\
e_6 &= -7.27509\times10^{-3}\\
e_8 &= 3.81786\times10^{-4}\\
e_{10} &= -1.09839\times10^{-5}\\
e_{12} &= 1.47731\times10^{-7}
\end{aligned}$$