ECE 6345
Spring 2015

Homework 1

Please do Probs. 1, 3, 4, 5, 7, 8, 9, 10.

1) Use the method of separation of variables to show that the electric field inside a rectangular patch resonator that has PMC walls on the sides has the form

$$E_z(x,y) = \cos\left(\frac{m\pi x}{L}\right)\cos\left(\frac{n\pi y}{W}\right),$$

where

$$k_1^2 = k_0^2\varepsilon_r = \left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2.$$

(The dimensions and coordinate system are the same as that used in class, where the origin is at the lower left-hand corner of the patch, and the length of the patch in the *x* direction is *L*.)

Start by assuming that the electric field has the form

$$E_z(x,y) = X(x)Y(y)$$

and insert this into the scalar Helmholtz equation for E_z.

Note: The method of separation of variables was covered in ECE 6340 for a rectangular waveguide with PEC walls. You can also find it in many standard EM and math textbooks.

2) Assume a perfectly conducting rectangular patch with the same geometry and field as in the problem above, operating in the usual (1,0) mode. Apply the equivalence principle to show how the patch may be replaced by equivalent currents radiating in the presence of the ground plane and substrate (without the patch metal).

   a) **Electric current model:** Use an equivalence surface that hugs the patch metal, and show how that patch may be replaced by an equivalent surface current $\underline{J}_s$ that is the sum of the surface currents that exist on the top and bottom surfaces of the metal patch. Note: You do not have to assume PMC walls. This equivalence is exact. Next, assume that the patch has PMC walls on the edges, and justify that the equivalent surface current then has the form

$$\underline{J}_s = \underline{\hat{x}}\, A \sin\left(\frac{\pi x}{L}\right).$$

b) **Magnetic current model:** Assume PMC walls and use an equivalence surface that surrounds the entire patch cavity to show how the patch may be removed and replaced by an equivalent magnetic surface current that exists at the edges of the patch (i.e., where the patch edges used to be). This magnetic current is given by

$$\underline{M}_s = -\underline{\hat{y}}, \quad x = 0 \text{ and } L \quad \text{(radiating edges)}$$

$$\underline{M}_s = \pm\underline{\hat{x}}\cos\frac{\pi x}{L}, \quad y = 0 \text{ and } W \text{ (non-radiating edges).}$$

(The + and - signs go along with y = 0 and y = W, respectively.)

3) Using the CAD formulas in the Jackson short-course notes, design a probe-fed rectangular patch antenna on a substrate having a relative permittivity of 2.2 and a thickness of 60 mils (0.1524 cm). Choose an aspect ratio of W/L = 1.5. The patch should resonate at the operating frequency of 1.575 GHz (the GPS L1 frequency). Ignore the probe inductance in your design, but account for fringing at the patch edges when you determine the dimensions. At the operating frequency the input impedance should be 50 Ω (ignoring the probe inductance). Assume an SMA connector is used to feed the patch along the centerline (at y = W / 2), and that the inner conductor of the SMA connector has a radius of 0.635 mm. The copper patch and ground plane have a conductivity of $\sigma = 3.0 \times 10^7$ S/m and the dielectric substrate has a loss tangent of $\tan\delta = 0.001$.

Your design should include the following (please summarize your final values in a table):

a) The final patch dimensions L and W (in cm).

b) The feed location x_0 (distance of the feed from the closest patch edge, in cm).

c) The bandwidth of the antenna (SWR< 2 definition, expressed in percent).

d) The radiation efficiency of the antenna (accounting for conductor, dielectric, and surface-wave loss, and expressed in percent).

e) The probe reactance X_p at the operating frequency (in Ω).

f) The expected complex input impedance (in Ω) at the operating frequency, accounting for the probe inductance.

4) Calculate numerical values for the four fundamental circuit elements (R, L, C, L_p) that characterize the patch in the above problem.

5) Plot the real and imaginary parts of the input impedance versus frequency for the patch in the above problem, using the circuit model and the values found from the problem above. Plot over the range from 1.53 to 1.62 GHz.

6) Starting with the CAD formulas in the short-course notes, obtain a CAD formula for the gain at broadside of a rectangular patch operating in the usual (1,0) mode. (Gain is the product of directivity and radiation efficinecy.)

   Plot the broadside gain vs. normalized substrate thickness h / λ_0. Assume $W/L = 1.5$. The copper patch and ground plane have a conductivity of $\sigma = 3.0 \times 10^7$ S/m and the dielectric substrate has a relative permittivity of 2.2 and a loss tangent of $\tan\delta = 0.001$. Assume an operating frequency of 1.575 GHz. Ignore fringing effects. (Hence, the length L is assumed to be fixed in this problem, and does not depend on the substrate thickness.)

7) Assume a linear one-dimensional N-element E-plane microstrip array along the x direction, with a spacing p between the centers of adjacent elements. The elements are numbered 1, 2, 3…N as x increases. Assume that the TM_0 surface wave propagates with a wavenumber β_{TM0}, where $k_0 < \beta_{TM0} < k_1$. Also, assume that the array is scanned to produce a beam in the E-plane at an angle θ_0 with respect to broadside (with a positive angle θ_0 corresponding to a beam pointing in the region $x > 0$). This means that element number n has a phase of $-n\Delta$ radians, where $\Delta = k_0\, p \sin\theta_0$. That is, Δ is the phase delay between adjacent patches as we move in the positive x direction.

   Scan blindness will occur when

$$\pm\beta_{TM0}\, p = \Delta + 2\pi m ,$$

   where m is an integer (positive or negative). The scan-blindness condition means that the surface-wave field excited by each patch adds up in phase from one patch to the next (as the surface wave moves in either the x or $-x$ direction).

   Using the diagram below as a suggested aid, explain why scan blindness will not occur for any scan angle θ_0, provided the period p is chosen small enough so that

$$\frac{2\pi}{p} > \beta_{TM0} + k_0 .$$

   Hint: Divide the scan blindness equation by p and note that $|\Delta / p| = |k_0 \sin\theta_0| < k_0$. Consider $m = 0$ and $m = 1, -1$.

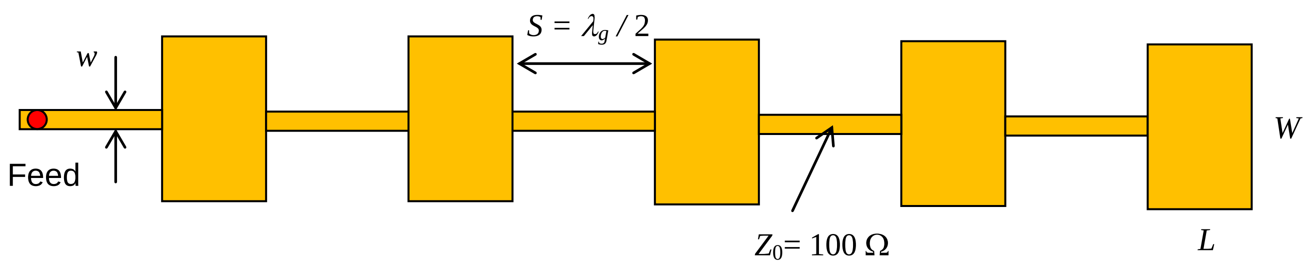
8) As a continuation of the previous problem, calculate the answers to the following questions.

a) Assume that $\beta_{TM0} = 1.2\ k_0$ for a particular substrate, at a frequency of 10 GHz. What is the largest value that the period p can be chosen to be in order to completely avoid scan blindness at any scan angle θ_0?

b) Now assume that the period is 0.75 λ_0 and that $\beta_{TM0} = 1.2\ k_0$. What is the maximum scan angle θ_0 that we can scan the array to before we hit scan blindness?

9) Design a 5 element series-fed array operating at 1.575 GHz, using the substrate and patches in Prob. 3. Determine the length and width of the lines that connect the patches using a CAD program such as TX Line. Assume that there is no line or load connected to the right of the last patch, while the first patch has a $\lambda_g/2$ microstrip line connected to its left edge, as shown below. All patches and lines have identical dimensions.

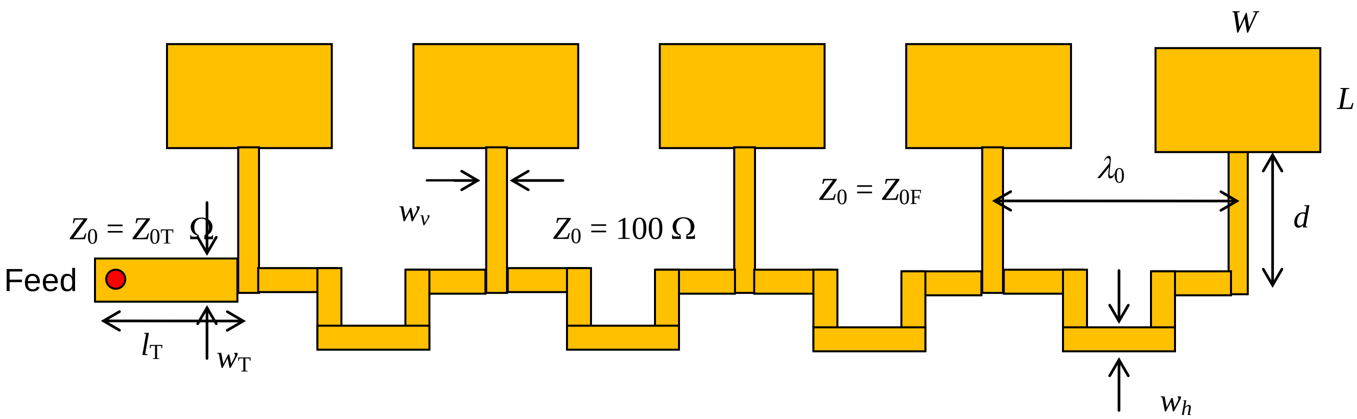
Make a table that shows all of the final dimensions: S, w, L, W.



Series-fed patch array.

10) Design a 5 element series/parallel array operating at 1.575 GHz, using the substrate and patches in Prob. 3. Determine all line lengths and widths using a CAD program such as TX Line. All patches have identical dimensions, and all connecting lines (to the right of the first patch) have Z_0 = 100 Ω. The vertical lines that connect to the patches have a characteristic impedance $Z_0 = Z_{0F}$ that matches to the edge resistance of the patches (you need to find this). A quarter-wave transformer ($Z_0 = Z_{0T}$) is used at the beginning of the array to match the array to 50 Ω at the feed location. The transformer has length l_T and width w_T. The spacing between the centers of the patches is one free-space wavelength. The total length w_h of the 100 Ω line connecting a patch to its neighbor should be two guided wavelengths on the 100 Ω microstrip line (the exact shape of the bend in the line is not important). The distance d is one-quarter of a guided wavelength on the vertical line.

Make a table that summarizes all final dimensions: L, W, d, w_v, S, w_h, l_T, w_T.



Series/parallel patch array.