ECE 6345
Spring 2015

Homework 2

1) A certain probe-fed microstrip antenna has a resistance of 50 [Ω] at resonance. The bandwidth (defined by SWR < 2) is 2%. The resonance frequency is 2.0 GHz. Assume that the probe inductance is negligible. Use the CAD circuit model to make the following plots.

    a) Plot the input resistance and reactance versus frequency for this antenna, from 1.9 to 2.1 GHz.

    b) Plot $|S_{11}|$ versus frequency over the same range, assuming that the antenna is fed by a 50 [Ω] feed line.

    c) Plot the normalized input impedance for this antenna on the Smith chart, over the same frequency range.

2) Redo the previous problem, assuming now that the probe reactance is $j10$ [Ω]. (The probe reactance may be assumed to be constant over the frequency range of interest.) Comment on how the probe reactance affects the various plots.

3) A probe-fed microstrip antenna has an input resistance of 50 [Ω] at resonance. The probe reactance may be neglected. What is the input impedance of this antenna at the frequencies f_1 and f_2? These are the two frequencies (lower and upper) at which the SWR is 2.0, when the antenna is fed by a 50 [Ω] feed line.

4) Show that when the probe inductance is accounted for, the resonance frequency f_{res} (where in the input impedance is purely real) is given by

$$x_{res} = \frac{1-\sqrt{1-4\bar{X}_p^2}}{2\bar{X}_p}$$

where

$$x_{res} = Q\left(f_r^{res} - \frac{1}{f_r^{res}} \right)$$

$$f_r^{res} = \frac{f_{res}}{f_0}$$

$$\bar{X}_p = X_p / R.$$

Explain why who make any sign choice in your derivation.

5) As a continuation of the previous problem, denote f_0 as the resonance frequency of a probe-fed microstrip antenna when the probe inductance is neglected. Let $f_{res} = f_0 + \Delta f$ be the resonance frequency when the probe inductance is included. That is, f_{res} is the frequency for which the input impedance of the microstrip antenna (including the probe inductance) is a real number. Derive the following approximate formula for the shift in resonance frequency due to the probe inductance:

$$\frac{\Delta f}{f_0} = (\mathrm{BW})\left(\frac{1}{\sqrt{2}}\right)\left(\frac{X_p}{R}\right),$$

where BW is the fractional bandwidth of the antenna (SWR < 2 definition), X_p is the probe reactance (assumed to be constant over the frequency range of interest), and R is the input resistance at frequency f_0.

6) A rectangular microstrip antenna has an edge resistance of 200 [Ω] at the frequency f_0 of the cavity resonance, where we have maximum input resistance. The probe reactance is $j20$ [Ω]. Determine the normalized feed position x_0/L so that the antenna has a real input impedance of 50 [Ω] at the resonance frequency f_{res} (the frequency where the input reactance is zero). Do not make any approximations (i.e., do not assume that the probe reactance is small).

7) Consider a microstrip patch antenna that is gap coupled (with a small gap) to a microstrip line (i.e., there is a capacitive gap between the line and the patch). The gap is represented as a series capacitor C_g that has a reactance X_c. What will the input resistance R_{in}^{res} seen by the feeding line be (at the gap location) when the patch is operating at the resonance frequency where the input impedance is purely real? Put your answer for R_{in}^{res} in terms of X_c, assuming that X_c is small. Assume that the patch is modeled as an RLC circuit with known values of (R, L, C) when the patch is fed by a direct contact at the edge. From your result, explain how the gap coupling can be used to lower the input resistance seen by the feed line, from R to a lower value R_{in}^{res}.

8) As we saw in the short-course notes, using a double-tuned resonator effect is a good way to increase the bandwidth of a microstrip antenna. Consider one microstrip antenna that is capacitively coupled to another one. An approximate model for this is shown below. The capacitor C_0 is the coupling capacitance between the two antennas.

Show that the input admittance of this circuit is approximately

$$Y_{in} = Y_1 + \frac{Y_2}{1 - j\left(Y_2 / B_c\right)},$$

where Y_1 is the input admittance of antenna tank circuit one, Y_2 is the input admittance of antenna tank circuit two, and $B_c = \omega_0 C_0$, where ω_0 is the center frequency of the double-tuned circuit.

The input admittances of the antenna tank circuits can be written as

$$Y_1 = \left(\frac{1}{R_1}\right)\left[1 + jQ_1\left(f_{r1} - 1/f_{r1}\right)\right]$$

$$Y_2 = \left(\frac{1}{R_2}\right)\left[1 + jQ_2\left(f_{r2} - 1/f_{r2}\right)\right]$$

where

$$f_{r1} = \frac{f}{f_{01}}$$

and

$$f_{r2} = \frac{f}{f_{02}}.$$

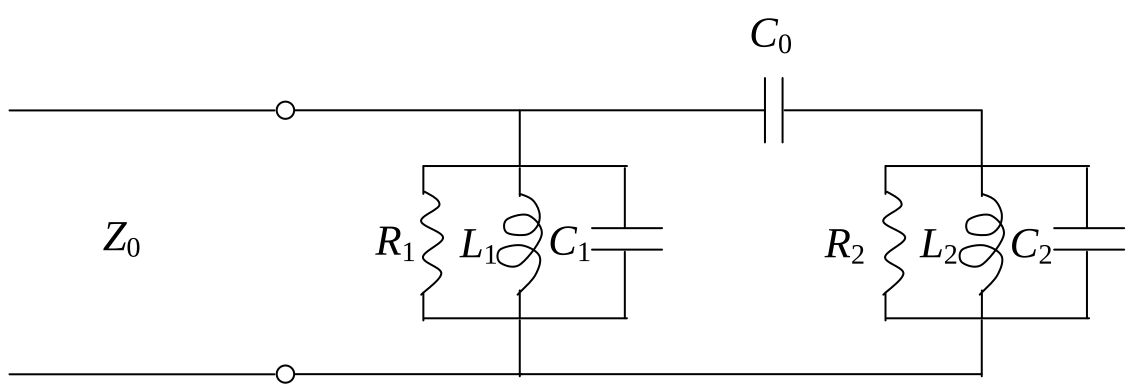
Note that we can write

$$f_{r2} = \frac{f_{r1}}{1+\Delta},$$

where

$$\Delta = \frac{f_{02} - f_{01}}{f_{01}}$$

is the normalized shift in resonance frequencies between the two antenna tank circuits (you should show this).



9) Make a plot of SWR vs. f_{r1} for the following case: $R_1 = 50$ [Ω], $Q_1 = 25$, $R_2 = 100$ [Ω], $Q_2 = 25$, $B_c = 0.0001$ [S], $\Delta = 0.009$, and $Z_0 = 50$ [Ω]. Because of the very small value of B_c, This corresponds to the response of the single resonator 1. Determine the bandwidth from your plot. Compare with the bandwidth from the formula

$$BW = \frac{1}{\sqrt{2}Q_1}.$$

10) Make a plot of SWR vs. f_{r1} for the following case: $R_1 = 75$ [Ω], $Q_1 = 25$, $R_2 = 100$ [Ω], $Q_2 = 25$, $B_c = 0.016$ [S], $\Delta = 0.009$, and $Z_0 = 50$ [Ω]. Verify that a double-tuned response is obtained. Determine the bandwidth from your plot. How much larger is the bandwidth compared to that of the single resonator in the previous problem (i.e., what is the ratio of the two bandwidths)?

Mini Project

(This problem will be worth more than a typical HW problem.)

Try varying the values R_1, R_2, B_c, and Δ in the previous problem, to see what the maximum bandwidth is that you can obtain, assuming that bandwidth is defined from SWR < 2. Keep both Q values at 25, and the feed-line impedance Z_0 at 50 [Ω]. Show the corresponding plot of SWR vs. f_{r1}.

(There is not necessarily a unique best solution to this problem. See how large of a bandwidth you can get!)