ECE 6345
Spring 2015

Homework 6

Do Probs. 1, 2, 3, 4, 5, 8, 9, 10

1) A dielectric slab has a relative permittivity $\varepsilon_r = 2.2$. Plot the normalized wavenumber of the TM_0 surface-wave mode, β_{TM0} / k_0, versus the normalized thickness of the slab, h/λ_0. Plot up to a maximum of $h/\lambda_0 = 0.1$. On the same graph, add a plot of the result from the closed-form CAD formula. The exact wavenumber comes from solving the transcendental equation given on slide 4 of Notes 21. The CAD formula is given on slide 6 of Notes 21. Solve for enough points to make a reasonably smooth plot.

2) Plot the exact radiation efficiency of an infinitesimal horizontal electric dipole on a lossless substrate with a relative permittivity of $\varepsilon_r = 2.2$, versus the normalized thickness of the substrate h/λ_0. Plot up to a maximum of $h/\lambda_0 = 0.1$. The formula for the surface-wave power is given on slide 6 of Notes 23. You may calculate the residue of the function $V_i^{TM}\left(k_t\right)$ numerically in the calculation of the surface-wave power, using a numerical derivative (central-difference approximation) of the D^{TM} function, if you wish (see slide 6 of Notes 23). Or, you can use the expression given on slide 7 of Notes 23 to calculate the derivative term exactly. (You might want to do both and compare as a validation.) Use the exact space-wave power in the calculation, found by integrating the Poynting vector (i.e., the formula on slide 5 of Notes 11).

   On the same graph, add a plot of the radiation efficiency of the dipole obtained by the closed-form approximate CAD formula for the surface-wave efficiency. This formula is on slide 14 of Notes 23.

3) Consider a resonant rectangular patch on a lossless substrate with a relative permittivity ε_r = 2.2. The width to length ratio is $W / L = 1.5$. (Neglect fringing, so that the length is one-half wavelength in the dielectric.) The patch current is taken as

$$J_{sx}\left(x,y\right) = \cos\left(\frac{\pi x}{L}\right),$$

   with x measured from the center of the patch. Plot the exact surface-wave radiation efficiency of the patch, and the approximate surface-wave radiation efficiency using the approximate

CAD formula for the radiation efficiency of the patch. (The surface-wave efficiency of the patch is taken to be the same as that of the dipole in the CAD formula; see slide 2 of Notes 23). For the exact radiation efficiency of the patch, use the two paths shown on slide 12 of Notes 22 to calculate the space-wave power and the total radiated power. Plot versus the normalized thickness of the substrate, h/λ_0, up to a maximum of $h/\lambda_0 = 0.1$. (Note: The surface-wave radiation efficiency e_r^{sw} is the radiation efficiency that accounts only for surface-wave loss, and not conductor or dielectric loss.)

4) A rectangular patch is on a substrate with a relative permittivity $\varepsilon_r = 2.2$ and a thickness of 0.1524 cm (corresponding to 60 mils). The width to length ratio is $W / L = 1.5$. The length of the patch is $L = 6.255$ cm (this gives a resonance frequency of $f_0 = 1.575$ GHz, when using the Hammerstad formula). The patch is fed at a distance of 1.85 cm from the edge in the x (resonant) direction, and along the centerline of the patch in the y direction. The radius of the feed probe is 0.635 mm (corresponding to a standard SMA connector). The loss tangent of the dielectric is 0.001. The patch and the ground plane are both made of copper having a conductivity of 3.0×10^7 S/m.

   Plot the input impedance of the patch (real and imaginary parts) versus frequency using the CAD model (that is, the RLC circuit model that has the probe inductance added in series). Use the CAD formulas for the probe inductance, the input resistance, and the Q value. In other words, use CAD formulas for everything in the formula for the input impedance, which is:

$$Z_{in} \approx jX_p + \frac{R}{1 + jQ\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}.$$

   Although it should not make too much of a difference one way or the other, use the actual physical dimensions of the patch (L and W) in the CAD formula calculations (instead of the effective dimensions) to calculate R and Q. (This will keep the calculation simpler since the actual dimensions of the patch have already been specified above.) However, note that the resonance frequency of the cavity f_0 (1.575 GHz) still corresponds to using the effective patch length. Plot from 1.53 GHz to 1.63 GHz. For the vertical axis, please plot from -30 to 70 Ω.

5) Plot the input impedance for the same patch as in Prob. 4 (real and imaginary parts) versus frequency using the transmission line model. Assume an effective loss tangent that accounts

for all losses, obtained from the CAD formula for Q (the same total Q found in Prob. 4). Include the probe inductance in the calculation, obtained from the CAD formula. Use the same plotting scale as in Prob. 4.

6) Repeat the previous calculation and plot for the same patch as in Prob. 5, now assuming that the transmission line has an effective loss tangent that accounts only for dielectric and conductor losses, and that edge conductances are added on the ends of the line in order to account for radiation losses. Use the same plotting scale as in Prob. 4.

7) Plot the input impedance of the patch (real and imaginary parts) versus frequency using the cavity model with the eigenfunction expansion method. Assume a uniform strip probe current model. Use the same plotting scale as in Prob. 4.

8) Plot the input impedance of the patch (real and imaginary parts) versus frequency using the cavity model with the mode-matching method. Again, assume a uniform strip probe current model. Use the same plotting scale as in Prob. 4.

9) Assume a phased array of rectangular microstrip patches is built using a grounded substrate having $\varepsilon_r = 2.2$ and $h = 0.1524$ cm (corresponding to 60 mils). The frequency is 12 GHz. At this frequency the normalized wavenumber of the TM_0 surface wave is $\beta_{TM0} / k_0 = 1.0225$. Assume that the element spacing is $0.75\ \lambda_0$ in both the x and y directions. Make a Pozar circle diagram for this case that applies for either scan blindness or grating lobes. (Because the normalized wavenumber of the surface wave is so close to unity, the same diagram should apply for both cases.) For ease of plotting, choose a plotting scale so that k_0 corresponds to a convenient dimension (e.g., 4 cm). Please use a drawing tool (e.g., what is in Word or PowerPoint) to make nice circles, so the diagram looks accurate. It is sufficient to draw the visible space circle and the four circles that are the nearest neighbors to it.

10) Use the Pozar circle diagram above to answer the following questions graphically and/or exactly, as indicated in each part.

    - What is the maximum scan angle θ_0 that one can have in the E plane to avoid both scan blindness and grating lobes? Give an exact answer.

    - Assume that one is scanning the main beam at $\phi_0 = 30^{\circ}$. What is the maximum scan angle θ_0 that one can have in this plane to avoid both scan blindness and grating lobes? Give a graphical answer.

- Assume that one is scanning the main beam in the plane $\phi_0 = 30^o$, and that one has scanned the beam angle θ_0 so that scan blindness occurs. At what angle ϕ (with respect to the x axis) will the surface wave field be adding up in phase along the substrate? Give a graphical answer. (For this can calculate the k_{xp} and k_{yq} wavenumbers for the Floquet wave of interest that is causing the scan blindness, using the graphical solution to first get k_{x0} and k_{y0}. Note that the graphical solution can easily tell you which Floquet wave (i.e., which (p,q) values) is causing the scan blindness. Alternatively, you can measure the angle directly from the center of the circle corresponding to the Floquet mode of interest (see if you can convince yourself of this.)

- Assume that one is scanning the main beam in the plane $\phi_0 = 120^o$, and that one has scanned the beam angle to $\theta_0 = 60^o$. At what angles (θ_g, ϕ_g) (in spherical coordinates) will a grating beam point? Give an exact answer. (For this you can calculate the exact k_{xp} and k_{yq} wavenumbers for the Floquet wave of interest that is causing the grating beam, from first calculating k_{x0} and k_{y0}. Although your answer should be exact, use the graphical solution to help you see which Floquet wave is causing the grating beam. Alternatively, you can measure the distance and angle directly from the center of the circle corresponding to the Floquet mode of interest (see if you can convince yourself of this).