NAME: ________________________________

ELEE 6382
Fall 2006

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book and open-notes. You may use your class notes, and a calculator. Please show *all steps of your work* and *write neatly* in order to receive credit.

Please write all of your work on the sheets attached.

Problem 1 (25 pts)

Find the analytic function $w(x, y) = u(x, y) + iv(x, y)$ whose real part is

$$u(x, y) = x^3 - 3xy^2 .$$

Problem 2 (25 pts)

For

$$f(z) = \frac{1}{(z+1)(z+2)}$$

find all the following: in the specified regions: (Hint: Some part of each subproblem below may be used in the next or succeeding subproblems!)

a) A Laurent series representation of $f(z)$ about the origin valid for $|z|<1$.

b) A Laurent series representation of $f(z)$ about the origin valid for $1<|z|<2$

c) A Laurent series representation of $f(z)$ about the origin valid for $|z|>2$.

d) The value of the integral $\oint_C f(z)\,dz$ where C is a circle of radius $r = \frac{1}{2}$ with center at $z = 0$.

e) The same as d) but with $r = \frac{3}{2}$.

f) The same as d) but with $r = e^{\alpha}$, $\alpha > 1$

Problem 3 (25 pts)

Calculate the value of each of the following definite integrals:

a) $\int_0^{2\pi} \frac{d\theta}{a+\cos\theta}, \quad a>1.$

b) $\int_0^\infty \frac{dx}{\left(x^2+1\right)^2\left(x^2+2\right)}$

Problem 4 (25 pts)

Recall that when mathematicians say that *proposition A is true if and only if proposition B is true*, it means that not only does 1) *A imply B* but also 1) *B implies A* **.** (Sometimes "if and only if" is abbreviated "iff".) Alternatively, they say that for proposition *A* to be true it is *necessary and sufficient* that proposition *B* be true. Note that the implication *must* be proved both ways to use the terms "if and only if" or "necessary and sufficient".

Prove that two functions $f_1(x),\ f_2(x)$ are linearly dependent if and only if their Wronskian vanishes. This requires that you prove *both* statements below:

a) Prove that if $f_1(x),\ f_2(x)$ are linearly dependent (i.e., if $f_1(x) = C\,f_2(x)$ for some *C*), then their Wronskian vanishes.

b) Prove that if the Wronskian $W\left[f_{1,}f_2\right]$ vanishes, then $f_1(x),\ f_2(x)$ are linearly dependent.

(Hint: consider writing $W\left[f_{1,}f_2\right]=0$ in the form of a logarithmic derivative, $\frac{d}{dx}\ln f\left(x\right)$.
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ROOM FOR EXTRA WORK