NAME: _____

ELEE 6382
Fall 2007

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book and open-notes. You may use your class notes, and a calculator. Please show *all steps of your work* and *write neatly* in order to receive full credit.

Please write all of your work on the sheets attached.

Problem 1 (40 pts)

Consider the function

$$f(z) = \frac{1}{z^2\left(z^2+1\right)}.$$

a.) Determine the *locations* and *classify* all the singularities of the function in the finite plane.

b.) Determine the residue of the function at each singularity in part a.).

c.) Determine all Laurent series about the point $z = 0$ (write out at least the first three non-vanishing terms) and specify their regions of convergence.

d.) Determine the value of the contour integral $\oint_C f(z)\,dz$ if C is the circle of radius 5 centered at the origin.

e.) Repeat d.) for the contour $C:\ |z+i| = \frac{1}{2}$

Problem 2 (20 pts)

The imaginary part of an analytic function $f(z) = u(x, y) + iv(x, y)$ is $v(x, y) = e^x \cos y$.

a) Find $u(x, y)$ and hence determine $f(z)$ to within an unknown (real) constant.

b) Determine the constant from the condition $f\left(i\frac{\pi}{2}\right) = 0$.

Problem 3 (40 pts)

Calculate the value of each of the following **three** definite integrals:

a) $\int_0^{\infty} \frac{x^2 dx}{\left(x^2+1\right)\left(x^2-1\right)}$

b) $\int_0^{2\pi} \frac{d\theta}{a+\cos\theta}, \quad a>1.$

c) $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2 + a^2}\, dx$

ROOM FOR EXTRA WORK