

NAME: _____

**ELEE 6382
Fall 2008
Oct. 23, 2008**

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book and open-notes. You may use your class notes, and a calculator. Please show *all steps of your work* and *write neatly* in order to receive full credit.

Please write all of your work on the sheets attached.

Problem 1 (40 pts)

Consider the function

$$f(z) = \frac{1}{z^2(z^2 + 4)}.$$

a.) Determine the locations and classify by *kind* (simple pole, pole of order ?, branch point, essential singularity, etc.) all the singularities of $f(z)$ in the *finite* plane.

b.) Determine the residue of the function at each singularity in part a.).

c.) Determine all Laurent series about the point $z = 0$ (write out at least the first three non-vanishing terms) and specify their regions of convergence.

d.) Determine the value of the contour integral $\oint_C f(z) dz$ for the contour $C : |z| = 5$.

Sketch the contour.

e.) Repeat d.) for the contour $C : |z + 2i| = 1$. *Sketch the contour.*

Problem 2 (20 pts)

The imaginary part of an analytic function $f(z) = u(x, y) + iv(x, y)$ is

$$v(x, y) = x^2 - y^2 + 2xy - 3x - 2y.$$

a) Find $u(x, y)$ and hence determine $f(z)$ to within an unknown (real) constant.

b) Determine the constant from the condition $f(0 + i0) = 1$.

Problem 3 (40 pts)

Calculate the value of each of the following **three** definite integrals:

a) $\int_0^{\infty} \frac{dx}{(x^2 + 9)(x^2 + 4)}$

b) $\int_0^{2\pi} \frac{2d\theta}{5-4\cos\theta}$.

c) $\int_{-\infty}^{\infty} \frac{e^{ix}}{x} dx$; use the result to evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$.

ROOM FOR EXTRA WORK