NAME: ______________________________________

ELEE 6382
Fall 2009
Oct. 22, 2009

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book (Arfken and Weber) and open-notes. You may also use your class notes, and a calculator. Please show *all steps of your work* and *write neatly* in order to receive full credit.

Please write all of your work on the sheets attached.

Problem1 (25 pts)

The imaginary part of an analytic function $f(z) = u(x, y) + iv(x, y)$ is

$$v(x, y) = 6xy + e^x \sin y \, .$$

a) Find $u(x, y)$ and hence determine $f(z)$ to within an unknown (real) constant.

b) Determine the constant from the condition $f\left(0 + i0\right) = 0$ and check that your solution satisfies the Cauchy-Riemann conditions.

Problem 2 (25 pts)

Obtain the Laurent or Taylor series, as appropriate, of the function $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the regions given.

a) $|z| < 1$

b) $1 < |z| < 2$

Problem 3 (25 pts)

Calculate the value of each of the following **two** definite integrals:

a) $\int_0^\infty \frac{dx}{(x^2-1)(x^2+4)}$

b) $\int_0^\infty \frac{x \sin ax}{1+x^2} dx, \;\; a > 0$ (Hint: *Both* x and $\sin ax$ are odd functions of *x*.)

Problem 4 (25 pts)

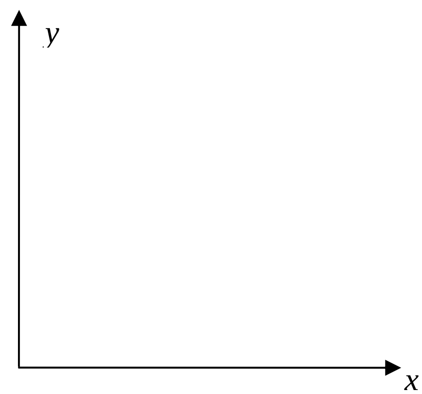
Consider the function

$$f(z) = \frac{z^{\frac{1}{2}}}{\left(z^2 + 4\right)}$$

a.) Determine the locations and classify by *kind* (pole with order, branch point, essential singularities, etc.) all the singularities of $f(z)$ in the *finite* plane. Using the axes given below, sketch the locations in the z-plane of the singularities. For any branch points present, define a *top sheet* by specifying a range of θ in the polar representation of $z = re^{i\theta}$. Show the resulting cut on your sketch.

Singularities:

Range of θ, top sheet:



b.) Determine the *top sheet residues* of the function of part a.).

c.) Determine the value of the contour integral $\oint_C f(z)\,dz$ for the contour $|z-2i|=1$.

Draw the contour on your sketch in part a). ($f(z)$ is the same function considered in parts a) and b).)

ROOM FOR EXTRA WORK