NAME: ____

ELEE 6382
Fall 2010
Oct. 28, 2010

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book (*Arfken and Weber* or approved substitute) and open-notes. You may also use your class notes, and a calculator. Please show *all steps of your work* and *write neatly and legibly* in order to receive full credit.

Please write all of your work on the attached sheets. If a problem is continued onto the workspace pages at the end, please indicate this.

Useful identities:

$$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$$

$$\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2$$

$$\sinh(z_1 \pm z_2) = \sinh z_1 \cosh z_2 \pm \cosh z_1 \sinh z_2$$

$$\cosh(z_1 \pm z_2) = \cosh z_1 \cosh z_2 \pm \sinh z_1 \sinh z_2$$

$$\sin iz = i \sinh z, \qquad \cos iz = \cosh z$$

$$\sinh iz = i \sin z, \qquad \cosh iz = \cos z$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$\frac{d}{dz} \sinh z = \cosh z, \ \frac{d}{dz} \cosh z = \sinh z,$$

Problem 1 (25 pts)

Verify that the Cauchy-Riemann conditions are satisfied for the function $w(z) = \sin 4z$.

Problem 2 (25 pts)

Obtain the Laurent or Taylor series, as appropriate, of the function $f(z) = \dfrac{1}{(z-1)(z+1)}$ in the regions specified below:

a. $0 < |z-1| < 2$. Sketch the locations of any singularities as well as the region of convergence in the *z*-plane.

b. $|z-1| > 2$

c. Evaluate the closed contour integral $\oint_C f(z)dz$, with $f(z)$ as defined above, about the following contours. Assume C has a counter-clockwise orientation.

a) C: $|z-1|=1$. Sketch the contour.

b) C: $|z-1|=3$. Sketch the contour.

Problem 3 (25 pts)

Calculate the value of the following **three** definite integrals. Sketch any contours used, including their orientation, closures, and singularity locations, if any, for each case.

a) $\int_{-\infty}^{\infty} \frac{e^{imx} dx}{\left(x^2+1\right)}$, two cases: $m > 0, m < 0$

$\boxed{m > 0}$

$\boxed{m < 0}$

b) $\int_{-\infty}^{\infty} \frac{e^{imx} dx}{\left(x^2-1\right)}$, two cases: $m > 0, m < 0$.

$\boxed{m > 0}$

$\boxed{m < 0}$

c) $\int_0^\infty \frac{dx}{\left(x^2+1\right)^2\left(x^2+4\right)}$

Problem 4 (25 pts)

Consider the function

$$f(z) = \frac{z^{\frac{3}{2}}}{(z+1)^2}$$

Give the locations and classify by *kind*, i.e., pole (with order and residue), branch point, essential singularity, etc., all the singularities of $f(z)$ in the *finite* complex plane. *Sketch the locations* in the *z*-plane of the singularities. For any branch points present, define a *branch cut* by assuming $0 < \theta < 2\pi$ in the polar representation of $z = re^{i\theta}$. Show the resulting cut on your sketch.

Essential singularities:

Location(s):

Branch point singularities

Location(s):

Branch definition: $0 < \theta < 2\pi$

Pole singularities

Location(s):

Order:

Residues:

ROOM FOR EXTRA WORK

ROOM FOR EXTRA WORK