**NAME:** ________________________________________

# ELEE 6382
# Fall 2011
# Oct. 27, 2011

# MIDTERM EXAM

**INSTRUCTIONS:**

This exam is open-book (*Arfken and Weber* or approved substitute) and open-notes. You may also use your class notes, and a calculator. Please show *all steps of your work* and *write neatly and legibly* in order to receive full credit.

Please write all of your work on the attached sheets. If a problem is continued onto the workspace pages at the end, please indicate this.

**Useful identities and integrals:**

$$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2 \qquad \int x \sin x \, dx = \sin x - x \cos x$$

$$\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2 \qquad \int x \cos x \, dx = \cos x + x \sin x$$

$$\sinh(z_1 \pm z_2) = \sinh z_1 \cosh z_2 \pm \cosh z_1 \sinh z_2 \qquad \int x e^x dx = x e^x - e^x$$

$$\cosh(z_1 \pm z_2) = \cosh z_1 \cosh z_2 \pm \sinh z_1 \sinh z_2$$

$$\sin iz = i \sinh z, \qquad \cos iz = \cosh z$$

$$\sinh iz = i \sin z, \qquad \cosh iz = \cos z$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$\frac{d}{dz} \sinh z = \cosh z, \quad \frac{d}{dz} \cosh z = \sinh z,$$

Problem 1 (25) ____________________ Problem 3 (25) ____________________

Problem 2 (25) ____________________ Problem 4 (25) ____________________

Total (100) ____________________

**Problem 1 (25 pts)**

The vectors $\mathbf{a} = (\alpha_1, \alpha_2, \alpha_3)$, $\mathbf{b} = (\beta_1, \beta_2, \beta_2)$ are in $\mathrm{C}^3$ with inner product defined as

$$< \mathbf{a}, \mathbf{b} > \; = (\alpha_1 \beta_1^* + \alpha_2 \beta_2^* + \alpha_3 \beta_3^*)$$

where the asterisk ($*$) denotes complex conjugate. Construct a vector $\mathbf{b}'$ that is 1) a linear combination of $\mathbf{b}$ and $\mathbf{a}$ and 2) is orthogonal to $\mathbf{a}$. That is, let $\mathbf{b}' = \mathbf{b} - c\mathbf{a}$ and find constant $c$ such that $< \mathbf{b}', \mathbf{a} > \; = 0$. (You may want to refer to the Gram-Schmidt process.)

a. Using inner product notation, derive an expression for $c$.

b. If $\mathbf{a} = (1, i, 2)$, $\mathbf{b} = (1, i, 0)$, determine $\mathbf{b}'$ and check that $< \mathbf{b}', \mathbf{a} >= 0$

**Problem 2 (25 pts)**
A function $w(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$ has imaginary part $v(x, y) = \left(y\cos y + x\sin y\right)e^{x}$. Find $u(x, y)$ if $w(0 + i0) = 0 + i0$.

**Problem 3 (25 pts)**

Calculate the value of the following **two** definite integrals. Sketch any contours used, including their orientation, closures, and singularity locations, if any, for each case.

a) $\int_{-\infty}^{\infty} \frac{x \sin mx}{\left(x^2+1\right)} dx = \operatorname{Im} \int_{-\infty}^{\infty} \frac{xe^{imx}}{\left(x^2+1\right)} dx$ ; two cases: $m > 0, m < 0$

$\boxed{m > 0}$

$\boxed{m < 0}$

b) $\int_0^\infty \frac{dx}{\left(x^2+1\right)^2\left(x^2-1\right)}$

**Problem 4 (25 pts)**

For each of the following functions, give the indicated information for each singularity in the complex plane:

i. Location of singularity
ii. Singularity type, e.g., pole (give order and residue), removable singularity, branch point, essential singularity, etc.
iii. Residue for any poles or essential singularities

a) $(z-1)^2 e^{\frac{1}{z-1}}$

b) $\dfrac{\sin^2(z-1)}{(z-1)^2}$

c) $\dfrac{z^{\frac{2}{3}}}{(z+1)}$

d) $\cot \pi z$

**ROOM FOR EXTRA WORK**

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