Name: ____

Student Number: ____

ELEE 6382
Fall 2013
Oct. 21, 2013

MIDTERM EXAM

INSTRUCTIONS:

This exam is open-book (*Arfken, Weber, Harris* or approved substitute) and open-notes. You may also use your class notes. Please **show all steps of your work** and **write neatly and legibly** in order to receive full credit.

Please write all of your work on the attached sheets. If a problem continues onto the workspace pages at the end, please indicate this.

Useful identities and integrals:

$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$

$\int x \sin x \, dx = \sin x - x \cos x$

$\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2$

$\int x \cos x \, dx = \cos x + x \sin x$

$\sinh(z_1 \pm z_2) = \sinh z_1 \cosh z_2 \pm \cosh z_1 \sinh z_2$

$\int x e^x dx = x e^x - e^x$

$\cosh(z_1 \pm z_2) = \cosh z_1 \cosh z_2 \pm \sinh z_1 \sinh z_2$

$\sin iz = i \sinh z, \qquad \cos iz = \cosh z$

$\sinh iz = i \sin z, \qquad \cosh iz = \cos z$

$\cosh^2 z - \sinh^2 z = 1$

$\frac{d}{dz} \sinh z = \cosh z, \; \frac{d}{dz} \cosh z = \sinh z,$

1. (25 pts) (Show all work!)

Expand $\frac{1}{z(z-1)(z-2)}$ in a Laurent series in the region $1 < |z| < 2$; also, **sketch the region** of convergence of the series.

2. **(25 pts)** **(Show all work!)**

Calculate the value of the following definite integral. **Sketch any contours used**, including their orientation, closures, and singularity locations, if any. Note there are two cases.

$$\int_{-\infty}^{\infty} \frac{x \sin mx}{\left(x^2+1\right)\left(x^2-1\right)} dx = \operatorname{Im} \int_{-\infty}^{\infty} \frac{x e^{imx}}{\left(x^2+1\right)\left(x^2-1\right)} dx \;\; ; \;\; \textbf{Two cases:} \;\; m>0, m<0$$

a. $\boxed{m>0 \text{ case}}$

b. $\boxed{m < 0 \text{ case}}$

3. (25 pts) (Show all work!)

Calculate the value of the following **two** definite integrals ((a) and (b)). Sketch any contours used, including their orientation, closures, and singularity locations, if any, for each case.

a) $\int_{0}^{2\pi} e^{e^{i\theta}} d\theta$

b) $\int_{-\infty}^{\infty} \frac{dx}{\left(x^4+1\right)}$ $\quad$ $\left(\text{It may be helpful to note that } e^{\pm i3\pi/4} = -e^{\mp i\pi/4}.\right)$

4. (25 pts) (Show all work!)

For each of the following functions (in (a), (b), and (c)) , give the following information for *each* singularity in the *finite* complex plane:

i. Location of singularity
ii. Singularity type: pole (*specify order and residue*), removable singularity, branch point (*specify order*), essential singularity (*specify whether isolated or non-isolated*),
iii. Residue for any poles or essential singularities

a) $z^2 \sin\left(\frac{\pi}{z}\right)^3$

b) $\frac{z}{\sin z}$

c) $\frac{z^{\frac{1}{3}}}{(z+27)}$

ROOM FOR EXTRA WORK

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