# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 29, 2023

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed only for looking up class notes or other material. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

a) Consider the function

.

* Draw the complex plane showing the singularities of the function.
* Give the first four terms of the Taylor series of the function expanded about the origin.
* Indicate what the radius of convergence is for the Taylor series.

b) Consider the function

.

* Give the complete Laurent series of the function expanded about the point *z* = 1.
* Indicate what the region of convergence is for the Laurent series.
* Find the residue at the pole by using your Laurent series.
* Find the reside at the pole by using the “recipe” for a pole of order *m*.

**Room for Work**

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**Problem 2 (25 pts.)**

1. Evaluate the following integral:

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In each case, show clearly the path that you are using.

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Problem 3 (25 pts.)

Consider the following function:

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The branch cut is chosen to be along the negative imaginary axis. Also, we have.

1. Where in the complex plane is ?
2. Now imagine that we have a Riemann surface, where the top sheet is the one where . The escalator (where the branch cut used to be in part (a)) runs along the negative imaginary axis. Find the value of the function at the following points:

*  on the top sheet.
*  on the bottom sheet.
*  on the top sheet.
*  on the bottom sheet.

1. Evaluate the integral



where

.

Use path *C* shown below. This path starts on the top sheet at point *a* and ends on the bottom sheet at point *b*.

Put your answers in rectangular format, keeping at least 5 significant figures.



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Problem 4 (25 pts.)

Consider the following electrostatic problem (which was examined in the class notes).



1. Find a formula for the magnitude of the electric field along the *x* axis. The formula can have both *x* and *u* in it, but show how *x* and *u* are related.
2. Assume that we are on the *x* axis and *x* is getting very large. See if you can obtain an approximate closed-form expression for *u* in terms of *x*, and therefore get an approximate formula for the magnitude of the electric field along the *x* axis that has only *x* in it.

**Room for Work**

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