# Name: \_\_\_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 2, 2016

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality must have the Internet access disabled.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

Problem 1 (40 pts.)

Consider the following function:

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a) Find the first two terms of the Laurent series expansion about the point *z* = 0, valid in the region 0 < |*z|* *<*1.

b) Identify the residue of the function *f* (*z*) at *z* = 0 from the Laurent series.

c) Evaluate the integral of *f* (*z*) clockwise around a circle in the complex plane that is centered at the origin with radius 1/2.

Part (a)

Use the following Taylor series expansion:





We then have

.

Using the geometric series, we have

.

**Room for Work**

The first two terms of the Laurent series are



or

.

Part (b)

To identify the residue, we need one more term in the Laurent series:





or

.

Hence we have



Part (c)



so



Problem 2 (30 pts.)

Consider the following function:

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A branch cut is chosen to lie along the negative real axis. The top sheet corresponds to *f* (1) = 1.

The following integral is then defined:

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Note that the indefinite integral of the function *f* is pretty easy to obtain.

Find the exact value of the integral *I* for the following three cases. In each case, draw a figure showing the path in the complex plane. Note that in all cases the function changes smoothly on the path.

a) *A* = -1 on the top sheet, *B* = -1 on the top sheet. The path from *A* to *B* goes around the branch cut and does not cross it.

b) *A* = -1+*i* on the top sheet, *B* = -1-*i* on the top sheet. The path from *A* to *B* is a path that goes clockwise around the branch cut and does not cross it.

c) *A* = -1+*i* on the top sheet, *B* = -1-*i* on the bottom sheet. The path from *A* to *B* is a vertical path that crosses the branch cut and descends to the lower sheet of the Riemann surface in order to connect to *B*.

**Room for Work**

The integral is evaluated as



so that

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The important point here is that we need to get the correct values of the square roots.

Part (a)





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Part (b)



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Part (c)



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Problem 3 (30 pts.)

Consider the electrostatic problem shown below. The semicircular arcs are perfect magnetic conductors. The *x*-axis parts of the structure are perfect electric conductors. One PEC boundary is at 1V and the other is at 0V.

1) Solve for the potential *φ* inside the structure (*a* < *ρ* < *b*) using conformal mapping. As part of your solution, draw the mapped region in the *w* plane.

2) Solve for the capacitance (per unit length in the *z* direction) between the two PEC conductors, using your conformal mapping.

Hint: Consider the following mapping:

.



**Room for Work**

The mapped region is shown below.





Part (a)

In the *w* plane we have

.

Note that

.

Hence, we have

.

Part (b)

The capacitance (per unit length in the *z* direction) is the same in both planes.

Hence, we have

.