# Name: \_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 23, 2020

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are not used to communicate in any way with anyone other than the instructor. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.
6. At the end of the exam you will be asked to scan your pages and convert them to a single pdf file, and then email this file to the instructor (djackson@uh.edu).

**Problem 1 (25 pts.)**

Consider the following function:

.

a) Derive the complete Laurent series expansion of this function about the point . Where will this Laurent series converge?

b) Derive the first three terms of the Taylor series of this function, expanding about the point . What will this Taylor series converge? Do not use the “calculus 101” formula (the formula that gives the Taylor series coefficients in terms of the derivatives of the function.

**Solution**

**Part (a)**



.

This Laurent series will converge in the region .

**Part (b)**



Collecting terms, we have

.

The region of convergence is .

**Problem 2 (25 pts.)**

Evaluate the following integral:

.

Note that there are simple poles on the real axis, and the integral is defined in the Cauchy principal value sense.

**Solution**

We can choose a small semicircle path to detour above the poles (though we could also choose to detour below if we wish) and close the contour in the upper half plane. We then have



so



We have

We then have



or

.

Problem 3 (25 pts.)

a) Evaluate the following integral along the path  as shown below:

.

The points *A* and *B* are both at *z* = -1, but are on opposite sides of the branch cut.

b) Evaluate the same integral as above, but using the path 



**Solution**

**Part (a)**



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**Part (b)**



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Problem 4 (25 pts.)

Consider the problem shown below. The total charge per unit length (in the *z* direction) on the right conductor in the *z* plane is [C/m] and the total charge per unit length on the left conductor is  [C/m].

Find an expression for the magnitude of the electric field along the *x* axis between the two conductors, as a function of *x*. You can leave your answer in terms of *a*. (The constant *a* can be evaluated in terms of the dimensions, but you can just leave *a* in your answer without having to evaluate it.)

Note that the right conductor in the *z* plane gets mapped into the inner conductor in the *w* plane.







**Solution**

We have



and

.

Hence, we have



or

.