# Name: \_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 1**

#### Nov. 14, 2022

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are in “airplane” mode and are not used to communicate in any way with anyone. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

a) Give the first three terms of the Taylor series of the following function expanded about the point *z* = 0:

.

Also, indicate what the radius of convergence is for the Taylor series.

Note: Do not use the “calculus formula” to get the coefficients of the Taylor series.

b) Give the first three terms of the Laurent series of the following function expanded about the point *z* = 1:

.

Also, indicate what the region of convergence is for the Laurent series.

**Solution**

**Part (a)**





Hence, we have



The radius of converges is the distance to the closest singularity. Hence, the radius of convergence is to where the sin(*z*) term is unity. Hence,

.

**Part (b)**



.

Next, use a Taylor series expansion for :



We then have



This gives us



There is a branch point at *z* = 0. Hence, the region of convergence is

.

**Problem 2 (25 pts.)**

Evaluate the following integral:

.

**Solution**

We can write

.

(The integral with the exponential function in the numerator is called *Ie*.)

We have a simple pole at *z* = 1 and a double pole at *z* = -1. We close the contour with a large semicircle of radius *R* in the upper half plane. For the poles on the real axis, we can detour above them or below them. Let’s assume that we go above them. We then have



so

.

For the simple pole at *z* = 1 we have

.

For the double pole at *z* = -1 we have



Therefore,

.

We can write this as



or

.

Hence, we have

.

Problem 3 (25 pts.)

Consider the following function:

.

On the top sheet, we have. The branch cut is chosen to be along the negative real axis.

Evaluate the integral



where

.

(a) Use path *C*1 shown below.

(b) Use path *C*2 shown below.

(c) Use path *C*3 shown below.

(d) Use path *C*4 shown below.

(e) Use path *C*5 shown below.

Put your answers in rectangular format, keeping at least 6 significant figures.







































**Solution**

For parts (a)-(d)**,** the path of integration is moving smoothly on the Riemann surface, and the integrand is analytic on the path. Therefore, according to the fundamental theorem of calculus, we have

.

Therefore,

.

The point *A* is on the top sheet, so we have

.

We have



and

.

We choose the + sign if point *B* is on the top sheet, and the minus sign if point *B* is on the bottom sheet. Therefore, we have:

a) 

b) 

c) 

d) 

This gives us

a) 

b) 

c) 

d) 

For part (e), the path does not change smoothly on the Riemann surface, so we break it up into two parts.

.

The +/- superscript denotes that the point *z* = -1 is slightly above or below the branch cut.

We then have



or



or

.

We then have

e) .

Problem 4 (25 pts.)

Consider the mapping

.

This function will map between two regions as shown below.





Use this mapping to solve for the potential  inside of the following geometry.







 **Solution**

In the *w* plane, we have

.

From the mapping we also have

.

Therefore,

.

We then have for the potential in the *z* plane

.

Expressing this in terms of *x* and *y*, we have

.