# Name: \_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 15, 2014

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality must have the Internet access disabled.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown (this includes showing all relevant paths in the complex plane that you use to solve a problem).
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

Problem 1 (25 pts.)

Evaluate the following integral as Ω → ∞. The path *C* is shown below, which lies along the real axis from -∞ to ∞.

.

As part of your solution, clearly identify and sketch the path of steepest descent and the path of steepest ascent.

**Room for Work**

We identify



.

The saddle point is at

.

 The SDP and SAP are described by

.

This gives us



which gives us

.

The *u* function is

.

From this we see that



We then have

,

where

.

This gives us

.

.

Hence, we have

.

There are also poles at

.

We need to add the pole contribution from the pole at -*i*, so that

.

The reside is



Hence, we have

.

Problem 2 (25 pts.)

Asymptotically evaluate the following integral as *x* → ∞.

.

The first two terms of the asymptotic expansion are sufficient.

Hint: It might be helpful to recall that

 .

**Room for Work**

We first use

.

We note that

.

Integrating by parts, we have



We thus have

.

Problem 3 (25 pts.)

Consider the Frobenius series solution about the point *x* = 0 of the following differential equation:

.

a) Classify the point *x* = 0 as an ordinary point, a regular singular point, or an irregular singular point.

b) Classify the differential equation as case 1, case 2, or case 3.

c) Find the Frobenius series solution for the solution that has the largest value of *α*. Find the first three terms of the series solution. Take *a*0 = 1.

**Room for Work**

The point *x* = 0 is a regular singular point.

The Frobenius solution gives us





.

Substituting into the differential equation, we have

.

The indicial equation is



or

.

Hence, we have

.

Hence, we have case 1 (*α*1 - *α*2 ≠ integer).

Shifting the indices on the first two series, we have

.

If we choose *n* = -1 we obtain

.

Hence, we have

.

The recurrence equation for the *an* coefficients comes from

.

Picking the largest value of *α* gives us

,

or

.

Hence, we have



We then have



or

.

Problem 4 (25 pts.)

Consider the differential equation

,

with the boundary conditions



a) Solve for the Green’s function G(*x*,*x*′) using “method 1” (the method of boundary matching).

b) Solve for the Green’s function G(*x*,*x*′) using “method 2” (the method of eigenfunction expansion).

Hint (for part (b)): It might be useful to recall that

.

**Room for Work**

We have



The Wronskian is

.

The general solution is then



Hence, we have



For the eigenvalue problem, we have

.

The general solution is

.

To satisfy the boundary conditions, we choose



The general solution is

.

We have

.

We then have



or

.