# Name:\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 8, 2017

1. This exam is open-book and open-notes. Any electronic devices (laptops, etc.) that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown (this includes showing all relevant paths in the complex plane that you use to solve a problem).
3. Perform all your work on the exam in the space allowed.
4. Write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (30 pts.)**

Consider the following integral as Ω → ∞.

.

a) Derive the first three terms of the asymptotic series as *x* → ∞.

b) Derive the complete asymptotic expansion of *I* (*x*) as *x* → ∞. That is, give a general formula for the *n*th term in the asymptotic series.

**Room for Work**

**Solution**

**Part (a)**

Integrate by parts three times:

.

**Part (b)**

After integrating by parts *n* times we have:



This may be written as

,

where the “double factorial” function is defined (for odd integers) as



We can also write



or

.

**Problem 2 (35 pts.)**

Evaluate the following integral as Ω → ∞:

.

The path *C* is along the real axis as shown below.

As part of your solution, clearly identify and sketch the path of steepest descent and the path of steepest ascent. Explain how you are determining the departure angle *θ*SDP.

**Hint:** When solving for the SDP, try factoring the left-hand side of the equation into a product of terms. This might allow you to identify the SDP more easily.

**Room for Work**

**Solution**

The saddle point is at

.

The SDP and SAP are defined by



or

.

The SDP is a horizontal line through the point *z*0. The departure angle is thus 0. We also have







.

The departure angle is thus 0.

A simple pole is located at *z* = *i*/2. The residue of the integrand at the pole is

.

We then have

.

**Problem 3 (25 pts.)**

The Laplace transform of a function *f* (*t*) is defined as

.

Consider the Laplace transform of the following function:

.

a) Evaluate the leading term of the asymptotic expansion of *F*(*s*) as *s* → ∞.

b) Derive the complete asymptotic expansion of *F*(*s*) as *s* → ∞. That is, give a general formula for the *n*th term in the asymptotic series.

**Room for Work**

**Solution**

**Part (a)**

At *t* → 0 we have

.

Hence,

.

Hence, we have



.

Therefore, we have

.

**Part (b)**

We use

.

Therefore, we have



.

Hence, we have

.

**Problem 4 (10 pts.)**

This is a continuation of Prob. 3. Assume that we know that the Laplace transform of the function *f* (*t*) in Prob. 3 is actually given by

,

where the complementary error function is defined by

.

(This formula for the Laplace transform was found in a math handbook.)

Evaluate the leading term of the asymptotic expansion of *F*(*s*) as *s* → ∞ using this formula for the Laplace transform from the math handbook.

Hint: You may use the fact that

.

**Room for Work**

**Solution**

At *s* → ∞ we have



and



so that

.

EXTRA CREDIT (20 pts.)

Consider an infinite transmission line that runs from *z* = -∞ to *z* = ∞. The transmission line is fed by a parallel (shunt) 1A current source at *z* = *z*′, as shown in the figure below.

It is known that the voltage on the transmissions line to due this parallel current source must have the following mathematical form:

 

where .

Solve for the voltage *V* (*z*) on the transmission line (for both *z* < *z*′ and *z* > *z*′) using “method 1”.



**Room for Work**

**Solution**

We have





and therefore

.

We also have

.

We thus have



or

