# Name: SOLUTION

#### ECE 6382

#### Engineering Analysis I

**Exam 2**

#### Dec. 12, 2022

1. This exam is open-book and open-notes. Calculators are allowed. Computers are allowed as long as they are not used to communicate in any way with anyone other than the instructor. Cell phones or any other devices that have communication functionality are not allowed.
2. Show all of your work. No credit will be given if the work required to obtain the solutions is not clearly shown.
3. Please perform all your work on the exam in the space allowed if possible, though you can attach extra pages if necessary.
4. Please write neatly. You will not be given credit for work that is not **easily** legible.
5. Circle your final answers.

**Problem 1 (25 pts.)**

Consider the following three functions:

 

a) Find the Wronskian for these three functions. Simply as much as you can.

b) Are these functions linearly dependent or independent? Explain clearly your reasoning using the Wronskian.

c) Can any of the three functions be expressed as a combination of the other two? If so, please show how to do it. (Do this for any of them for which your answer is yes.)

**Helpful identity**: 

**Solution**

**Part (a)**

The Wronskian is

.

This expands to

 

We then see that

.

**Part (b)**

Because these three functions are analytic, we the know that the Wronskian being identically equal to zero implies that these three functions are linearly dependent.

**Part (c)**

Any of the three functions can be expressed as a combination of the others:



Hence, we have



**Problem 2 (25 pts.)**

Find the leading term of the asymptotic series for the following integral, as Ω gets large:

.

where .

The original path is along the imaginary axis from  to .

As part of your solution, show what the SDP and SAP paths look like.

**Note:** A helpful identity might be 

**Solution**

We have



and

.

The saddle point is located at . We have

.

or

 

or

.

Hence, we have



The SAP and SDP therefore come from

.

The complex plane is shown below. The SDP is shown in red.

We then follow the SDP recipe.

We have

,

.

Hence,

.

From looking at the SDP, we conclude that

.

The SDP integral then is asymptotically approximated as

.

This gives us

.

Now, we must also capture the residue at the simple pole at  when we deform the path from *C* to the SDP.

The residue of the integrand is

.

We have

.

We then have

 .

**Problem 3 (25 pts.)**

a) Find the first three nonzero terms of the asymptotic series for the following integral as Ω gets large:

.

b) Assume that we keep only the first term of the asymptotic series. Give a formula that asymptotically predicts what the error is in using this leading term to estimate the integral, as Ω gets large.

**Solution**

**Part (a)**

We use

.

We then have



or

.

We have

.

We thus identify



We then use Watson’s Lemma:

.

Hence, we have

.

**Part (b)**

If we stop at *N* terms, the error is asymptotically predicted by the *N*+1 term.

Hence if we keep only the leading term, we have

.

**Problem 4 (25 pts.)**

A parallel 1 Amp current source is on a semi-infinite transmission line as shown below. The line is open-circuited at *z* = 0, and extends to infinity in the positive *z* direction.



a) Find the solution for the voltage  at any point on the line. This is the same as finding the Green’s function . (The subscript “*v*” denotes that we are solving for voltage.)

b) Find the solution for the current  at any point on the line.

c) Now assume that the line is excited by a distributed surface current source (units of [A/m]) of width *w* that is centered at *z* = *z*0 and is given as:

 

Find the voltage  on the line for .

**Solution**

**Part (a)**

Using “Method 1”, we have



.

We then have



where

.

This gives us



The Wronskian is

.

This can be written as

 

or



or



or

.

We then have



or



**Part (b)**

From the telegrapher’s equations we have

.

Therefore, we have

.

 This gives us



 **Part (c)**

From Green’s function we have

.

Hence, we have (using the form of the Green’s function for )

 .

This gives us



so that

.

We then have

.