# ECE 2100H Experiment I

# Measurement Techniques and Experimental Error

## Introduction

Often in electrical engineering we solve "textbook" problems designed to illustrate theoretical concepts. In these problems the values of circuit components and sources are assumed to be exact, and the problems have exact solutions, which can usually be expressed analytically. For example, if a "textbook" problem indicates that a voltage source of 20[V] is applied across a 5[k] resistor, the value of the current determined using Ohms law is exactly 4[mA].

In the laboratory, circuit parameters are obtained by measurement. In that case, the value obtained for the current through a 5[k] resistor with 20[V] across it will depend in part on the meter used to make the measurement. In addition, some error is to be expected in the setting of the voltage source and in the value of the resistor; such errors will cause a deviation of the current from 4[mA].

It is important to be able to estimate how much error to expect in your experimental results. If you cannot estimate error, you will not recognize situations in which the experiment has not been set up correctly, for example, or that the theory you are using does not apply. Experience in the laboratory will allow you to decide how much of a deviation is expected and whether a particular deviation is within reason.

Laboratory measurements are often compared to theoretical predictions or to the calculated value, and we will do that here. The degree of correspondence between measurement and theory is subject to limitations of the measurement equipment and errors in circuit component values. These limitations are expressed in terms of accuracy, precision, and error.

## Research Question

# For this experiment we pose the following research question.

# How can error due to resistor tolerance be estimated for simple resistive circuits?

# **Methods**

In this lab we begin to learn to estimate the error in electrical circuit measurements. Specifically, we will be examining simple resistive circuits built using resistors with a 5% tolerance, and the voltage source available in the lab. We will also make measurements designed to test KVL and KCL. That will necessitate a different kind of error calculation.

***Data***

We will measure circuit voltages and currents in simple resistive circuits using the lab multimeter.

***Data Analysis***

We will calculate the error in the voltage and current measurements by comparing them with the predictions of circuit theory. Additionally, we will estimate worst-case errors expected from the 5% resistor tolerance.

A common way to measure error is by percent difference, expressed by the formula

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where “measured” is the experimentally measured value. By “reference” we mean whatever it is we want to compare our measurement to. This may be a calculation, or another measurement that serves as a comparison to the measured value. In this lab, we will compare measured values of voltage and current with theoretical predictions made from the laws of circuit theory. In that case, the reference value will be that predicted by the theory.

In this lab we will also measure the sum of the voltages around a closed path. In that case, the theoretical value is of course 0! But if our reference value is 0, we are going to run into trouble using our percentage difference formula. In that case, we will redefine the error to be



where N is the number of voltage measurements in a given path. That is, percent error is the difference between the measured sum and ‘0’, as a fraction of the average of the absolute values of the voltage measurements. For example, if the values in a particular case are v1 = 0.22 [V],   
v2 = 1.28 [V], and v3 = -1.54 [V], then the average of the absolute values is 1.01 [V], the difference of the sum from 0 is -0.04 [V], and the error is (-0.04 - 0)/1.01 x 100% = -3.9%.

We will also measure the sum of currents at a node, and make use of this formula adapted for current instead of voltage.

# **Procedure and Results**

***Voltage divider with equal nominal resistor values***

1. Construct the voltage divider circuit shown in Figure 1 using the 5% resistors in your lab kit and the power supply in the lab. Set the power supply *vP* to 10.0[V]. Use the multimeter to be certain that the supply voltage is as close to 10.0[V] as you can reasonably make it. Choose resistors whose *nominal* values are different, but by no more than 10x. In other words, if one of your resistors is 2.2 [kW], the other should be no bigger than 22 [kW] or smaller than 220 [W]. (The *nominal* value of a resistor is the value expressed by its color code.)

**Important!!** In selecting your resistor values, keep in mind the resistor power limitations. Do not allow power dissipated in any resistor to exceed its rated value.

**Figure 1:** Voltage Divider Circuit to be used in the experiment.

2. Measure the voltage *vO* using the lab multimeter. Copy Table 1 to your Lab Notebook, and in the notebook record *vO* calculated from the voltage divider rule. Also include the measured value and the error between the calculated and measured values.

Table 1: Measurement of Output Voltage

|  |  |  |
| --- | --- | --- |
| *v*O in [V] calculated | *v*O in [V] measured | % error |
|  |  |  |

3. Assuming the source voltage to be exactly 10[V], calculate the error in *vO* that would arise if the resistor values were different from their nominal values by ±5%. Assume that everything else in your measurement was accurate. Assume the resistors differ from their nominal values in such a way that the error takes its largest positive value; repeat for values such that the error takes its largest negative value. Copy Table 2 to your Lab Notebook and record the values there. For help on this step, refer to ***Discussion of max and min error calculation*** in the Appendix at the end of this handout.

### **Table 2:** Maximum Error Values

|  |  |
| --- | --- |
| maximum positive % error | maximum negative % error |
|  |  |

***Actual Resistor Values***

4. Use the multimeter to measure the *actual* values of the resistors *R1* and *R2*. Using these values, re-calculate *vO* using the voltage divider rule and the actual resistances. Then, determine the %error between the value of *vO* measured in Part 2, and the re-calculated value of *vO*. Use the re-calculated value as the reference. Copy Table 3 into your Lab Notebook and record the values there.

### **Table 3:** Recalculation of Output Voltage Using Measured Resistances

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| R1 measured | R2 measured | *vO* calculated | *vO* measured  (from part 2) | % error |
|  |  |  |  |  |

*Voltage Source Errors*

5. Disconnect the power supply from your circuit. Using the voltage readout on the power supply, set the voltage output *vP* to 8.50[V]. Measure *vP* with the multimeter and find the error in the voltage using 8.50[V] as a reference. Repeat this procedure for values of 12.3 [V] and 15.0 [V]. Copy Table 4 to your Lab Notebook and record your values there.

# **Table 4:** Voltage Source Errors

|  |  |  |
| --- | --- | --- |
| set in [V] | measured in [V] | % error |
| 8.5 |  |  |
| 12.3 |  |  |
| 15.0 |  |  |

*Current Measurement*

6. Construct the circuit shown in Figure 2. Calculate and measure the current in each of the resistors. Determine the error in the readings, using the calculated value as a reference. Use Table 5 to record your values in your Lab Notebook.



**Figure 2:** Circuit to be used in measurement of current.

# **Table 5:** Measurement of Currents

|  |  |  |  |
| --- | --- | --- | --- |
| Current | Current calculated | Current measured | % error |
| *i1* |  |  |  |
| *i2* |  |  |  |
| *i3* |  |  |  |

7. Based on 5% resistor tolerance, analyze the maximum positive and negative error for *i2* in the circuit in Figure 2. Record your calculated errors in your Lab Notebook using Table 6.

# **Table 6:** Calculated Errors in *i2*

|  |  |  |  |
| --- | --- | --- | --- |
|  | Nominal value | max positive % error | max negative % error |
| *i2* |  |  |  |

***KVL: First Circuit***

8.Construct the circuit shown in Figure 3 on a prototyping board.



**Figure 3.** Construction of Resistive Circuit Using Resistors in the [] Range.

9. Figure 3 contains several closed paths around which Kirchhoff’s voltage law could be applied. Choose two different, independent KVL paths, and write the corresponding KVL equations in terms of voltages. To do this you will need to define the resistor voltages.

Use the multimeter to measure the voltages around the closed paths you chose. Be sure to measure the polarity you indicated in your diagram. In Table 7, report the measured voltages indicated in your diagram. Also record the sum of the voltages, making sure that you have entered them into the KVL equation with the proper sign. Finally, enter the percent error obtained for the sums. Note that the table may contain more entries than you need for your paths; just leave the other entries blank.

***Important:*** Don’t forget to include the labels for the voltages you record. For example, if you labeled the voltage across the 2.7 [kW] resistor “*v*2”, and your measurement for *v*2 is – 1.63 [V], you should have a table entry that says “*v*2 = – 1.63”. Note that units have been indicated in the table heading, so you don’t need to include them in the table entries.

**Table 7**: Kirchhoff's Voltage Law for the Circuit of Figure 3

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Voltages [V] | | | | | | [V] | *%Error* |
| 1. |  |  |  |  |  |  |  |  |
| 2. |  |  |  |  |  |  |  |  |

10. Choose two of the essential nodes in Figure 3, label currents entering or leaving the nodes (as you choose), and write the corresponding KCL equations. Measure the currents at each of the nodes you have identified. In Table 8, record the individual branch currents. As you did for the voltage measurements, be sure to measure the polarity you labeled in each case. Also, include the current label along with the measurement in Table 8.

In Table 8, indicate the sum of the currents (), and the percent error. When you make the summation, make sure that you use appropriate signs, as required in Kirchhoff’s current law.

**Table 8:** Sum of Currents in Kirchhoff's Current Law for the Circuit of Figure 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Branch Currents [mA] | | | | [mA] | *%error* |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |

***Second Circuit: Large Resistances***

11. Construct the circuit shown in Figure 4. Note that it is similar to the circuit of Figure 3 but contains larger resistor values. As in part 9, choose two closed paths around which to verify KVL, label the voltages, and write the corresponding KVL equations.



**Figure 4.** Circuit with Larger Resistances.

12. Measure the voltages around the closed paths you chose. In Table 9, record the individual voltages (including their labels), the sum of the voltages, and the percent error.

**Table 9:** Sum of Voltages in Kirchhoff's Voltage Law

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Voltages [V] | | | | | | [V] | *%Error* |
| 1. |  |  |  |  |  |  |  |  |
| 2. |  |  |  |  |  |  |  |  |

13. As in part 10, choose two nodes at which to verify KCL, label the currents, and write the corresponding equations. Measure the sum of the currents at the nodes you chose. In Table 10, record the individual branch currents (including the labels for each), the sum of the currents, and the percent error.

**Table 10:** Sum of Currents in Kirchhoff's Current Law

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Branch Currents [mA] | | | | [mA] | | *%error* | |
| 1. |  |  |  |  |  | |
| 2. |  |  |  |  |  | |

*Resistance Histogram*

In this section we will measure the resistance of 200 1 [k] and 200 10 [k] resistors, and plot their measured values on a histogram. We will then examine the histogram with the intention of discovering what “5% tolerance” means.

14. You will be given some 1 [k] and some 10 [k] resistors. Measure the value of each of them using the multimeter in the lab.

# 15. Construct a histogram for each of the two resistance values. Determine the mean and standard deviation, and enter those in Table 6. Also enter the %error in the mean value, i.e., the error in the mean value as compared to the expected mean value.

# **Table 11:** Mean and Standard Deviation of Resistance Values

|  |  |  |  |
| --- | --- | --- | --- |
| Resistance | Mean [] | Std Dev [] | % error of the mean |
| 1 [k] |  |  |  |
| 10 [k] |  |  |  |

# **Conclusions**

1. Go to the N.E.R.D. folder available on the ECE 2100 website. Open the document ***Basic Lab Procedure*** and read the discussion on ***Accuracy, Precision, and Significant Figures*** in that document.
   1. Based on that discussion, state whether it is possible to add two measured values with an equal number of significant figures to obtain a result with a greater number of significant figures (assuming the measured values are positive). Is it possible to obtain a result with a smaller number of significant figures?
   2. In subtracting two measured values with an equal number of significant figures is it possible to obtain a result with a greater number of significant figures (assuming the measured values are positive)? Is it possible to obtain a result with a smaller number of significant figures?
2. To evaluate a circuit built using 1 % tolerance components, a voltage measurement is required. A worst-case analysis like that in step 3 shows that the voltage should be within – 3.0% of the designed value. But in making the voltage measurement you obtain a value that is -5.0% of the designed value. What would you consider the possible causes of this discrepancy? List at least three possible causes in the order you think would be the most likely. State what steps you would take to correct the problem.
3. Using any source you like, do some investigation of basic statistics. In particular, research the following. Then, in a few sentences, answer these questions as part of the assignment.
4. What is a “normal” distribution function?
5. What do “mean”, “median”, “average”, and “standard deviation” refer to?
6. What can we say about the standard deviation for a normal distribution?
7. What is meant by “5% tolerance” when referring to resistors? What does this have to do with measurement error? State your answer in terms of the histogram you constructed in Step 8; in other words, explain what “5% tolerance” means in terms of the histogram.
8. In thinking about voltage and current measurements, which one should have fewer errors? Why? **Appendix: Discussion of the max and min error calculations**

Voltage Divider Rule (based on Figure 1):



***Discussion of “max” and “min” error in Tables 2 and 4***

Consider the formula above for *vO*, and note that the resistor values can be either larger or smaller than their nominal values by as much as 5% as specified by the resistor tolerance. In making a measurement of *vO*, if R2 is too high we expect to get a positive error (i.e., the measurement will be high), as compared with the value obtained with nominal resistor values. We will also get a positive error if R1 is too low. The “maximum positive error” will obtain if R2 is too high *and* R1 is too low at the same time, in both cases by 5%. This is highly unlikely, but it represents the maximum expected error due to resistor tolerance, and thus provides a useful measure of error. This is what is being asked in Table 2 for “maximum positive error”. The maximum negative error can be calculated in a similar fashion.