Signature: SOLUTION KEY

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

EXAM #3
ELEE 2335
NOVEMBER 23, 1985

INSTRUCTIONS:

1. Fill in the information required on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary, but indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless necessary work is shown.

3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

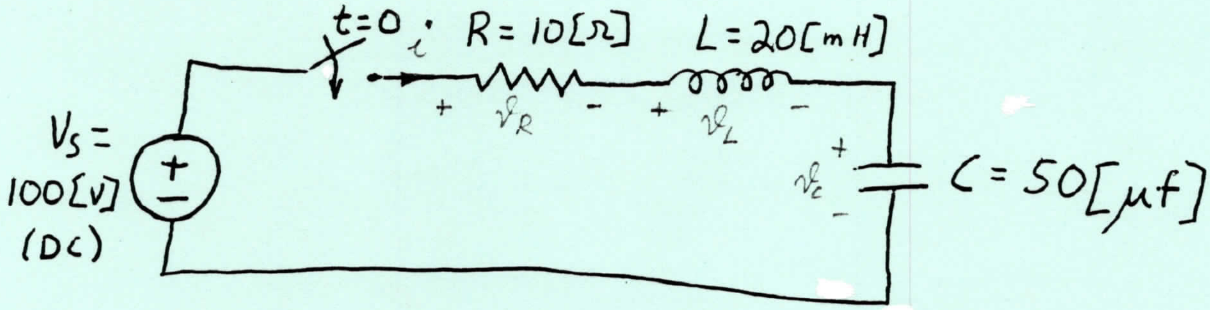
1. 40

2. 30

3. 30

(100)

1. For the following circuit, the switch has been open for a long time, and then is closed at t=0.



a) Write the circuit integral-differential equation that describes the circuit behavior for t≥0, using current as the variable. Transform it into a second order differential equation.

$$v_R + v_L + v_c - v_s = 0$$

$$Ri + L\frac{di}{dt} + V_{c0} + \frac{1}{C}\int_0^t i\,dt = V_s \qquad \Big|\ \frac{d}{dt}$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0 \Rightarrow \boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0}$$

with numerical values:

$$\frac{d^2i}{dt^2} + \frac{10}{20\times10^{-3}}\frac{di}{dt} + \frac{1}{20\times10^{-3}*50\times10^{-6}}i = 0$$

$$\Downarrow$$

$$\boxed{\frac{d^2i}{dt^2} + 500\frac{di}{dt} + 10^6 i = 0}$$

1. continued

b) Write the circuit characteristic equation, and the general expression for its roots. Calculate the root component values (neper and resonant radian frequency) and the total root values.

$$\left(i(t) = Ie^{st} ; \quad \frac{di(t)}{dt} = sIe^{st} ; \quad \frac{d^2 i(t)}{dt^2} = s^2 I e^{st} . \right.$$

Replaced into the differential equation:

$$\left. Ie^{st}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0 \right)$$

characteristic equation:

$$\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0} \; ; \text{ roots: } \boxed{s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

neper frequency:

$$\boxed{\alpha = \frac{R}{2L} = \frac{10}{2*20*10^{-3}} = 250\ [rad/s]}$$

resonant radian frequency :

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20*10^{-3}*50*10^{-6}}} = 1000\ [rad/s]}$$

$$\alpha^2 < \omega_0^2 : \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1000^2 - 250^2} = 968.24584\ [rad/s]$$

$$\boxed{\begin{array}{l} s_1 = -250 + j\,968.24584\ [rad/s] \\ s_2 = -250 - j\,968.24584\ [rad/s] \end{array}}$$

1. continued

c) Define the response type and write the response expression.

Roots are complex $\Rightarrow$ underdamped time response:

$$i(t) = I_{final} + \{ \text{Natural response} \}$$

Because: $-$ $v_s \Rightarrow$ d.c. source
$-$ capacitor in series in the circuit $\Big\} \Rightarrow \underline{I_{final} = 0}$

so: $i(t) = \{ \text{Natural response} \}$.

For the underdamped natural response: $\boxed{i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)}$

$$\boxed{i(t) = e^{-250t}(B_1 \cos 968.24584t + B_2 \sin 968.24584t) [A]}$$

d) Determine the response constants using the initial conditions.

At $t = 0^+$: *) $i(0^+) = 0$, because: $I_{L0} = 0$

that is: $i(0^+) = 0 = 1(B_1 \cos 0 + B_2 \sin 0) \Rightarrow \boxed{B_1 = 0} [A]$

$$\Rightarrow i(t) = e^{-250t} * B_2 \sin 968.24584t$$

*) $v_R(0^+) = R\, i(0^+) = 0$; $v_C(0^+) = v_C(0^-) = 0$; $\Rightarrow v_L(0^+) = v_S$

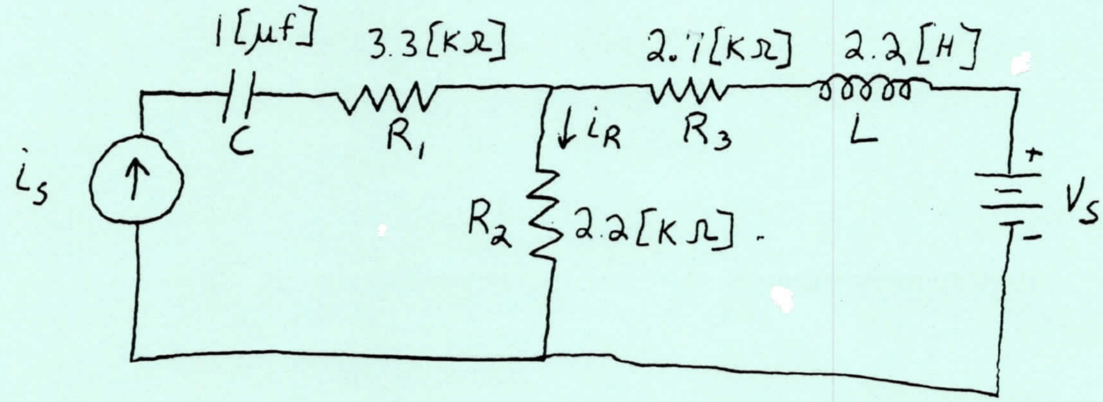
$$v_L(t) = L \frac{di(t)}{dt} = B_2 L \{ -250 e^{-250t} \sin 968.24584t + e^{-250t} * 968.24584 \cos 968.24584t \}$$

$$v_L(0^+) = L \frac{di(t)}{dt}\Big|_{0^+} = 968.24584 * B_2 * L = v_S$$

$$\boxed{B_2 = \frac{v_S}{968.24\ldots * L} = \frac{100}{968.24\ldots * 20 * 10^{-3}} = 5.1639778 [A]}$$

$$\boxed{i(t) = 5.1639778 * e^{-250t} * \sin 968.24584t \quad [A]}$$

2. For the circuit below, find $i_R(t)$. Assume that the circuit has been in this configuration for a long time. Note that there is an AC and a DC source in this circuit.



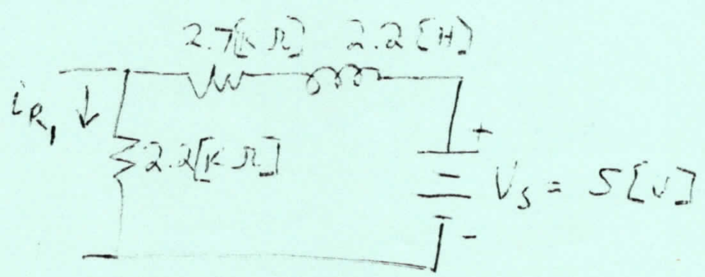
$i_s(t) = 3\sin(1000t)$ [mA]

$V_s(t) = 5$ [V]

Two sources with two different frequencies. Use Superposition.

(1) First take the DC Voltage Source. Replace the current source with an open circuit. (+2)

This leaves:



(+5)

The inductor behaves as a short circuit.

Thus,

$$i_{R_1} = \frac{5[V]}{(2.2+2.7)[K\Omega]} = 1.02[mA]$$ (+5)

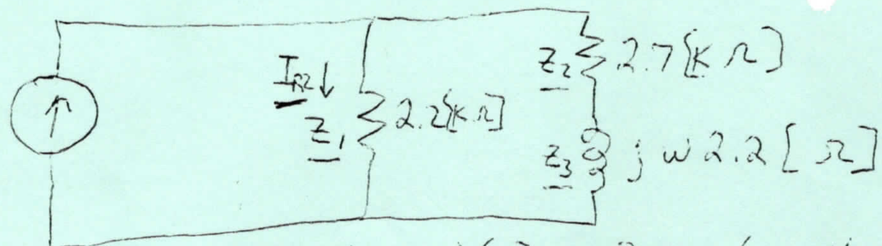
(Solution continues on next page.)

ROOM FOR EXTRA WORK

(2) Take the current source, and replace the voltage source with a short circuit. Since this is an AC circuit, replace the components with their phasor values. Note that C and R_1 have no effect since they are in series with a Current source

(+3)

$\underline{I_s}$



(+5)

$i_s(t) = 3\sin(1000t)$ [mA] $= 3\cos(1000t - 90°)$ [mA]

$\underline{I_s} = 3\angle -90°$ [mA] $\qquad \omega = 1000$ [rad/sec]

$$\underline{I_{R2}} = \underline{I_s}\,\frac{\underline{Z_2}+\underline{Z_3}}{\underline{Z_1}+\underline{Z_2}+\underline{Z_3}} = \frac{2700+2200j}{2{,}200+2700+2200j}\,3\angle -90° \text{ [mA]}$$

$$\underline{I_{R2}} = \frac{(3483\angle 39.17°)\,3\angle -90°}{5371\angle 24.18°} = 1.95\angle -75° \text{ [mA]}$$

$i_{R2}(t) = 1.95\cos(1000t - 75°)$ [mA]

(+5)

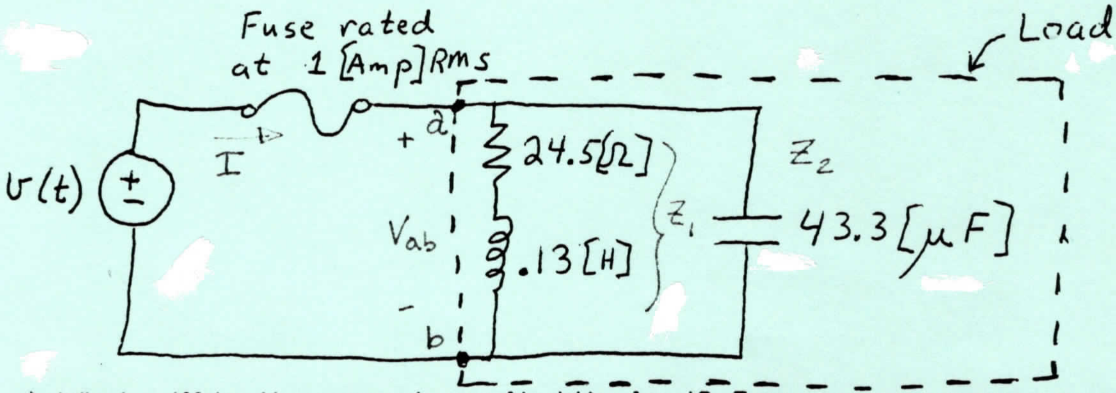
So

$$\boxed{i_R(t) = i_{R_1} + i_{R_2} = 1.02 + 1.95\cos(1000t - 75°) \text{ [mA]}}$$

(+5)

Solution using V_s as 1000 rad/sec source, -15 points

3. A load with the internal circuit shown is connected to a sinusoidal power source through a protecting fuse rated at 1.0 Amps. RMS. The voltage source has an amplitude of 163 Volts and a frequency of 60 Hz.



a) What will be the current supplied the load? Express your answer as a phasor in RMS.

$$V_{ab} = v(t) \Rightarrow \frac{163}{\sqrt{2}} \angle 0^\circ = 115.2 \angle 0^\circ \text{ [VOLTS RMS]}$$

$$Z_1 = R + j\omega L = 24.5 + j\,2\pi \times 60 \times .13 = 24.5 + j49 = 54.8 \angle 63.4^\circ [\Omega]$$

$$Z_2 = \frac{1}{j\omega C} = \frac{-j}{2\pi \times 60 \times 43.3 \times 10^{-6}} = -j61.2 = 61.2 \angle -90^\circ [\Omega]$$

$$I_1 = \frac{V_{ab}}{Z_1} = \frac{115.2 \angle 0^\circ}{54.8 \angle 63.4^\circ} = 2.1 \angle -63.4^\circ = .94 - j1.88 \text{ [AMPS RMS]}$$

$$I_2 = \frac{V_{ab}}{Z_2} = \frac{115.2 \angle 0^\circ}{61.2 \angle -90^\circ} = 1.88 \angle 90^\circ = j1.88 \text{ [AMPS RMS]}$$

$$\therefore I = I_1 + I_2 = .94 - j1.88 + j1.88 = \boxed{.94 \text{ [AMPS RMS]}}$$

$$I = \boxed{.94 \angle 0^\circ \text{ [AMP RMS]}}$$

3. continued

b) Determine the complex power delivered by the source to the circuit (or absorbed by the circuit) and its components: the real average power, and the reactive power.

$$S = V_{RMS} I_{RMS} \angle \theta_v - \theta_i$$

$$= 115.2 \times .94 \angle 0^\circ - 0^\circ = 108.28 \angle 0^\circ \text{ [VA]}$$

$$\boxed{S = 108.28 \text{ WATTS} + j0}$$

c) If the load capacitor fails such that it becomes an open circuit, will the fuse blow?

WHEN THE CAPACITOR "OPENS", THE CURRENT I_2 WILL BE ZERO. THEREFORE:

$$I = I_1 = 2.1 \angle -63.4^\circ \text{ [AMPS RMS]}$$

SINCE $|I| > 1$ THE FUSE WILL <u>BLOW</u>.

(FUSE RATING)