Signature: Solution Key

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

EXAM 2
ELEE 2335
March 15, 1986

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

3. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 20

2. 15

3. 15

4. 10

5. 10

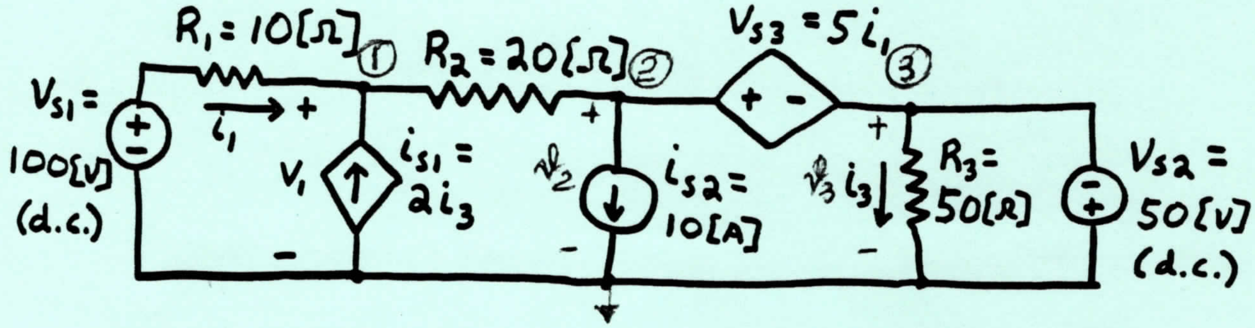
6. 15

7. 15

Bonus. 10

110

1. (20 Points) Apply the node voltage method to determine the voltage v_1 in the following circuit.



(1) Node #1: $\dfrac{v_1 - 100}{10} - i_{s1} + \dfrac{v_1 - v_2}{20} = 0.$

(2) -"- #2+3; $v_2 - v_3 - v_{s3} = v_2 - v_3 - 5i_1 = 0$

-"- #3: $v_3 = -v_{s2} = -50\ [V].$

i_{s1} : $i_{s1} = 2i_3 = 2\dfrac{v_3}{R_3} = 2\dfrac{v_3}{50} = \dfrac{v_3}{25} = -2\ [A].$

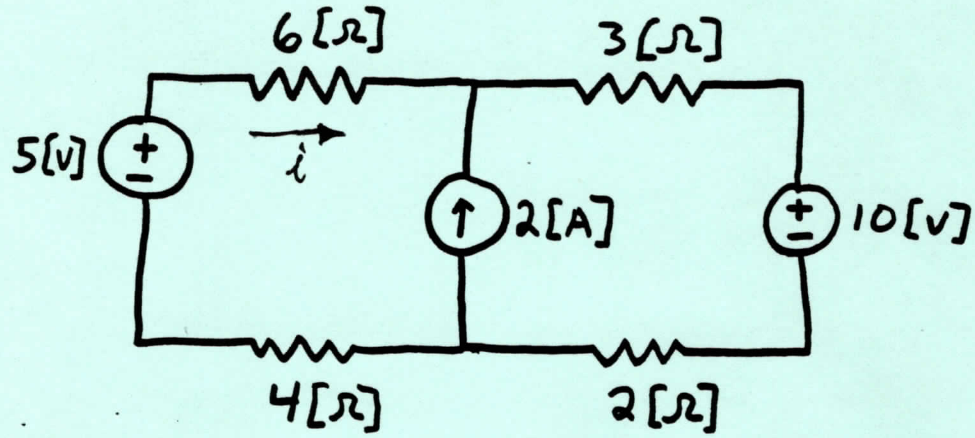
v_{s3} : $v_{s3} = 5i_1 = 5\dfrac{v_{s1} - v_1}{10} = 5\dfrac{100 - v_1}{10} = 50 - 0.5v_1$

$\Downarrow$.

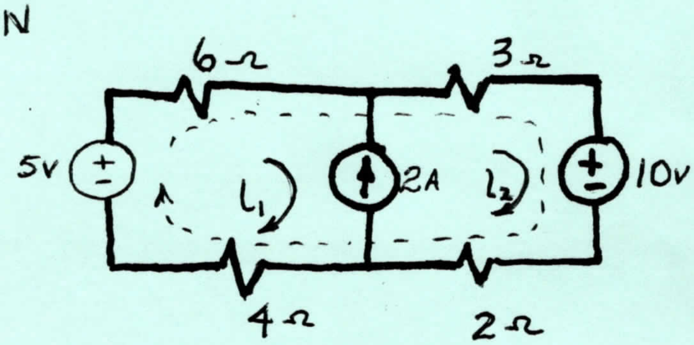
$$\begin{cases} (1)\quad \dfrac{v_1 - 100}{10} + 2 + \dfrac{v_1 - v_2}{20} = 0 \\ (2)\quad v_2 + 50 - (50 - 0.5v_1) = 0 \end{cases} \Leftrightarrow \begin{cases} 2v_1 - 200 + 40 + v_1 - v_2 = 0 \\ v_2 + 0.5v_1 = 0;\quad v_2 = -0.5v_1 \end{cases}$$

(1) $3v_1 - v_2 = 160 \Rightarrow 3v_1 - (-0.5v_1) = 160;$ $\boxed{v_1 = \dfrac{160}{3.5} = 45.714286\ [V]}$

2. (15 Points) In the circuit given, determine the value of the current i <u>using the mesh current technique.</u>



SOLUTION



USING THE SUPERMESH CONCEPT

$$5 - 6i_1 - 3i_2 - 10 - 2i_2 - 4i_1 = 0 \quad ; \quad i_2 - i_1 = 2A$$

$$10i_1 + 5i_2 = -5$$

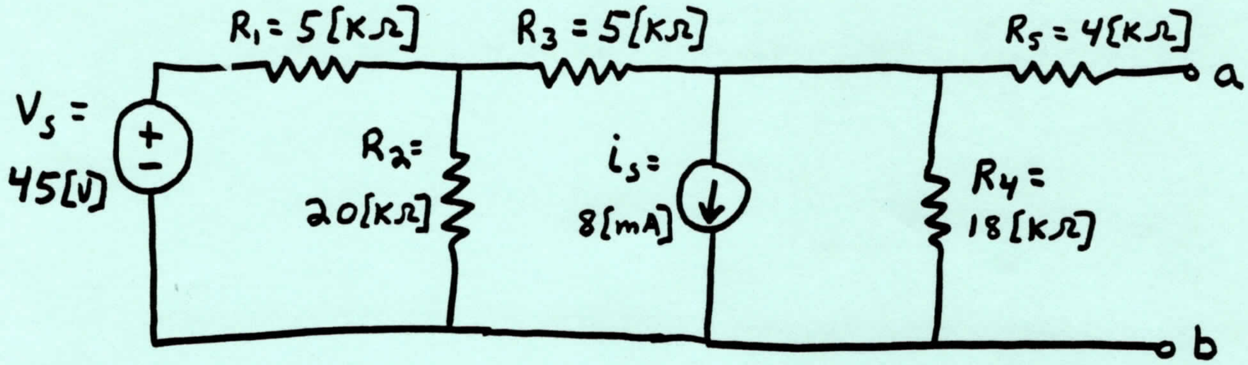
$$\therefore\ 10i_1 + 5(2 + i_1) = -5$$

$$15i_1 = -15$$

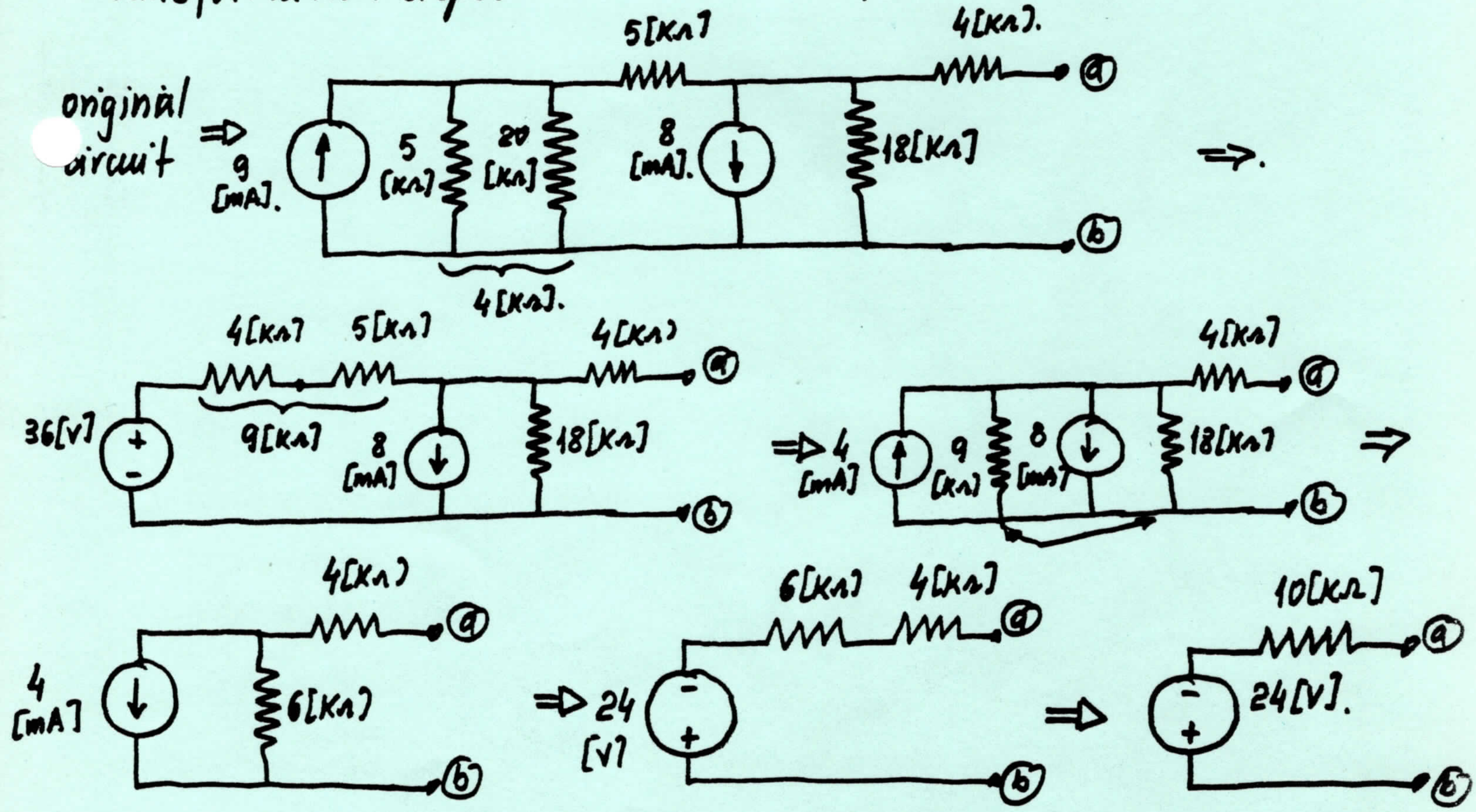
$$i_1 = -1\,A$$

SINCE $i = i_1$, $\boxed{i = -1 \text{ AMP}}$

3. (15 Points) Apply the source transformations to determine the Thevenin equivalent with respect to terminals a and b for the following circuit.



Transformation steps:



4. (10 Points) A circuit has the following Thevenin equivalent values,

$$V_{TH1} = 30[V]$$
$$R_{TH1} = 300[\Omega],$$

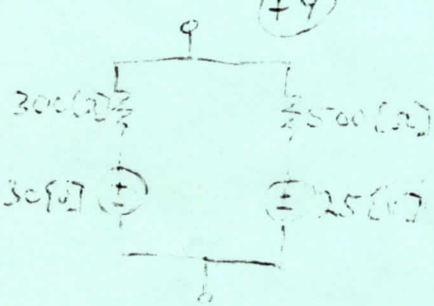
and a second circuit has the following Thevenin equivalent values,

$$V_{TH2} = 25[V]$$
$$R_{TH2} = 500[\Omega].$$

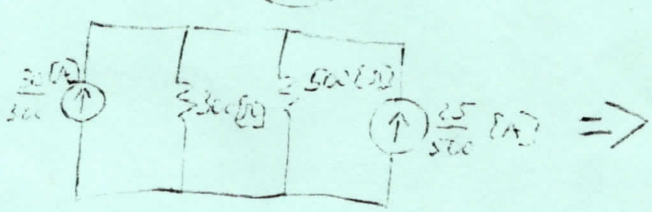
These two equivalent circuits are connected in parallel at their terminals. Find the Thevenin equivalent of one of the possible parallel combinations of these circuits, with respect to the common terminals.

Solutions

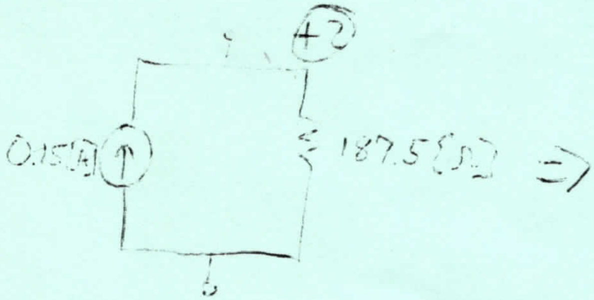
(+4)



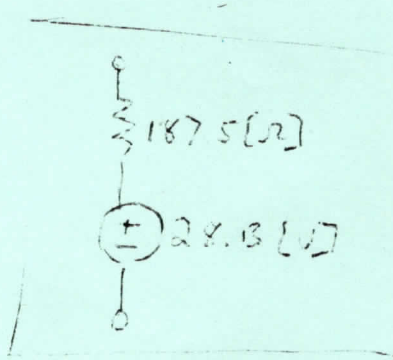
(+2)



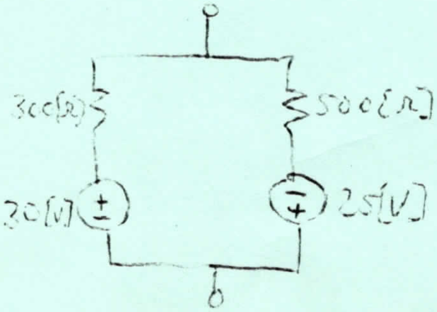
(+2)



(+2)



Other possibilities



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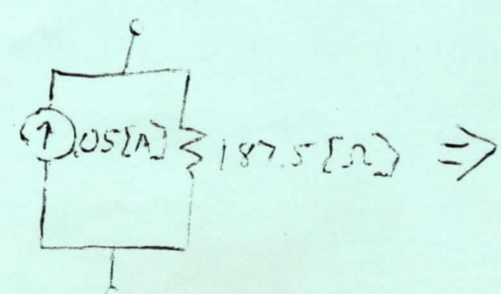
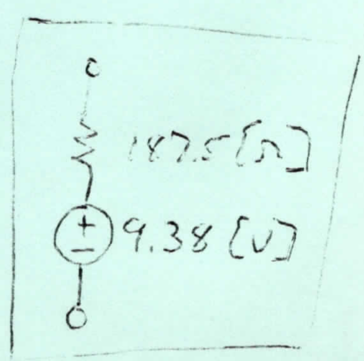
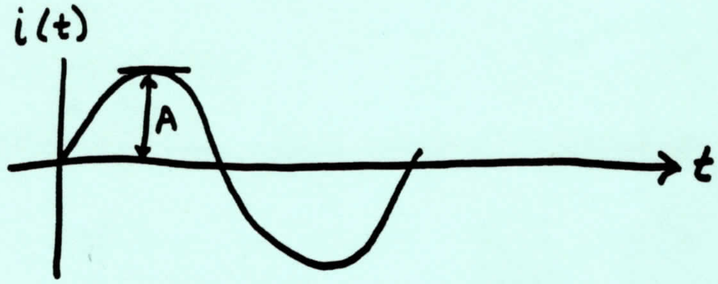




either source could have a minus sign.

5. (10 Points) If the current through a 90[mH] inductor is sinusoidal, as shown in the sketch below, what must the amplitude A be in order to generate an inductor voltage that has an amplitude of 10[kV]?



$i(t) = A \sin(1000t)$

$$v(t) = L\frac{di(t)}{dt} = LA\,1000\cos 1000t = 90 * 10^{-3} * 10^{3} * A \cdot \cos 1000t;$$

$$V_{max} = 90\,A \Rightarrow 10.10^{3}[V];\ \boxed{A = \frac{10.10^{3}}{90} = 111.11111\,[A]}$$

6. (15 Points) An inductor with inductance L = 10[mH] has a current at time t=1[s] which is equal to 5[A]. The voltage across the inductor is equal to $5e^{-3t}$[V] for the time period 1[s]<t<5[s]. Find the magnitude of the power absorbed by the inductor at t=3[s].

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau + i_L(t_0) \quad +4$$

$$P = v_L\, i_L \quad +3$$

$$P(3) = \left(5e^{-3t}\right)\left(\frac{1}{L}\int_1^t 5e^{-3\tau}\,d\tau + i_L(1)\right)\Bigg|_{t=3} \text{ [watts]} \quad +4$$

$$P(3) = \left(5e^{-9}\right)\left(\frac{1}{10^{-2}}\int_1^3 5e^{-3\tau}\,d\tau + 5\right) \text{ [watts]}$$

$$P(3) = \left(5e^{-9}\right)\left(100\frac{(5)}{(-3)}e^{-3\tau}\Big|_1^3 + 5\right) \text{ [watts]}$$

$$P(3) = \left(5e^{-9}\right)\left(\frac{-500}{3}\left\{e^{-9} - e^{-3}\right\} + 5\right) \text{ [W]}$$

$$P(3) = \left(6.17\times10^{-4}\right)\left(\frac{-500}{3}\left\{-4.97\times10^{-2}\right\} + 5\right) \text{ [W]}$$

$$P(3) = \left(6.17\times10^{-4}\right)\left(8.28 + 5\right) \text{ [W]}$$

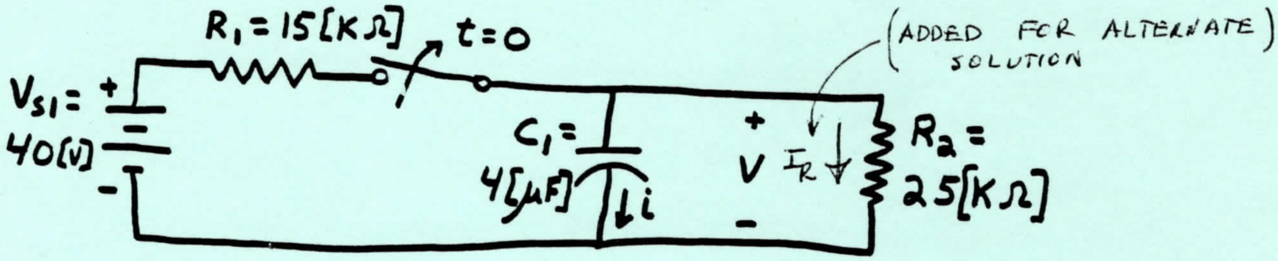
$$P(3) = \left(6.17\times10^{-4}\right)\left(13.28\right) \text{ [W]}$$

$$\boxed{P(3) = 8.19 \text{ [mW]}}$$

+4 work + answer

-3 for neglecting $i_L(1)$
-3 for wrong integral limits
-2 for math error
-2 for no units

7. (15 Points) In the network given, the switch has been closed for a long time. At time t=0 the switch is opened.



a) Determine the value of v for all time greater than zero.

SOLUTION: SINCE THE SWITCH HAS BEEN CLOSED FOR A "LONG TIME,"

$$v_c(0^-) = 40 \times \frac{25K}{40K} = 25_V = v_c(0^+)$$

AFTER SWITCH IS OPENED, THE EQUATION FOR $v(t) = v_c(t)$ CAN BE WRITTEN AS $v_c(t) = A_1 e^{-t/RC} + A_2$ VOLTS

NOW

$$RC = 25\times10^3 \times 4\times10^{-6} = .1 \text{ sec}$$

$$v_c(0^+) = 25v \; ; \therefore \; 25 = A_1 + A_2$$

$$v_c(t\to\infty) = 0 \; ; \therefore \; 0 = A_2$$

HENCE $v_c(t) = v(t) = \boxed{25 e^{-10t} \text{ VOLTS}}$

b) Determine the value of i for all time greater than zero.

NOW $i_c = i = C\dfrac{dv_c(t)}{dt} = C\left[-250 e^{-10t}\right] A$

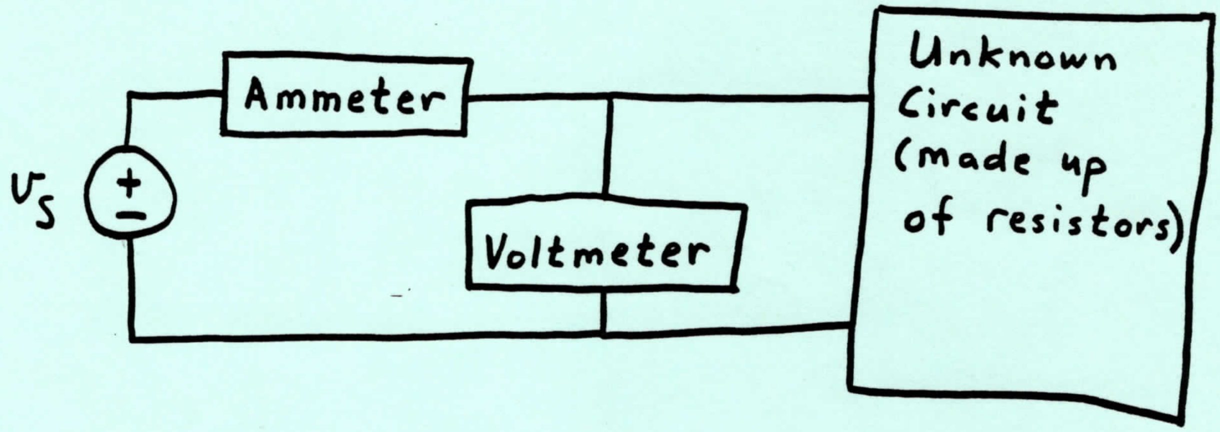
$$= -4\times10^{-6} \times 250 e^{-10t} = \boxed{-1\times10^{-3} e^{-10t} \text{ AMPS}}$$

ALTERNATE SOLUTION: (SEE CIRCUIT DIAGRAM)

$$I_R = \frac{v(t)}{R} = \frac{25 e^{-10t}}{25\times10^3} = 1\times10^{-3} e^{-10t} \text{ AMPS}$$

BUT $i = -I_R = \boxed{-1\times10^{-3} e^{-10t} \text{ AMPS}}$

Bonus question. (10 points) In the following circuit, the ammeter shown has a resistance of 10[Ω], and the voltmeter shown has a resistance of 20[kΩ]. The ammeter reads 2.5[mA], and the voltmeter reads 10[V]. What is the equivalent resistance of the unknown resistive circuit?



The current in the voltmeter is

$$\frac{10[V]}{20[k\Omega]} = 0.5[mA] \quad +3$$

so the current in the unknown circuit is

$$2.5 - 0.5 = 2[mA] \quad +3$$

$$\text{so } \boxed{R = \frac{10[V]}{2[mA]} = 5[k\Omega]} \quad +4$$