

Signature: Solution Key

**DO NOT OPEN THIS BOOKLET
UNTIL INSTRUCTED TO DO SO.**

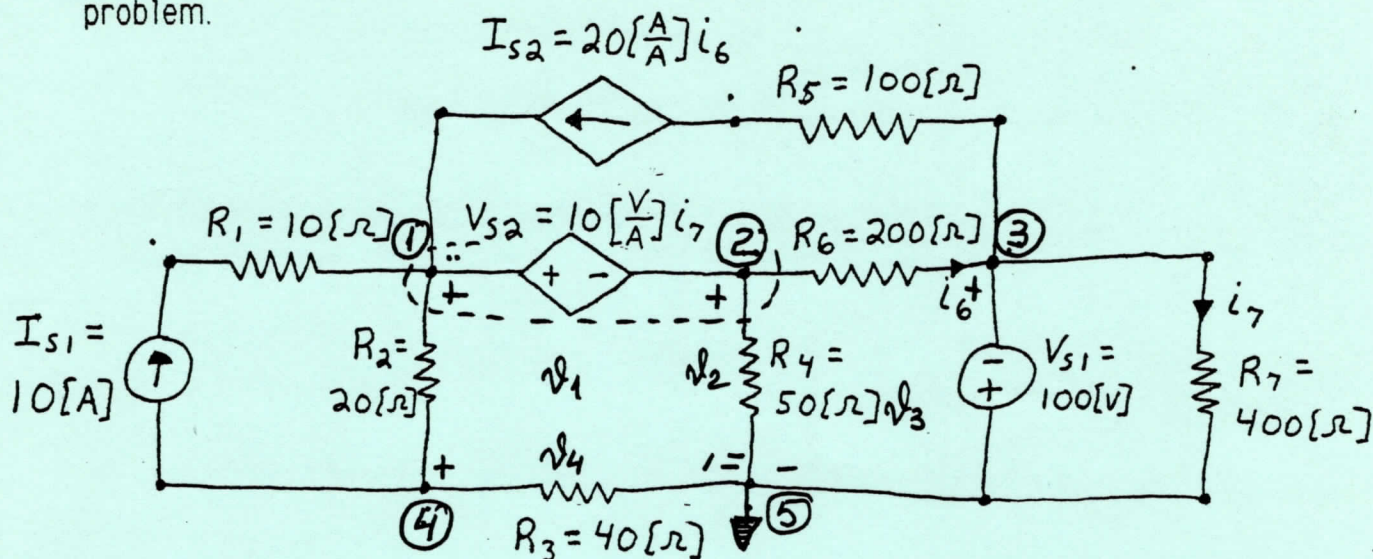
**EXAM 2
ELEE 2335
October 25, 1986**

INSTRUCTIONS:

1. Sign your name on the upper left of this page.
2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown.
3. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.
4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 21
2. 18
3. 10
4. 13
5. 23
6. 15
100

1. (21 Points) The following circuit should be used for both parts of this problem.



a) How many nodes, branches, essential nodes and essential branches are there in this circuit?

$$n = 7$$

$$b = 11$$

$$n_e = 5$$

$$b_e = 9$$

b) Supposing that the circuit is to be solved using the node voltage method, write all the equations that would be needed to solve for the node voltages. DO NOT SOLVE THEM.

Depending how the reference node is selected, could be more version.
It is shown the case with node # 5 as the reference.

Kirchhoff's current law applied to:

* node # 1+2 (supernode):

$$-I_{s1} - I_{s2} + \frac{v_1 - v_4}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_6} = 0 \quad (1)$$

* node # 4:

$$I_{s1} + \frac{v_4 - v_1}{R_2} + \frac{v_4}{R_3} = 0 \quad (2)$$

* Kirchhoff's voltage law on supernode:

$$v_1 - v_2 = v_{s2} \quad (3)$$

Dependent sources

$$I_{s2} = 20 \times 6 = 20 \frac{v_2 - v_3}{200} \quad (4)$$

$$V_{s2} = 10 \times 7 = 10 \frac{v_3}{R_7} \quad (5)$$

* Node # 3:

$$v_3 = -V_{s1} = -100[V] \quad (6)$$

with numerical values:

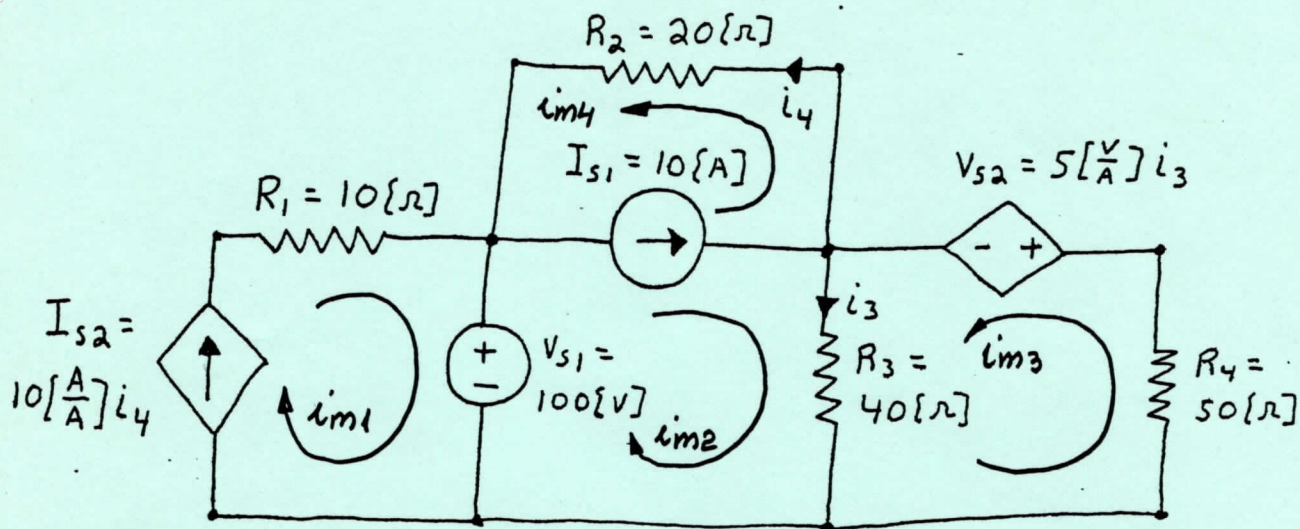
$$-10 - I_{s2} + \frac{v_1 - v_4}{20} + \frac{v_2}{50} + \frac{v_2 + 100}{200} = 0$$

$$10 + \frac{v_4 - v_1}{20} + \frac{v_4}{40} = 0$$

$$I_{s2} = \frac{v_2 + 100}{10}$$

$$v_{s2} = 10 \frac{-100}{400} = -2.5[V]$$

2. (18 Points) The following circuit should be used for both parts of this problem.



a) Using the number of essential branches and nodes, find the number of independent meshes in this circuit.

$$n_e = 3, \quad b_e = 6, \quad \# \text{ indep. meshes} = b_e - (n_e - 1) = 4$$

b) Supposing that the circuit is to be solved using the mesh current method, write all the equations that would be needed to solve for the mesh currents. DO NOT SOLVE THEM.

The positive direction for the mesh currents, can be chosen in different ways.

Equations are applied for directions shown in the figure.

ROOM FOR EXTRA WORK

Kirchhoff's voltage law, applied to:

mesh # 2+4 (supermesh):

$$-V_{s1} - R_2 i_{m4} + R_3 (i_{m2} + i_{m3}) = 0 \quad (1)$$

or

$$i_{m2} + i_{m4} = I_{s1} \quad (2)$$

mesh # 3:

$$R_3 (i_{m2} + i_{m3}) + R_4 i_{m3} + V_{s2} = 0 \quad (3)$$

mesh # 1:

$$i_{m1} = I_{s2} \quad (4)$$

Dependent sources

$$I_{s2} = 10 i_4 = 10 * i_{m4} \quad (5)$$

$$V_{s2} = 5 i_3 = 5 (i_{m2} + i_{m3}) \quad (6)$$

with numerical values:

$$-100 - 20 i_{m4} + 40 (i_{m2} + i_{m3}) = 0$$

$$i_{m2} + i_{m4} = 10$$

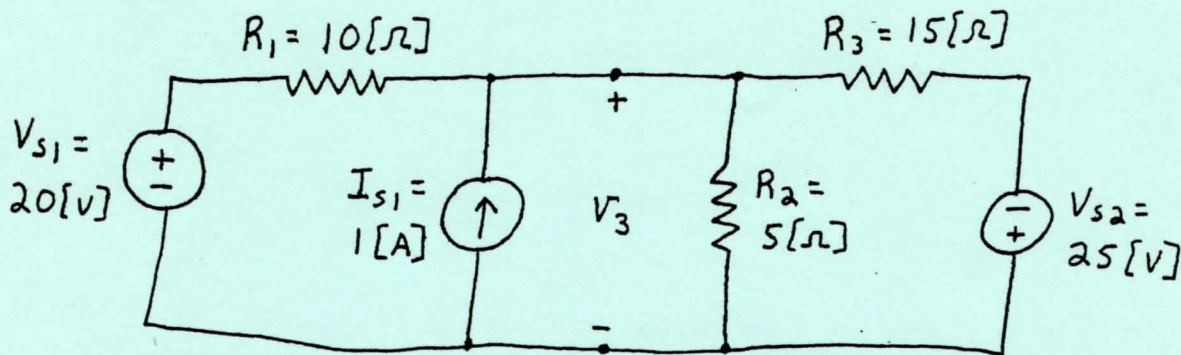
$$40 (i_{m2} + i_{m3}) + 50 i_{m3} + V_{s2} = 0$$

$$i_{m1} = I_{s2}$$

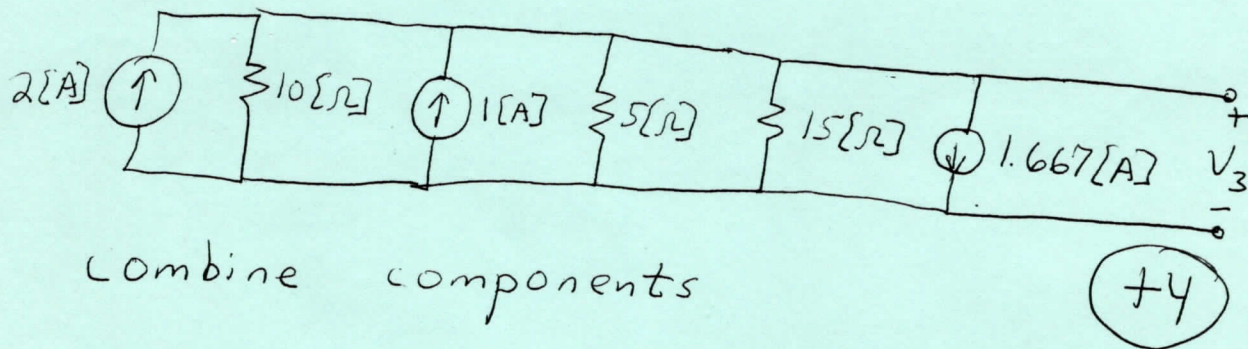
$$I_{s2} = 10 * i_{m4}$$

$$V_{s2} = 5 (i_{m2} + i_{m3})$$

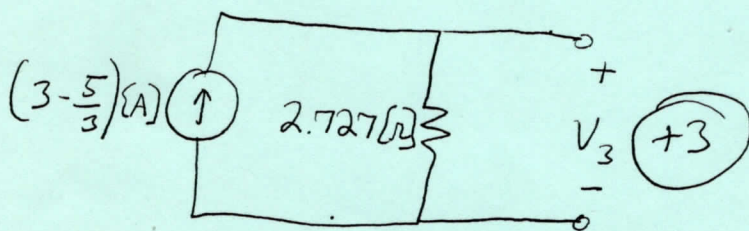
3. (10 Points) Find the voltage v_3 . Examine the circuit first, and use the most efficient method.



Solution: Use source transformations



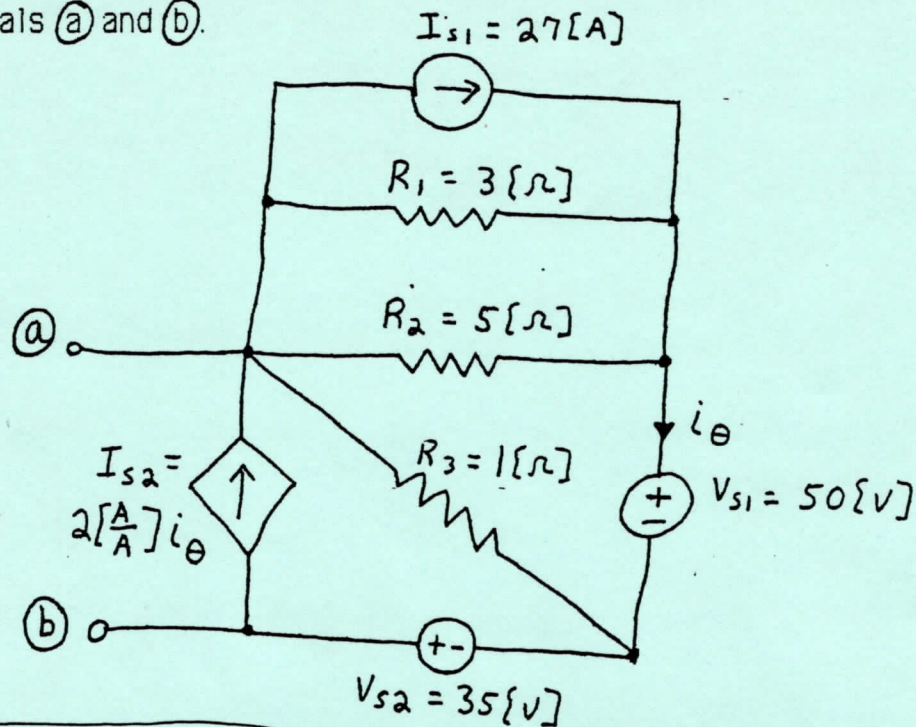
Combine components



$$V_3 = \frac{4}{3} [A] 2.727 [\Omega] = 3.636 [V]$$

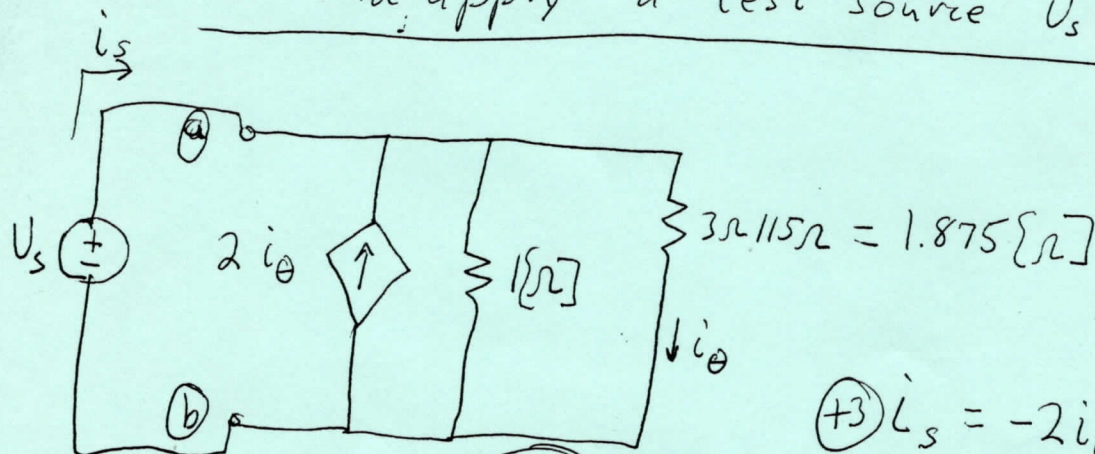
- 3 no units
- 2 math error
- +3 correct answer

4. (13 Points) Find the Thevenin equivalent resistance with respect to terminals (a) and (b).



Soln.

Set indep. sources to zero, redraw, and apply a test source V_s



$$+3 \quad i_s = -2i_\theta + \frac{V_s}{1[\Omega]} + \frac{V_s}{1.875[\Omega]}$$

$$+2 \quad i_\theta = \frac{V_s}{1.875[\Omega]}$$

$$i_s = -\frac{2V_s}{1.875[\Omega]} + \frac{V_s}{1[\Omega]} + \frac{V_s}{1.875[\Omega]}$$

$$i_s = .467 V_s$$

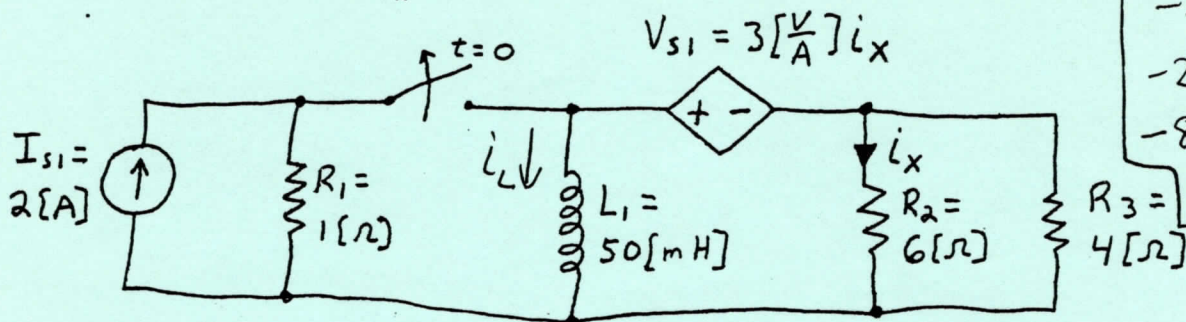
$$+2 \quad \boxed{\frac{V_s}{i_s} = 2.143 [\Omega]} = R_T$$

Math error (-2)

Sign error (-3)

If dep. source is ignored, then 5pts will be awarded, max.

5. (23 Points) The switch had been closed for a long time, and then was opened at $t = 0$. Find $i_x(t)$ for $t > 0$.



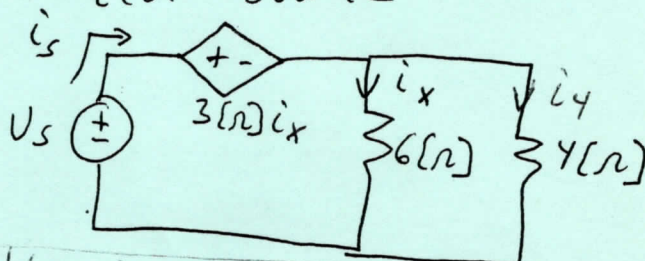
-3 sign error
-2 math error
-8 neglecting dep. source

$$i_L(0) = 2[A] \quad (+4)$$

$$\tau = \frac{L}{R_{eq}} \quad (+4)$$

$$R_{eq} = ?$$

To find R_{eq} , redraw ckt w/res to inductor, Apply test source



alt. method. $i_y = \frac{6}{4} i_x$ $i_s = 2.5 i_x$
 $R_{eq, source} = \frac{3[\Omega] i_x}{2.5 i_x} = 1.2[\Omega] \quad (\text{then solve})$

$$(1) \quad i_s = \frac{V_s - 3i_x}{6[\Omega]} + \frac{V_s - 3i_x}{4[\Omega]} \quad (\text{from KVL}) \quad (+3)$$

$$(2) \quad i_x = i_s \frac{4}{10} \quad (\text{from CDR}) \quad (+3)$$

Combine (1) + (2), yield

$$24 i_s = 4 V_s - 12(.4 i_s) + 6 V_s - 18(.4 i_s)$$

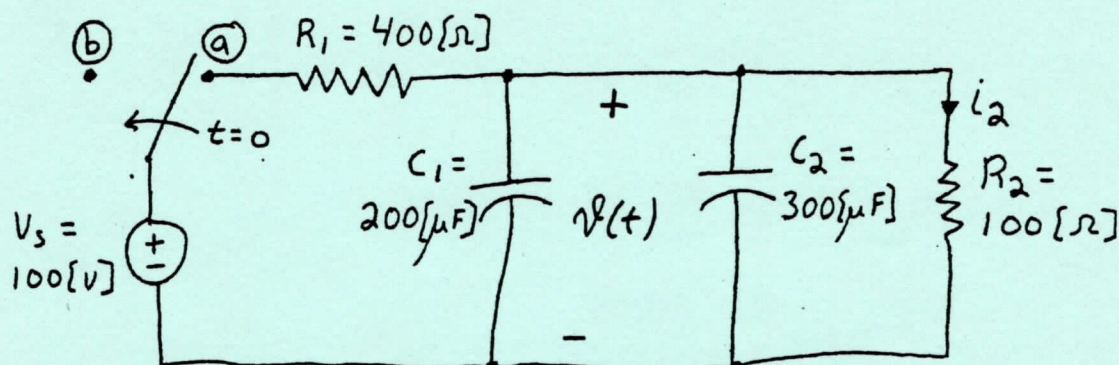
$$36 i_s = 10 V_s$$

$$3.6[\Omega] = \frac{V_s}{i_s} = R_{eq} \quad (+2)$$

$$\tau = \frac{.05}{3.6} [s] = 13.89 [ms]$$

$$i_L = 2[A] e^{-t/13.89[ms]} ; t \geq 0 \Rightarrow \begin{cases} \text{for } t > 0 \\ i_x = -.4 i_L = -0.8[A] e^{-t/13.89[ms]} \\ \quad \quad \quad = -0.8[A] e^{-72t} \end{cases} \quad (+4)$$

6. (15 Points) In the following circuit, the switch was in position (a) for a long time. At $t = 0$, the switch moves to position (b). Determine $i_2(t)$, for $t \geq 0$.

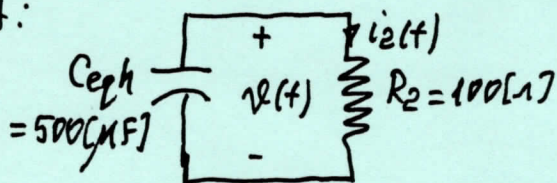


$$t < 0 \quad v(0^-) = V_s \frac{R_2}{R_1 + R_2} = 100 \frac{100}{500} = 20 \text{ [V]}$$

$$C_{eqh} = C_1 + C_2 = 500 \text{ [\mu F]}$$

$$t \geq 0 \quad v(0^+) = v(0^-) = 20 \text{ [V]} = V_0$$

Circuit:



natural response:

$$v(t) = V_0 e^{-\frac{t}{\tau}}$$

$$\tau = R_2 C_{eqh} = 100 \times 500 \times 10^{-6} = 0.05 \text{ [s]}$$

$$v(t) = 20 e^{-20t} \text{ [V]};$$

$$i_2(t) = \frac{v(t)}{R_2} = 0.2 e^{-20t} \text{ [A]}$$