Signature: solution!

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

EXAM 3
ELEE 2335
November 22, 1986

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

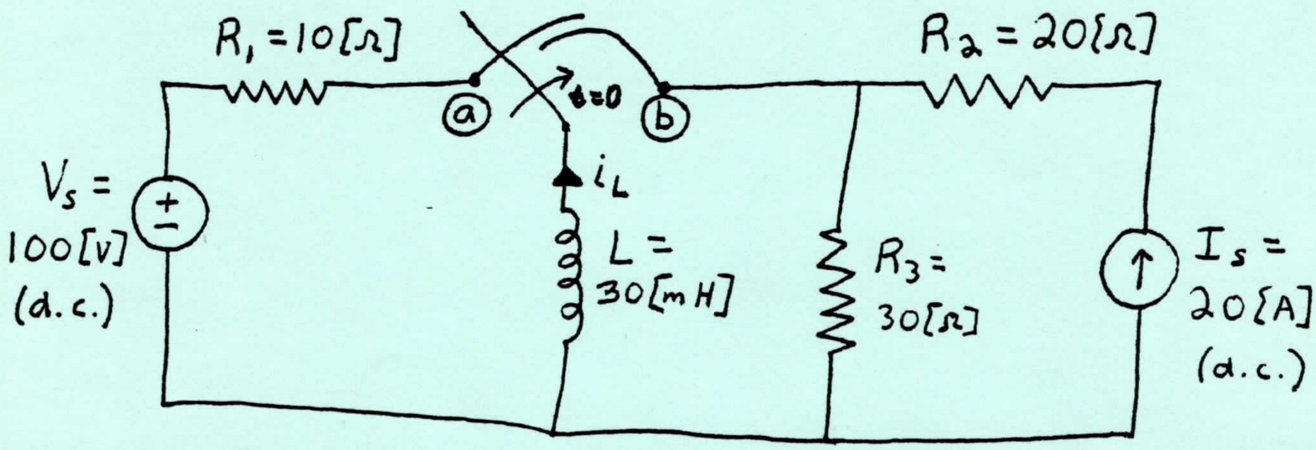
3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 23
2. 23
3. 20
4. 6
5. 28

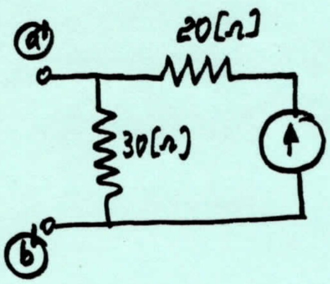
100

1. (23 Points) Assume that the circuit below has had the switch in position ⓐ for a long time. The switch is moved instantaneously to position ⓑ at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

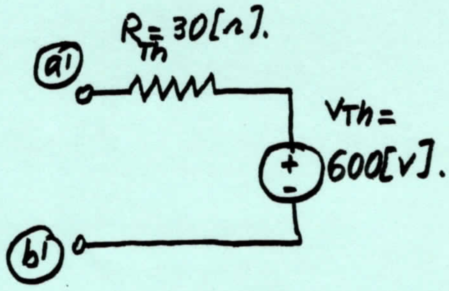


$t<0$; $i_L(0^-) = -\frac{V_s}{R_1} = -\frac{100}{10} = -10[A]$ 3

$t \geq 0$;



$I_s = 20[A] \Rightarrow$ Thévenin



6

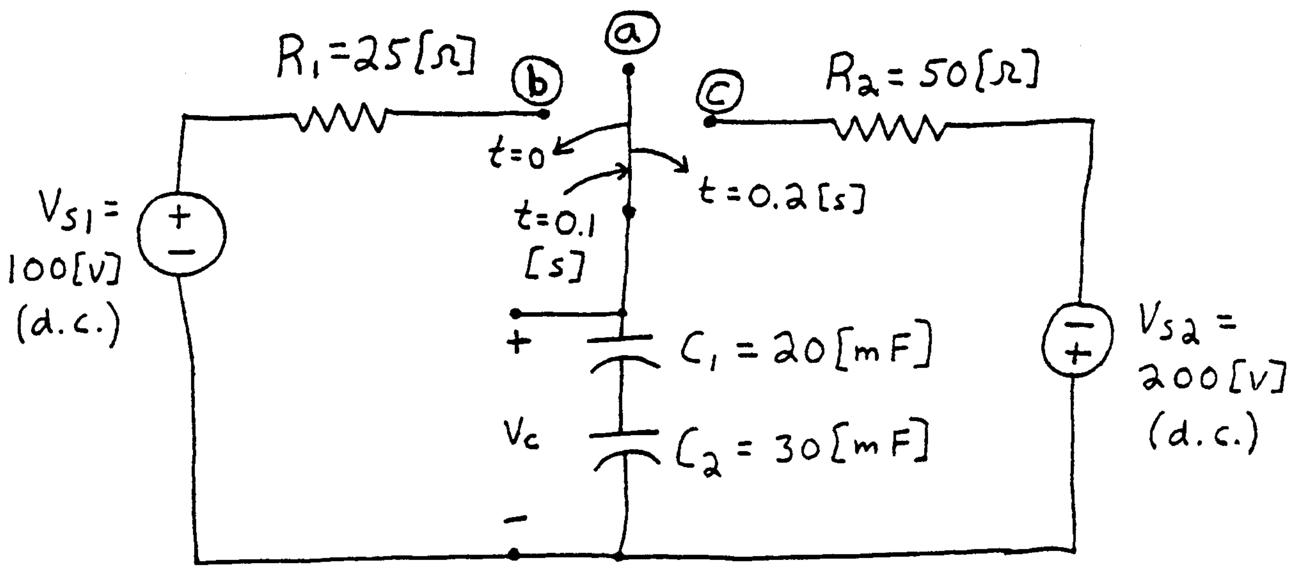
6 $i_L(t) = i_{Lfinal} + (i_{Linit} - i_{Lfinal})\, e^{-\frac{t}{\tau}}$; $\tau = \frac{L}{R} = \frac{30 \times 10^{-3}}{30} = 10^{-3}[s]$; 5

$i_{Lfinal} = -\frac{600}{30} = -20\,[A]$. 3

$$\boxed{i_L(t) = -20 + (-10-(-20))\, e^{-1000t} = -20 + 10\, e^{-1000t}\ [A]}$$

2. (23 Points) In the following circuit, the switch was in position ⓐ for a long time before t = 0. At t = 0, the switch is moved to position ⓑ and remains there for 0.1 [seconds]. At t = 0.1 [s] the switch is moved back to position ⓐ where it remains for another 0.1 [s]. At t = 0.2 [s], the switch is moved to position ⓒ where it remains indefinitely. The initial value of the capacitor voltage v_C is zero. Determine v_C for each of the time intervals listed below.

$t < 0$
$0 \le t \le 0.1$ [s]
0.1 [s] $\le t \le 0.2$ [s]
$t \ge 0.2$ [s]



$$C_{eqh} = \frac{C_1 C_2}{C_1 + C_2} = \frac{20*30}{50} = 12[mF]$$

$t<0$; $\boxed{v_c(0^-)=0;}$

$0 \le t \le 0.1[s]$; $v_c(t) = v_{final} + (v_{cin} - v_{cfin})\, e^{-\frac{t}{\tau}}$; $v_{cfinal} = 100[V]$; $\tau = R_1C = 12*10^{-3}*25 = 0.3[s]$.

$\boxed{v_c(t) = 100 + (0-100)e^{-\frac{t}{0.3}} = 100 - 100e^{-t/0.3}[V]}$; $v_c(0.1^-) = 100 - 100e^{-0.1/0.3} = 28.346867[V]$.

$0.1 \le t \le 0.2[s]$;

$\boxed{v_c(t) = V_c(0.1^-) = 28.346867[V]}$

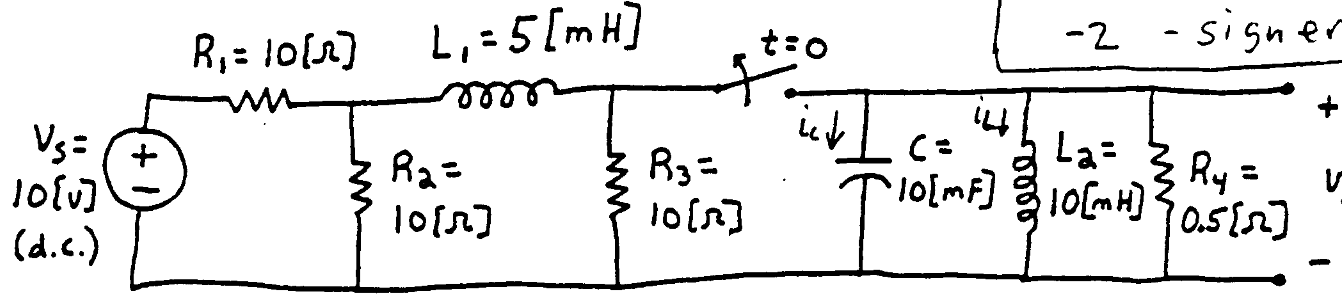
$t \ge 0.2[s]$: $v_{cfinal} = -200[V]$; $v_{cinitial} = 28.346867[V]$;

$\tau = R_2 C_{eq} = 50*12*10^{-3} = 0.6[s]$;

$$\boxed{v_c(t) = -200 + (28.\dots + 200)e^{-t/0.6} = -200 + 228.346867*e^{-\frac{t-0.2}{0.6}}[V]}$$

3. (20 Points) The switch in the circuit shown below had been closed for a long time, and was opened at $t = 0$. Find $v_X(t)$ for $t \geq 0$.

−3 − no units
−2 − math error
−2 − sign error



Solution: Find initial conditions first.

$v_x(0^-) = 0$ since inductor is a short circuit

$v_x(0^+) = v_x(0^-) = 0$ since this is a voltage across a capac. (+4)

$$\left.\frac{dv_x}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C} = \frac{-i_L(0^+)}{C}$$ {since current in resistor is zero}

$$i_L(0^+) = \frac{V_s}{R_1} = 1[A]$$

$$\left.\frac{dv_x}{dt}\right|_{t=0^+} = \frac{-1[A]}{10^{-2}[F]} = -100\left[\frac{V}{s}\right]$$ (+4)

This is a parallel RLC natural response

$$\alpha = \frac{1}{2RC} = \frac{1}{2(0.5)10^{-2}}\,[\text{rad/sec}] = 100 \text{ rad/sec}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{10^{-2}\,10^{-2}}}\,[\text{rad/sec}] = 100 \text{ rad/sec}$$ (+2)

$\alpha = \omega_0 \Rightarrow$ critically damped response

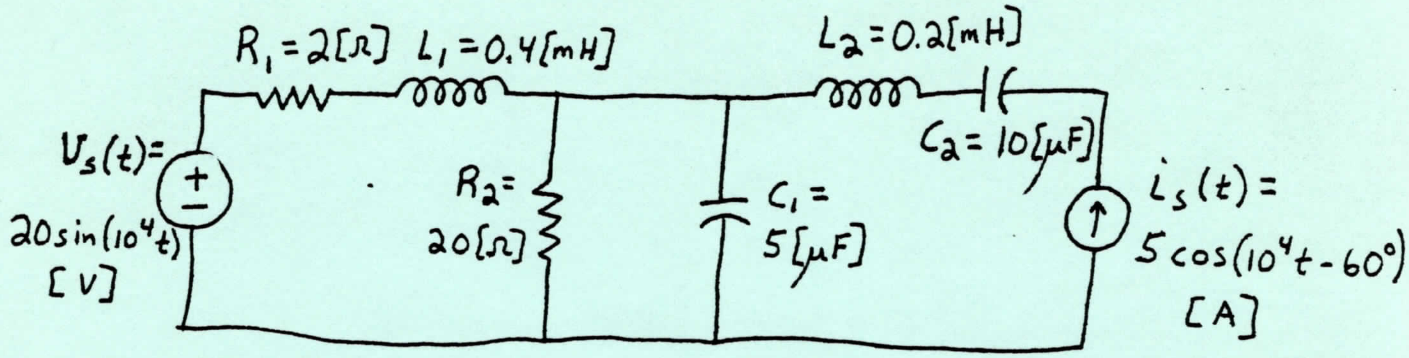
$$v_x(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$ (+4)

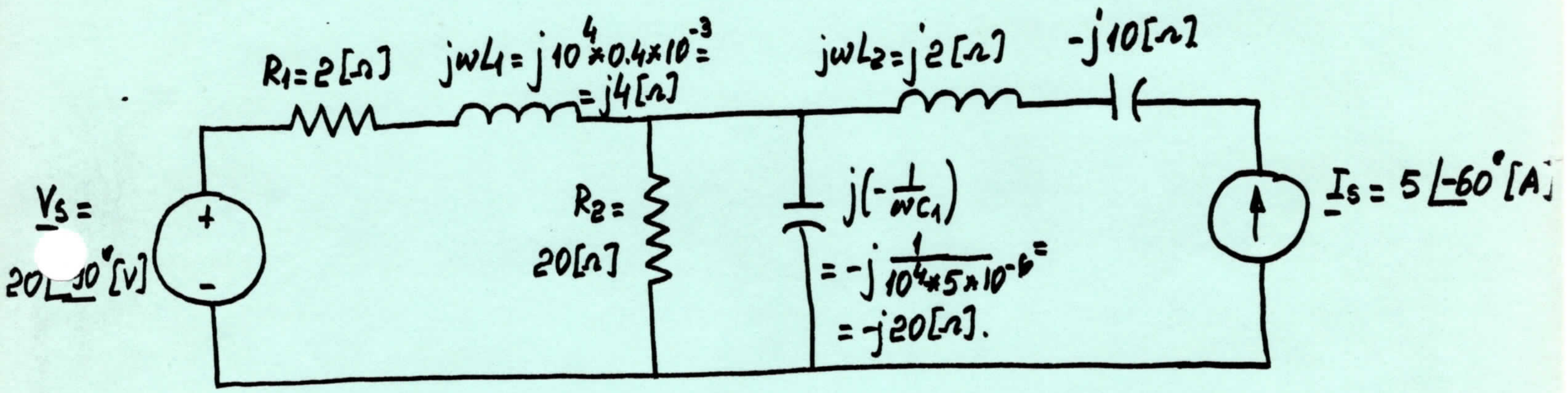
$$v_x(0) = 0 \Rightarrow D_2 = 0$$ (+2)

$$\frac{dv_x}{dt} = -100 = \left.\left(D_1 e^{-\alpha t} + D_1(-\alpha)\overset{0}{\cancel{t}}e^{-\alpha t} + \overset{0}{\cancel{D_2}}(-\alpha)e^{-\alpha t}\right)\right|_{t=0} = D_1$$ (+2)

$$\boxed{v_x(t) = -100\left[\frac{V}{s}\right] t\, e^{-100t}; \text{ for } t \text{ in [sec]}; \text{ for } t \geq 0}$$ (+2)

4. (6 Points) Take the circuit below and redraw it in the phasor domain.



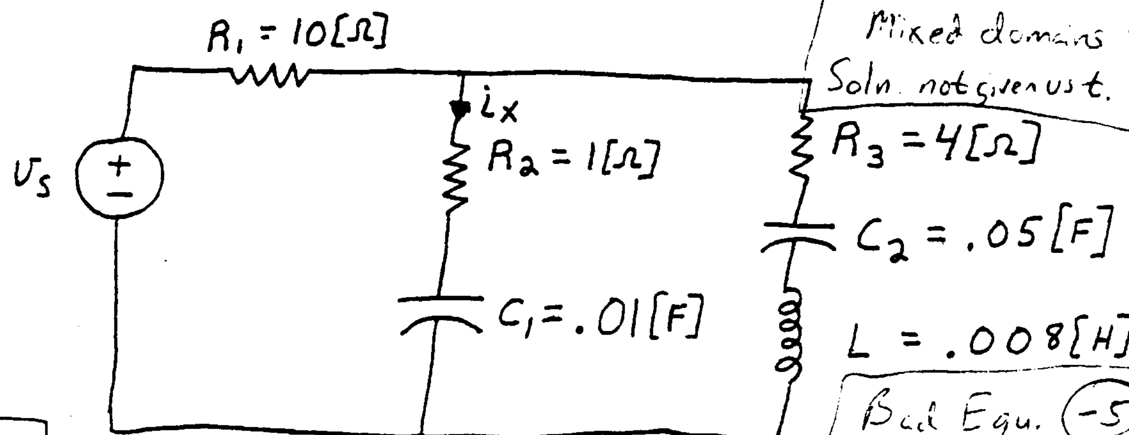


5. (28 Points) The circuit below is in the sinusoidal steady state. The value of the current i_x is known to be

$i_x(t) = 30 \cos (50t + 65°)$ [Amperes].

What is the steady state voltage $v_S(t)$?

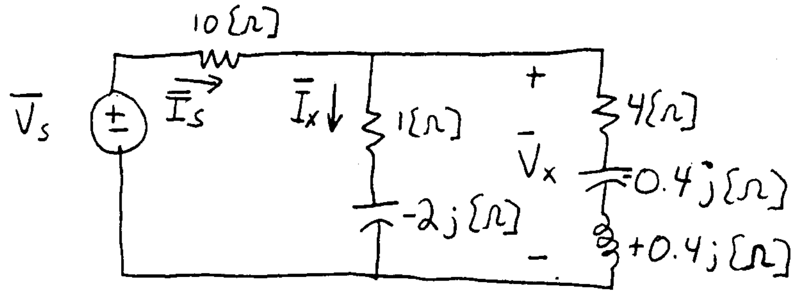
not distinct phasor (-3)
no units (-3)
math error (-2)
Sign error (-3)
Mixed domains (-10)
Soln. not given w/ t. (-8)



Bad Equ. (-5)

Solution

Redraw in phasor domain:



$$\bar{Z}_{C1} = \frac{1}{j\omega C_1} = -j\frac{1}{50(.01)} = -2j\ [\Omega]$$

$$Z_{C2} = \frac{1}{j\omega C_2} = -0.4j\ [\Omega]$$

$$Z_L = j\omega L = 0.4j\ [\Omega]$$

$$\bar{V}_x = \bar{I}_x(1-2j) = 30\angle 65°(2.23\angle -63.43°) = 67.1\angle 1.57°\ [V]$$

$$\bar{I}_x = \bar{I}_s\left(\frac{4[\Omega]}{4[\Omega]+(1-2j)[\Omega]}\right)$$ by Curr. Div. Rule

$$\bar{I}_s = \frac{30\angle 65°[A](5-2j)}{4} = \frac{(30\angle 65°)(5.39\angle -21.8°)}{4}\ [A]$$

$$\bar{I}_s = 40.39\ \angle 43.2°\ [A]$$

$$\bar{V}_s = \bar{I}_s R_1 + \bar{V}_x = (403.9\angle 43.2° + 67.1\angle 1.57°)\ [V]$$

$$\bar{V}_s = (294.4 + 276.5j + 67.07 + 1.84j)\ [V]$$

$$\bar{V}_s = 361.5 + 278.3j = 456\angle 37.6°\ [V]$$

$$\boxed{v_s(t) = 456\cos(50t + 37.6°)\ [V]}$$