Signature: Solution Key

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

FINAL EXAM
ELEE 2335
December 12, 1986

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

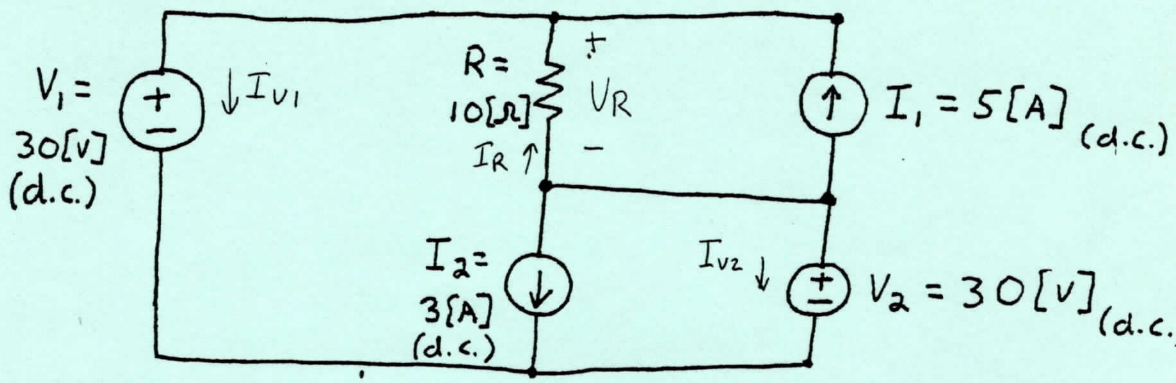
3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 14
2. 16
3. 30
4. 24
5. 19
6. 28
7. 50
8. 19

$\Sigma = 200$

1. (14 Points) How much power is absorbed by each of the five elements in this circuit?



Solution:

$V_R = 30[V] - 30[V] = 0$ (+2)

$I_R = 0$

$\therefore I_{V1} = 5[A]$ (+1)

$I_{V2} = -8[A]$ (+1)

sign error (-2)

math error (-1)

wrong KVL, KCL (-4)

$P_R = 0$ (+2)

$P_{I_1} = 0$ (+2)

$P_{V_1} = 5[A]\,30[V] = 150[W]$ (+2)

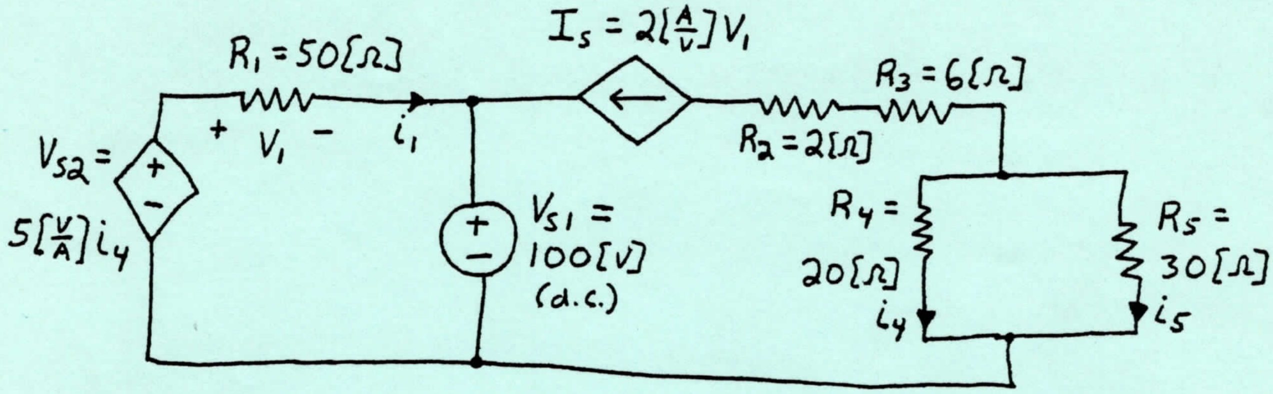
$P_{I_2} = 3[A]\,30[V] = 90[W]$ (+2)

$P_{V_2} = 30[V]\ -8[A] = -240[W]$ (+2)

check $P_R + P_{I_1} + P_{V_1} + P_{I_2} + P_{V_2} = 0$

checks ✓

2. (16 Points) Determine the values of i_1, i_4, i_5, I_S, and V_{S2} in the following circuit. Do not use the node voltage method or the mesh current method.



(1) $i_1 = \dfrac{V_{S2} - V_{S1}}{R_1} = \dfrac{V_{S2} - 100}{50}$; $I_s = 2V_1 = 2 * R_1 i_1 = 100 i_1$;

(2) $i_4 = -I_s \dfrac{30}{30+20} = -100 \dfrac{30}{50} i_1 = -60 i_1$; $V_{S2} = 5 i_4 = -300 i_1$

with (1): $i_1 = \dfrac{-300 i_1 - 100}{50}$ $\rightarrow$ $\boxed{i_1 = -\dfrac{100}{350} = -\dfrac{10}{35}[A] = -0.2857142[A]}$

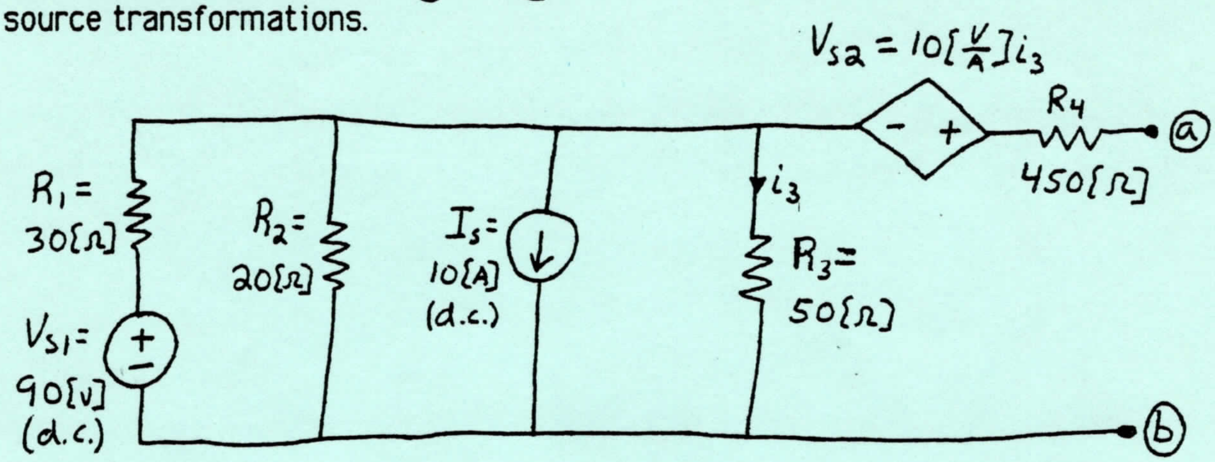
with (2):

$\boxed{i_4 = -60 * \left(-\dfrac{10}{35}\right) = \dfrac{600}{35} = 17.142857[A]}$; $\boxed{I_s = 100 i_1 = -\dfrac{1000}{35} = -28.571429[A]}$

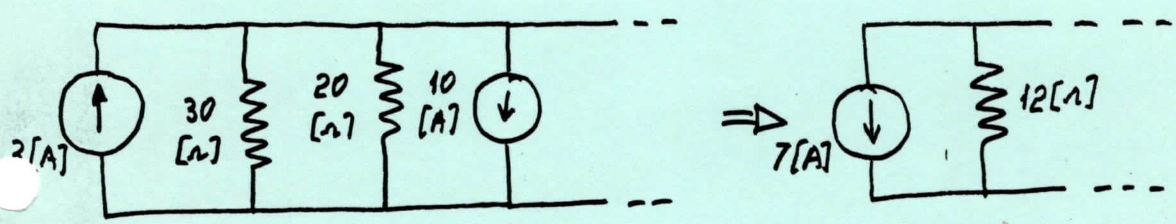
$\boxed{i_5 = -I_s - i_4 = \dfrac{400}{35} = 11.428571[A]}$

$\boxed{V_{S2} = -300 * \left(-\dfrac{10}{35}\right) = \dfrac{3000}{35} = 85.714286[V]}$

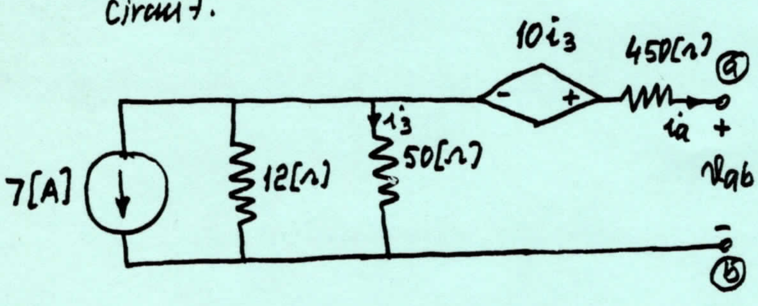
3. (30 Points) For the following circuit, determine the Thevenin equivalent with respect to terminals ⓐ and ⓑ. Simplify the circuit first using source transformations.



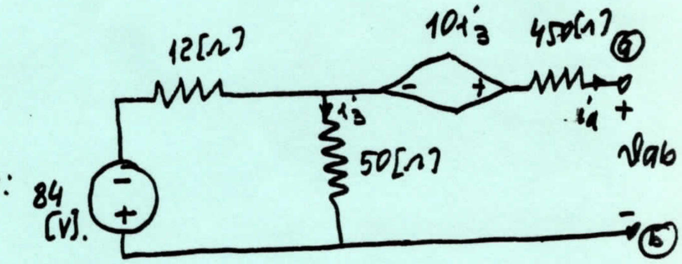
source transformations:



circuit:



or:



Thévenin equivalent:

version 1:

version 2:

Thévenin voltage: $i_a = 0$

$$i_3 = -7\,\frac{12}{12+50} = -1.3548387[A].$$

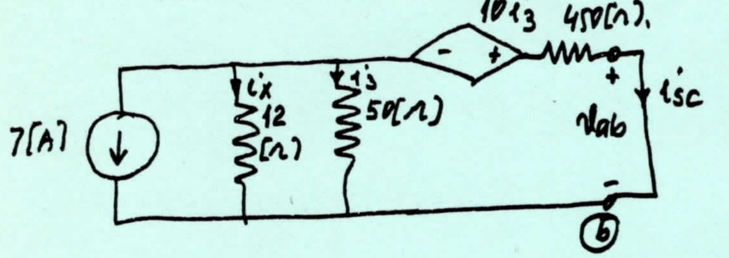
$$i_3 = -\frac{84}{62} = -1.3548387[A]$$

$$\boxed{v_{ab} = 10i_3 + 50i_3 = \underline{-81.290323} = v_{Th}}$$

ROOM FOR EXTRA WORK

Thevenin resistance:

short circuit test: $v_{ab} = 0$;



$$v_{12} = 10 i_3 = 450 i_{sc} - 50 i_3 \Rightarrow i_3 = \frac{45}{6} i_{sc}$$

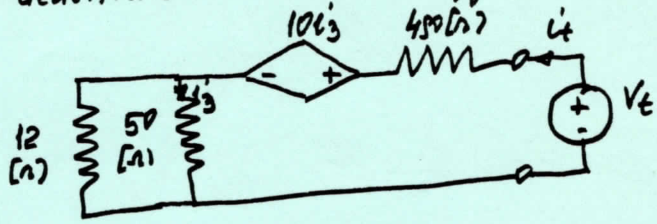
$$i_x = \frac{50 i_3}{12} = \frac{50 * 45}{12 * 6} i_{sc} = 31.25 i_{sc}.$$

KCL: $7 + i_x + i_3 + i_{sc} = 0 \Rightarrow$

$$7 + 31.25 i_{sc} + 7.5 i_{sc} + i_{sc} = 0 \Rightarrow i_{sc} = -\frac{7}{39.75} [A]$$

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{-81.29\ldots}{-\frac{7}{39.75}} = 461.61291 [\Omega].$$

deactivate the sources and apply external voltage:

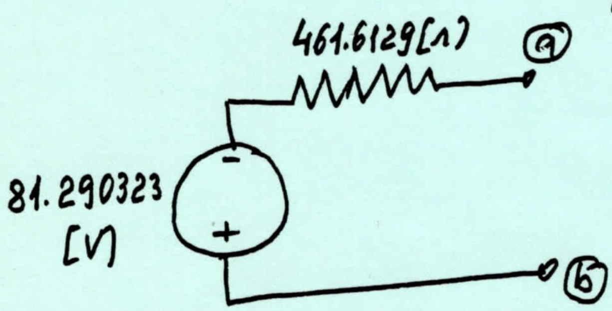


$$i_3 = i_t \frac{12}{62}; \quad V_t = 450 i_t + 10 i_3 + 50 i_3$$

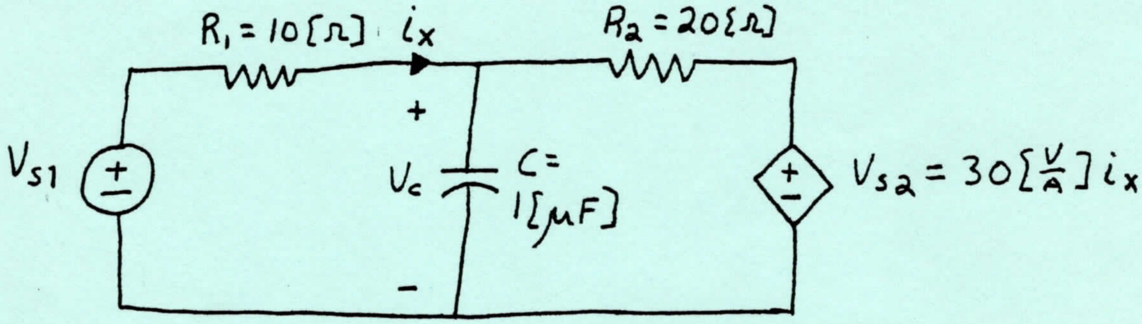
$$V_t = 450 i_t + 60 \frac{12}{62} i_t = 461.6129 i_t$$

$$R_{Th} = \frac{V_t}{i_t} = 461.6129 [\Omega].$$

Thevenin equiv. circuit:



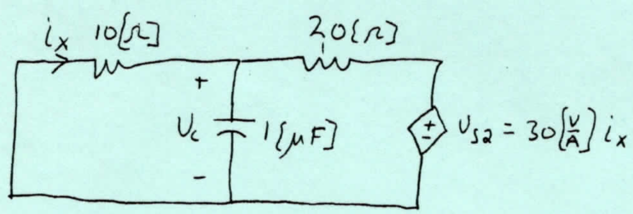
4. (24 Points) In the circuit below, the following information is given.
$V_{S1} = 0[V]$; for $t < 1[sec]$
$V_{S1} = 10[V]$; for $t > 1[sec]$
$v_C(t) = 0[V]$; for $t < 1[sec]$
Find $v_C(t)$ for $t > 1[sec]$.



matherror (−1)
no units (−3)

Solution

circuit for $t < 1[s]$



No independent sources $\Rightarrow$ $i_x = 0$, $v_c = 0$ $\Big\}$ for $t < 1[s]$

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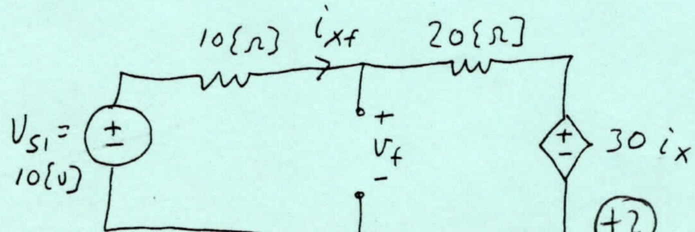
for $t > 1[s]$ $\rightarrow$ RC step response

(+4) $v_c(t) = v_f + (v_c(1) - v_f)\, e^{-(t-1)/\tau}$;

(+4) $v_c(1) = 0$

Need v_f and τ.

at $t = \infty$,



(+2)

$$\frac{V_{S1} - 30 i_x}{30[\Omega]} = i_{xf}$$

$$\frac{1}{3} - i_{xf} = i_{xf}$$

$$\frac{1}{3} = 2 i_{xf}$$

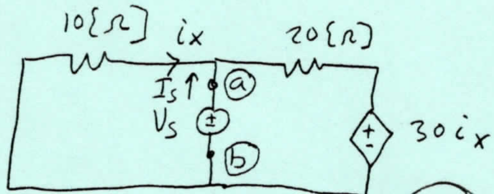
$$i_{xf} = \frac{1}{6}[A]$$

$$V_f = 10[V] - i_{xf}[10[\Omega]] = \left(10 - \frac{10}{6}\right)[V] = \frac{50}{6}[V] = 8.33[V]$$ (+4)

(con't next page)

ROOM FOR EXTRA WORK

to get τ, set independent sources to zero, and find the equiv. resistance seen by C, at terminals (a) and (b), by applying a test source V_s



(+4)

$$I_s = -i_x + \frac{V_s - 30 i_x}{20}$$

$$i_x = \frac{-V_s}{10}$$

$$I_s = \frac{V_s}{10} + \frac{V_s}{20} + 1.5\frac{V_s}{10} = (.1 + .05 + .15)\,V_s = .3\,V_s$$

$$\frac{V_s}{I_s} = \frac{1}{.3} = 3.33[\Omega] = R_{eq.} = 3.33\,[\Omega]$$ (+2)

$$\tau = R_{eq.}\,C = 3.33 \times 10^{-6}\,[s]$$ (+2)

So,

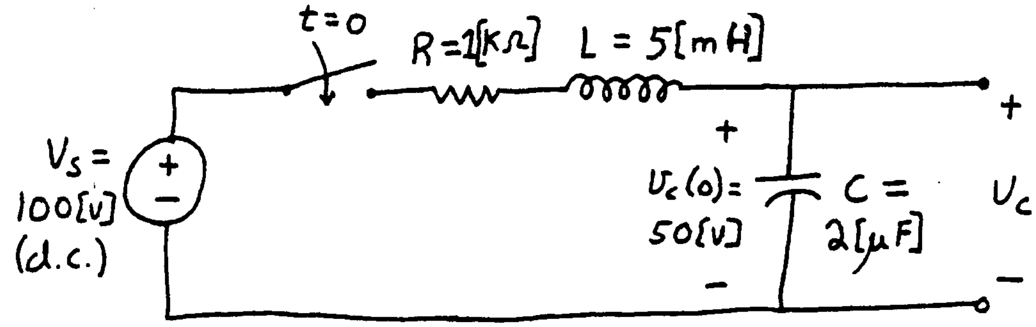
$$v_c(t) = 8.33[v] - 8.33\,e^{-(t-1)/3.33\times10^{-6}}\,[v]$$; for $t > 1$; for t in [s]

or

$$\boxed{v_c(t) = 8.33[v] - 8.33\,e^{-300{,}000(t-1)}\,[v] \text{ ; for } t > 1 \text{ ; for } t \text{ in } [s]}$$

(+2)

5. (19 Points) In the circuit shown, the switch is closed at t = 0. Determine $v_C(t)$ for $t \geq 0$. It is known that $v_C(0) = 50[V]$.



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step response:

$v_c(t) = V_{cfinal} + \{\text{natural response}\}$; $v_{cfinal} = 100\,[V]$.

$\alpha = \frac{R}{2L} = \frac{1000}{2*5*10^{-3}} = 100{,}000\ [rad/s]$;

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5*10^{-3}*2*10^{-6}}} = 10{,}000\ [rad/s]$;

$\alpha^2 > \omega_0^2$, overdamped:

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -501.257\ [rad/s]$; $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -199498.74\ [rad/s]$.

$v_c(t) = 100 + A_1 e^{s_1 t} + A_2 e^{s_2 t}$; $t = 0^+$, $v_c(0^+) = 50\,[V] \Rightarrow$

$50 = 100 + A_1 + A_2$: $\underline{A_1 + A_2 = -50}$ (1)

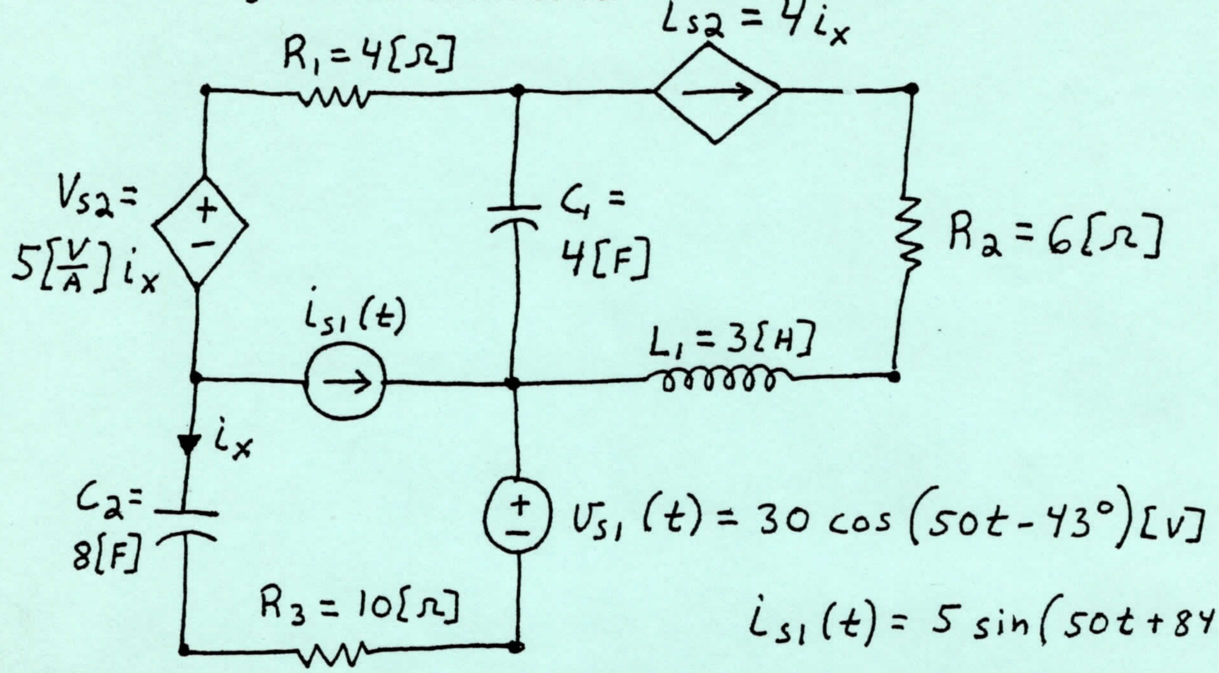
$i_c(t) = C\frac{dv_c(t)}{dt} = (s_1 A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t})C$; $i(0^+) = 0 = s_1 A_1 + s_2 A_2 = i_L(0^-)$

$\underline{-501.257 A_1 - 199498.74 A_2 = 0}$ (2): with (1)+(2), results;

$A_1 = -50.125945\,[V]$; $A_2 = 0.125945\,[V]$.

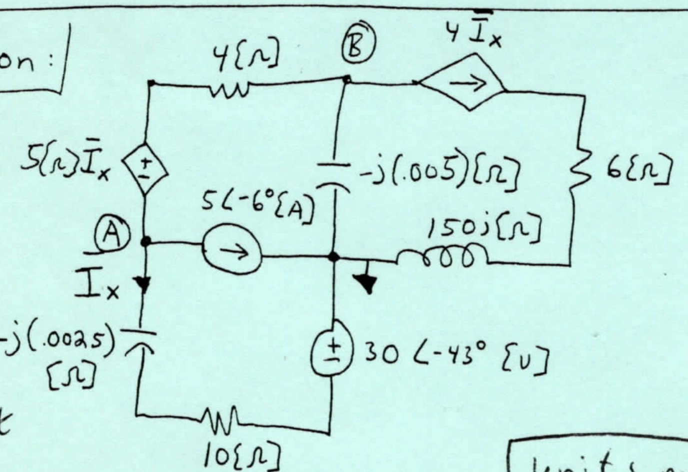
$$\boxed{v_c(t) = 100 - 50.125945\, e^{-501.257t} + 0.125945\, e^{-199498.74t}\ [V].}$$

6. (28 Points) a) Draw the following circuit in the phasor domain, and then write the equations that would be needed to solve this problem using the node voltage method. Do not solve.



$i_{s1}(t) = 5\sin(50t + 84°)[A]$

Solution:



$\bar{Z}_{L_1} = j\omega L_1 = j\,50(3) = 150j\,[\Omega]$

$\bar{Z}_{C_1} = \frac{1}{j\omega C_1} = -j(.005)[\Omega]$

$\bar{Z}_{C2} = \frac{1}{j\omega C_2} = -j(.0025)[\Omega]$

$\bar{I}_{s1} = 5\angle(84° - 90°)[A]$

$\bar{I}_{s1} = 5\angle -6°[A]$

diagram (+8)

(-2) for each wrong component

units missing (-3) one time

3 essential nodes

(+4) A

$$5\angle -6°[A] + \frac{\bar{V}_A + 30\angle -43°[V]}{(10 - j(0.0025))[\Omega]} + \frac{\bar{V}_A + 5[\Omega]\bar{I}_x - \bar{V}_B}{4[\Omega]} = 0$$

(+4) B

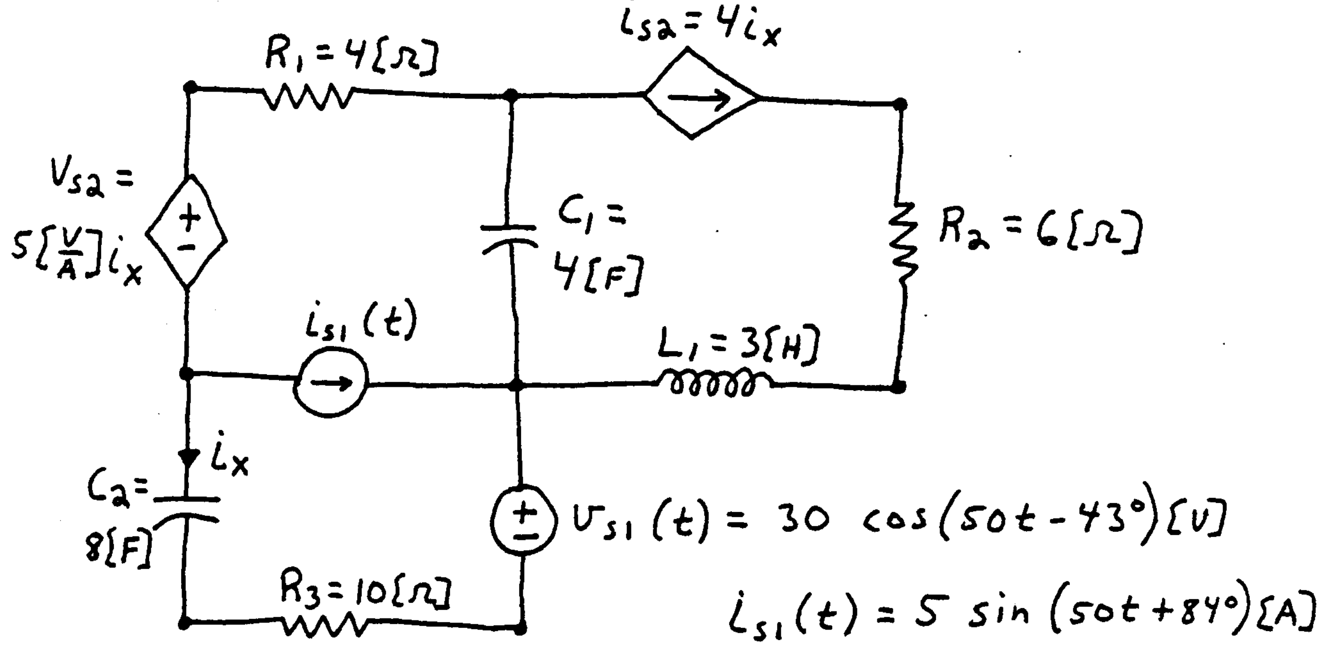
$$\frac{\bar{V}_B}{-j(.005)[\Omega]} + \frac{\bar{V}_B - 5[\Omega]\bar{I}_x - \bar{V}_A}{4[\Omega]} + 4\bar{I}_x = 0$$

Dep. Source

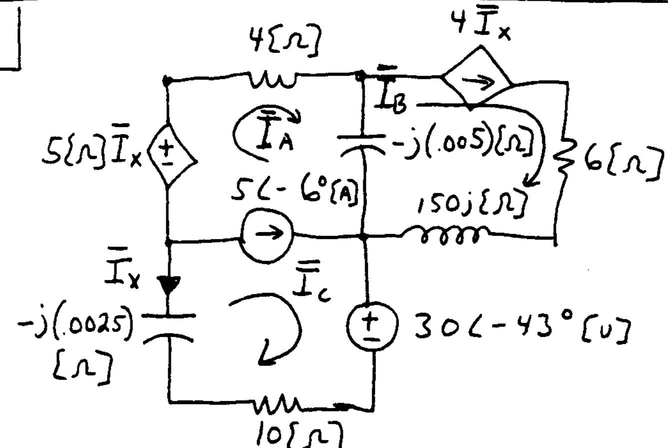
(+2)

$$\bar{I}_x = \frac{\bar{V}_A + 30\angle -43°[V]}{(10 - j(0.0025))[\Omega]}$$

6. (continued) b) Draw the following circuit in the phasor domain, and then write the equations that would be needed to solve this problem using the mesh current method. Do not solve.



Solution:



Loop B

(+2)

$$\bar{I}_B = 4\bar{I}_x$$

Supermesh A + B

(+4)

$$-30\angle -43°[V] - \bar{I}_c(10 - j(.0025))[\Omega] + 5[\Omega]\bar{I}_x - 4[\Omega]\bar{I}_A$$
$$-(\bar{I}_A - \bar{I}_c)(-j.005)[\Omega] = 0$$

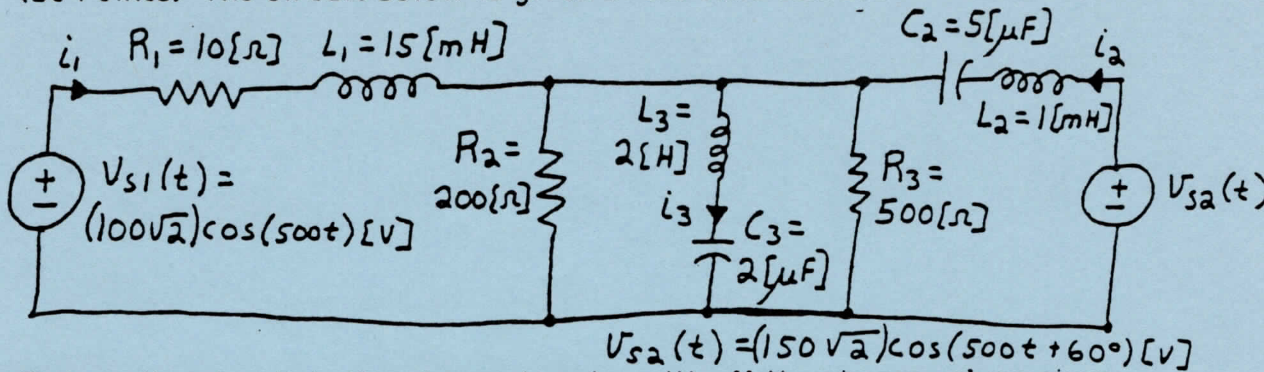
(+2) Mesh A + B

$$\bar{I}_c - \bar{I}_A = 5\angle 6°[A]$$

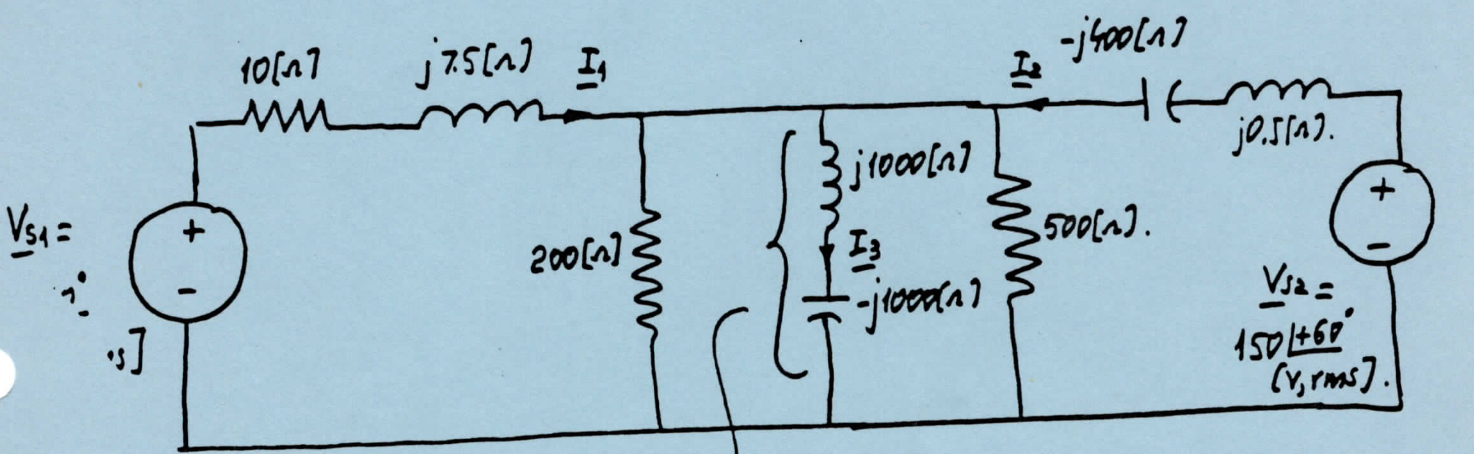
(+2) Dep. Source

$$\bar{I}_x = -\bar{I}_c$$

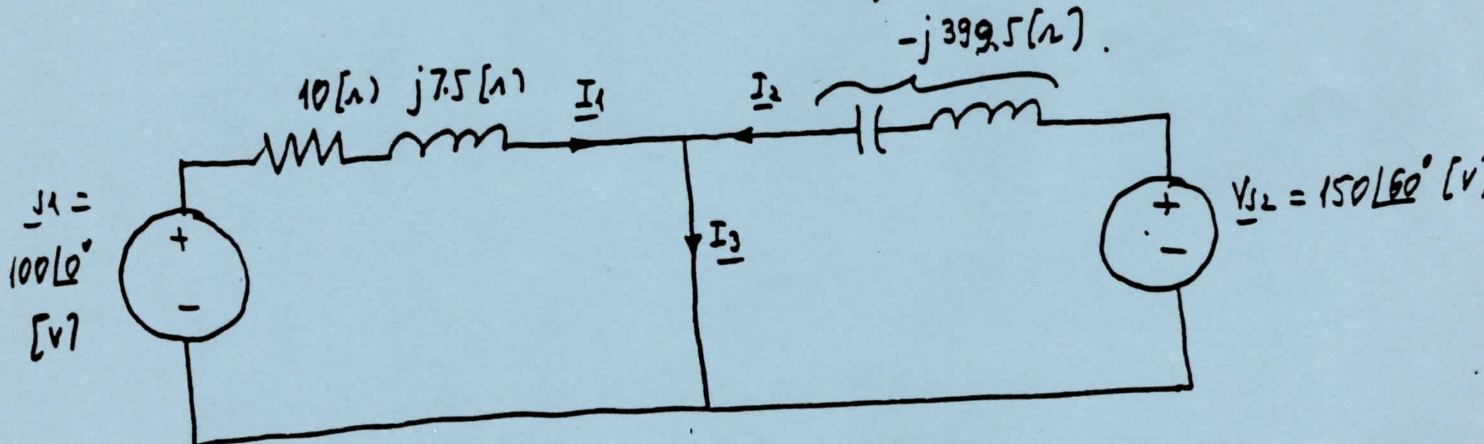
7. (50 Points) The circuit below is given in the time domain.



a) Redraw the circuit in the phasor domain, with all the phasor values given as RMS values.



because $Z_{eqh} = 0$, the circuit becomes:



7. (continued)

b) Determine the values of the phasors $\underline{I}_1$, $\underline{I}_2$, and $\underline{I}_3$.

$$\underline{I}_1 = \frac{\underline{V}_{s1}}{\underline{Z}_1}; \quad \underline{Z}_1 = 10 + j7.5 = 12.5 \angle 36.869897^\circ \, [\Omega];$$

$$\boxed{\underline{I}_1 = \frac{100\angle 0^\circ}{12.5\angle 36.\ldots} = 8\angle -36.869897^\circ \, [A]};$$

$$\underline{I}_2 = \frac{\underline{V}_{s2}}{\underline{Z}_2}; \quad \underline{Z}_2 = -j395.5 = 395.5 \angle -90^\circ \, [\Omega];$$

$$\boxed{\underline{I}_2 = \frac{150\angle +60}{399.5\angle -90^\circ} = 0.3754693\angle 150^\circ \, [A]}$$

$$\underline{I}_3 = \underline{I}_1 + \underline{I}_2 = (6.4 - j4.8) + (-0.3251659 + j0.1877346) =$$

$$= (6.074834 - j4.6122654) = 7.6273587 \angle -37.207273^\circ \, [A]$$

7. (continued)

c) Determine the complex power delivered by each source and the total delivered power. Also, calculate the average and reactive power consumed by each of the passive components.

complex power delivered:

$$\underline{S}_{S1} = \underline{V}_{S1} \cdot \underline{I}_1^* = 100\angle 0^\circ * 8 \angle 36{,}869\ldots = 800\angle 36{,}869\ldots =$$

$$= 800(0{,}8 + j0{,}6) = (640 + j480)\,[VA].$$

$$\underline{S}_{S2} = \underline{V}_{S2}\,\underline{I}_2^* = 150\angle 60^\circ * 0{,}37\ldots \angle -150^\circ = 56{,}320395\angle -90^\circ = (0 - j56{,}320395)[VA]$$

$$\sum \underline{S}_{sources} = \underline{S}_{S1} + \underline{S}_{S2} = (640 + 423{,}67961)\,[VA]$$

power consumed:

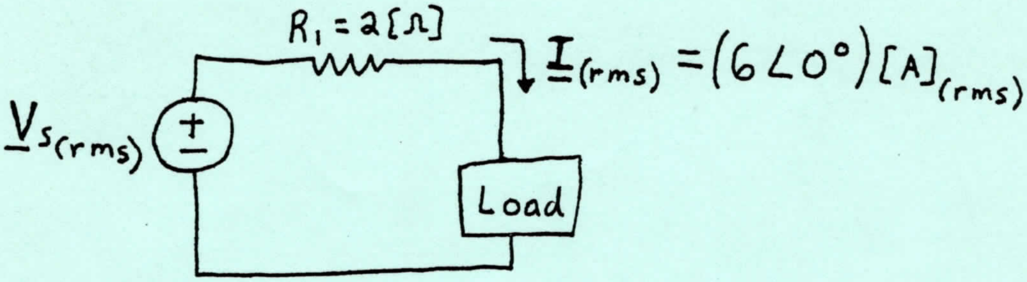
$$P_{R1} = I_1^2 R_1 = 8^2 * 10 = 640\,[W]\ ;\ Q_{R1} = 0\,[VAR].$$

$$P_{XL1} = 0\,[W]\ ;\ Q_{L1} = I_1^2 X_{L1} = 8^2 * 7{,}5 = 480\,[VAR]$$

$$P_{X(C2+X_{L2})} = 0\,[W]\ ;\ Q_{(X2+C2)} = I_2^2 X_{(X2+C2)} = 0{,}37\ldots^2(-399{,}5) = -56{,}320389\,[VAR]$$

$$\sum P_{cons} = 640\,[W]\ ;\ \sum Q_{consu} = 423{,}67961\,[VAR].$$

8. (19 Points) In the circuit below, the load absorbs 230.51 [VA] at a leading power factor of 0.6247. Find the magnitude and the phase of the voltage source, $\underline{V}_{S,(RMS)}$.



(+3) $|\bar{S}_L| = 230.51\ [VA]$

$\cos(\theta_v - \theta_i) = 0.6247$

(+3) $\theta_v - \theta_i = -51.34°$ → sign is known to be negative since it is a leading power factor → capacitive type load.

$\bar{S}_L = 230.51\angle -51.34° = (144 - j\,180)\,[VA]$

(+3) $P_L = 144\ [W]$ $\quad Q_L = -180\ [VAR]$

$P_L = |\underline{I}_{rms}|^2 R_L = 36 R_L = 144\ [W]$

(+2) $R_L = 4\ [\Omega]$

(+2) $Q_L = |\underline{I}_{rms}|^2 X_L = 36 X_L = -180\ [VAR]$

$X_L = -5\ [\Omega]$

(+1) $\bar{Z}_L = (4 - 5j)\ [\Omega]$

(+3) $\underline{V}_{S_{rms}} = \underline{I}_{rms} R_1 + \underline{I}_{rms}(\bar{Z}_L) = 6(2) + 6(4 - 5j) = 12 + 24 - 30j\ [V]_{rms}$

$\underline{V}_{S_{rms}} = 36 - 30j\ [V]_{rms}$

(+2) $\underline{V}_{S_{rms}} = 46.86 [V]_{rms} \angle -39.8°$

units (−3)
sign error (−2)
math error (−1)