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SOLUTIONS

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

FINAL EXAM
ELEE 2335
December 18, 1987

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. Two crib sheets, or four sides of 8.5" x 11" paper may be used. Print your name and student number on each sheet and submit them with your exam.

3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

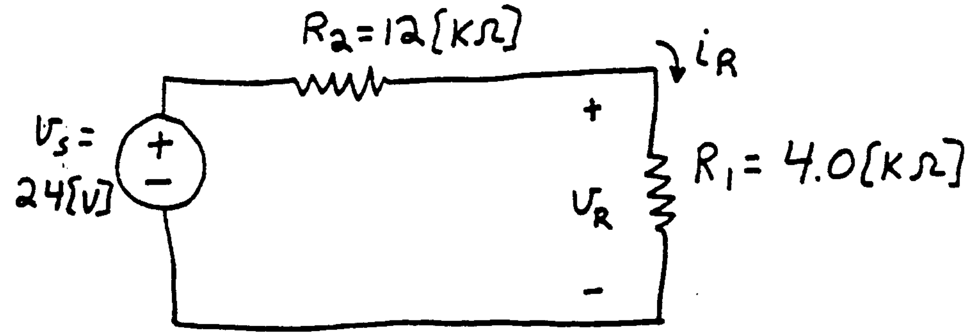
4. Do not use red ink.

1.	15
2.	12
3.	12
4.	15
5.	18
6.	18
7.	10
Bonus.	20

120

/100

1. (15 Points) You and your lab partner are asked to measure the voltage drop across a precision resistor, R_1, in the circuit below. The value of R_1 is 4.0[kΩ]. Your lab partner uses a meter with a 20[mV], 1[mA] d'Arsonval movement to measure v_R directly. He uses the meter as a voltmeter at 10[V] full scale. You use the same meter as an ammeter at 2[mA] full scale to measure the current i_R through the resistor. You then multiply your result by R_1 to get the voltage. Whose method gives the more accurate result?



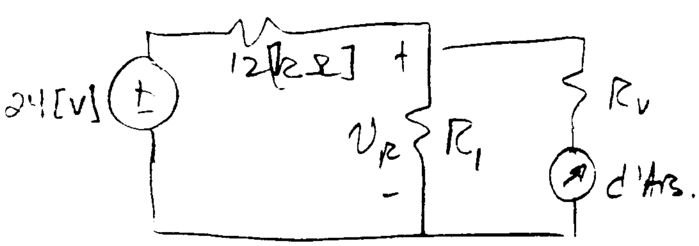
CALCULATED VALUES:

$$V_R = 24\,V \cdot \frac{4}{16} = 6\,[V]$$

(+1)

$$R_{d'Ar} = 20\,[\Omega]$$

VOLTAGE MSMT:



$$10\,V\ fs \Rightarrow 10 \cdot \frac{20}{R_V + 20} = 20\times10^{-3}$$

(+5

$$\therefore R_V = 9980\,[\Omega]$$

$$\Rightarrow R_{meter} = R_V + R_{d'Ar} = 10\,[k\Omega]$$

$$R_{meter} \,//\, 4\,[k\Omega] = 2.86\,[k\Omega]$$

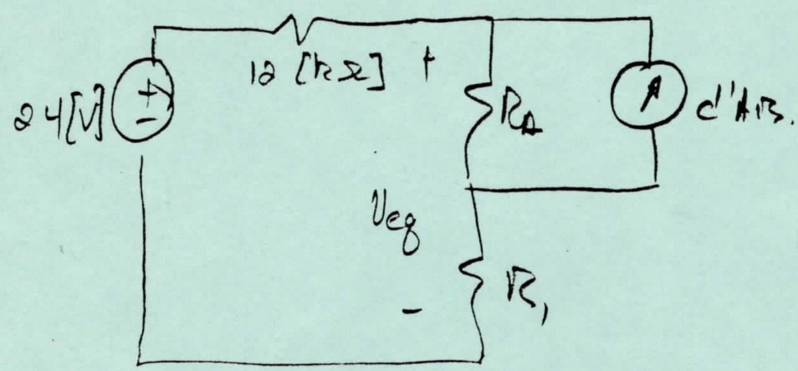
+7

$$\Rightarrow V_R = 24\,[V] \cdot \frac{2.86}{2.86+12} = 4.62\,[V]$$ (+2)

$$\therefore \%\ error = \frac{6-4.62}{6} = 23\%$$

ROOM FOR EXTRA WORK

CURRENT MSMT:



$R_{d'ARS} = 20\,[\Omega]$

$2\,mA\ fs \Rightarrow i_{R_A} = 1\,mA\ ;\ V_{R_A} = 20\,mV \Rightarrow R_A = 20\,[\Omega]$

$R_{meter} = 20 \parallel 20 = 10\,[\Omega]$

$R_{meter} + R_1 = 4.01\,[k\Omega]$ (+5)

$\Rightarrow i_R = \dfrac{24}{16.01k} = 1.499\,[A]$ +7

$\therefore V_R = 1.499 \times 4.0 = 5.996\,[V]$ (+2)

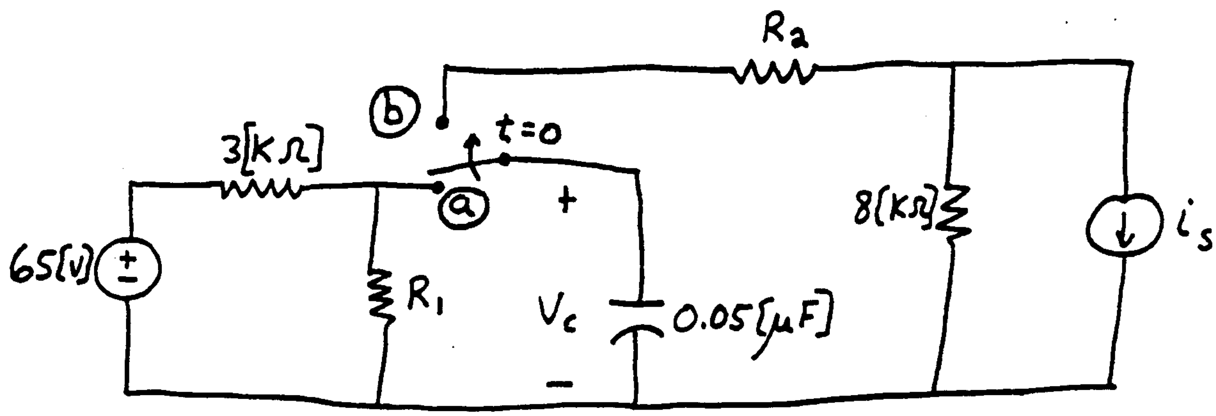
$\%\ error = \dfrac{0.004}{6} = 0.067\%$

So current measurement is more accurate in this case.

2. (12 Points) The switch has been in position ⓐ for a long time. Then, at $t = 0$, it is thrown from position ⓐ to position ⓑ. The voltage $v_C(t)$ is measured and found to be

$$v_C(t) = (18 + 17\exp(-t/0.66[\mu sec]))\ [V];\ \text{for } t \geq 0.$$

Find R_1, R_2, and i_S.



$$V_c(t) = 18 + 17 e^{-t/0.66\mu sec} \quad [V]$$

$$\Rightarrow \quad V_f = 18\ V \qquad V_i = 35\ V$$

$$(R_2 + 8k)C = 0.66 \times 10^{-6}$$

$$\Rightarrow R_2 = \frac{0.66 \times 10^{-6}}{C} - 8k$$

+4

$$\underline{\underline{R_2}} = 13.2 - 8000 = -\underline{\underline{17986.8\,[\Omega]}}$$

!! (OOPS)

(CREDIT GIVEN FOR EITHER SIGN)

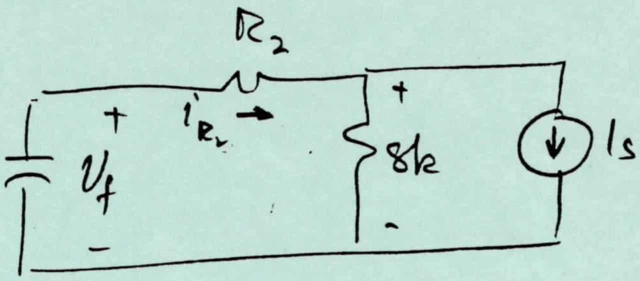
$$V_i = 35V \quad \Rightarrow \quad \frac{R_1 65}{R_1 + 3k} = 35V$$

+4

$$35R_1 + 105k = 65R_1$$

$$\therefore \quad \boxed{R_1 = \frac{105}{30} = 3.5[k\Omega]}$$

ROOM FOR EXTRA WORK



$V_f = 18V$

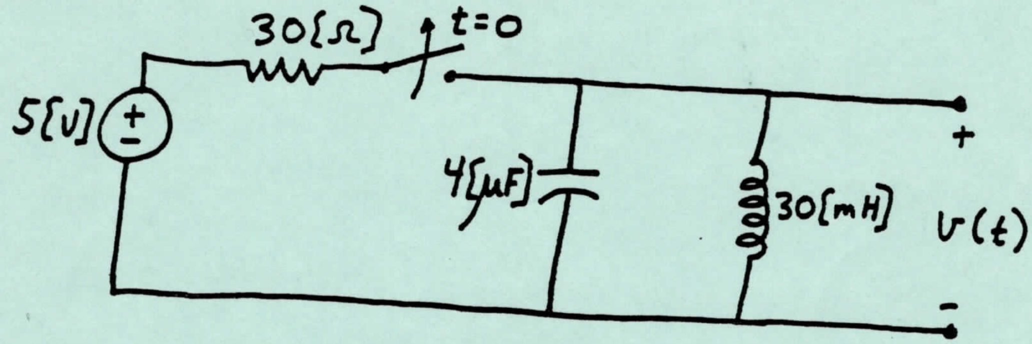
$i'_{R_2} = 0$

$\therefore \quad \boxed{i'_s = -\frac{18\,V}{8k} = -2.25\,mA}$

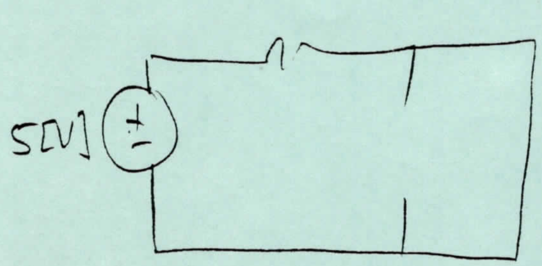
(+2)

+4

3. (12 Points) The switch in the circuit below had been closed for a long time, and then was opened at t = 0. Find v(t) at t = 1[s].



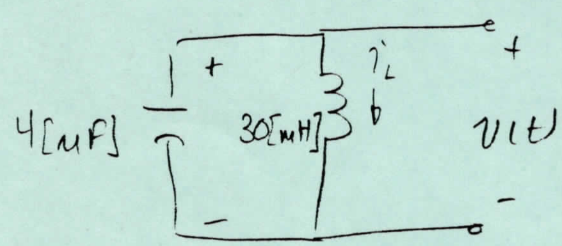
For $t<0$:



$$i_L(0) = \frac{5\,[V]}{30\,[\Omega]} = 167\,[mA]$$

$$v(0) = 0$$

For $0<t<1\,[s]$



Parallel RLC

w/ $R = \infty$

+ 1

+ 3

$$\therefore \quad \alpha = \frac{1}{2RC} = 0 \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}} = 2887\,[rad/sec]$$

General Solution is UNDERDAMPED response:

$$v(t) = B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t) \qquad + 3$$

$$\omega_d = \omega_0 = 2887\,[rad/sec] \quad ; \quad \alpha = 0$$

ROOM FOR EXTRA WORK

$v(0) = 0 = B_1$ + 1

$\frac{dv}{dt}(0) = \frac{-i_L}{C}(0) = \frac{-0.167}{4\times10^{-6}} = \omega_d B_2$ + 3

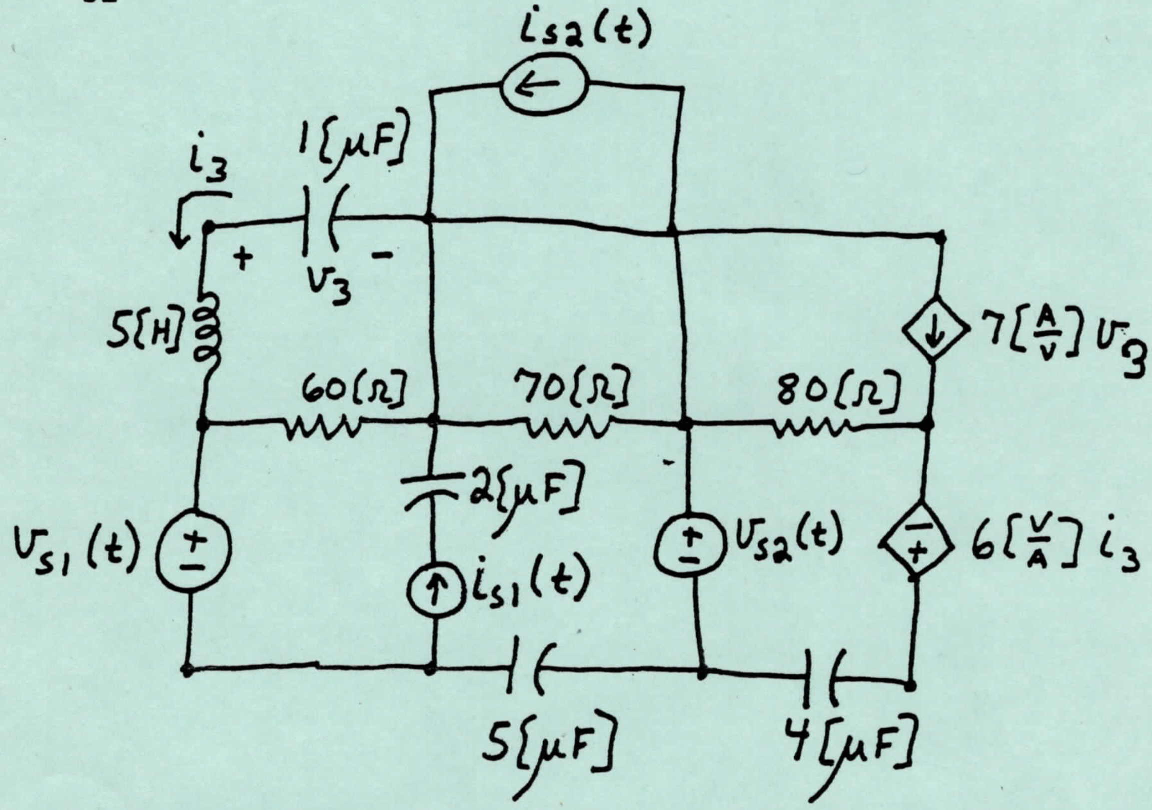
$\Rightarrow B_2 = -14.46$ SIGN +2

$\therefore\ v(t) = -14.46 \sin(2887\, t)$ [V]

$t = 1$ [s] $\Rightarrow$ $\boxed{v(1) = -1.783 \text{ [V]}}$ + 1

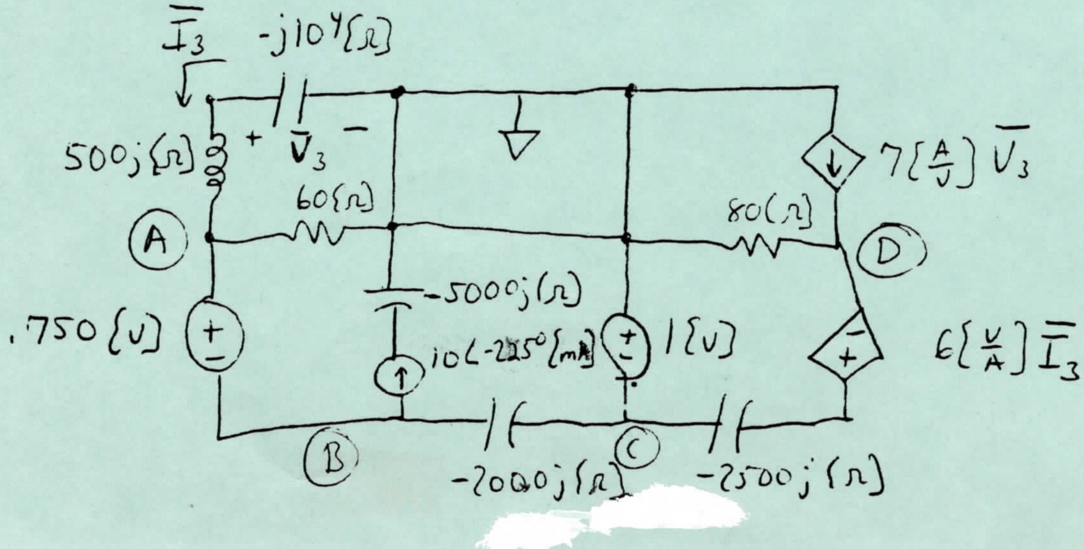
4. (15 Points) Write all of the complex node voltage equations that would be needed to solve for the steady state values of this circuit. Do not try to solve or simplify these equations. Make sure that some clear distinction is made between time domain and phasor quantities.

$v_{S1}(t) = (750\cos(100t))$ [mV]
$v_{S2}(t) = (1\sin(100t + 90°))$ [V] $= 1\cos(100t)$ [V]
$i_{S1}(t) = (10\sin(100t - 135°))$ [mA] $= 10\cos(100t - 225°)$ [mA]
$i_{S2}(t) = (50\cos(100t - 145°))$ [mA]



Solution.

First, redraw in complex phasor domain. Eliminate shorted elements. w = 100.



next page

ROOM FOR EXTRA WORK

$\boxed{\text{Equations}}$

A + B supernode

$$-\bar{I}_s + \frac{\bar{V}_A}{60\{\Omega\}} + \frac{\bar{V}_B - \bar{V}_c}{-2000j\{\Omega\}} + .010\angle -225°\{A\} = 0$$

A + B

$$\bar{V}_A - \bar{V}_B = .75\{V\}$$

C

$$\bar{V}_c = -1\{V\}$$

D

$$\frac{\bar{V}_D}{80\{\Omega\}} + \frac{\bar{V}_D + 6\{\frac{V}{A}\}\bar{I}_3}{-2500j\{\Omega\}} - 7\{\tfrac{A}{V}\}\bar{V}_3 = 0$$

$$\bar{V}_3 = \bar{V}_A\left(\frac{-j10^4}{500j - 10^4 j}\right)$$

$$\bar{I}_3 = -\frac{\bar{V}_A}{(500j - 10^4 j)\{\Omega\}}$$

points.

+5 transformations

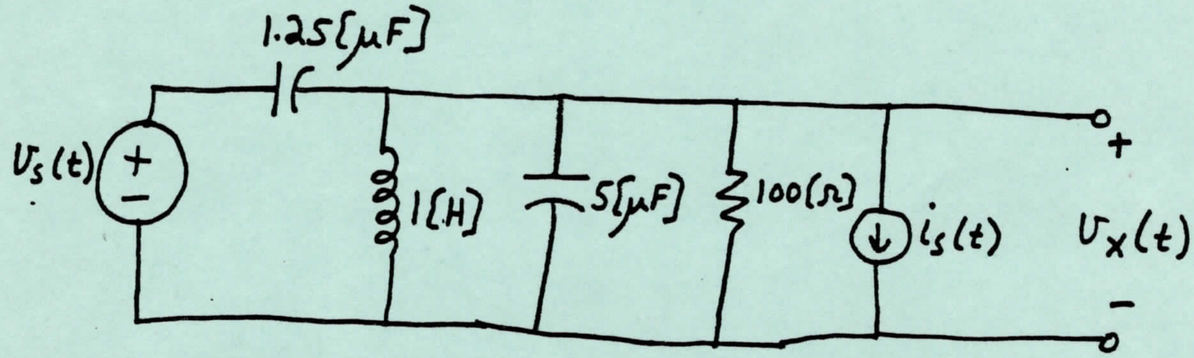
+10 equations

−4 for every wrong equation, up to 10 pts.

−5 mixed domains

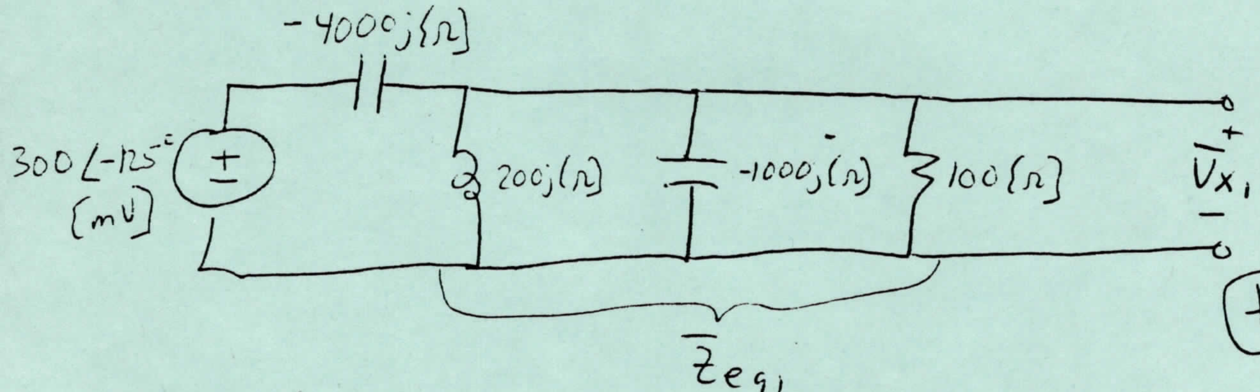
5. (18 Points) For the circuit below, find the steady state value of $v_X(t)$.

$v_S(t) = (300 \sin(200t - 35^\circ))$ [mV]
$i_S(t) = (5 \cos(400t + 30^\circ))$ [mA]



Solution | 2 different sources with different frequencies → Superposition (+4)

Take V_s first + transform. ⇒



(+2)

$$\bar{Z}_{eq_1} = \left(\frac{1}{200j} + \frac{1}{-1000j} + \frac{1}{100}\right)^{-1} = \left(0.01 - 4\times10^{-3}\,j\right)^{-1}$$

$$\bar{Z}_{eq_1} = \left(10.77\times10^{-3} \angle -21.8^\circ\right)^{-1} = 92.85 \angle +21.8^\circ\ [\Omega]$$ (+2)

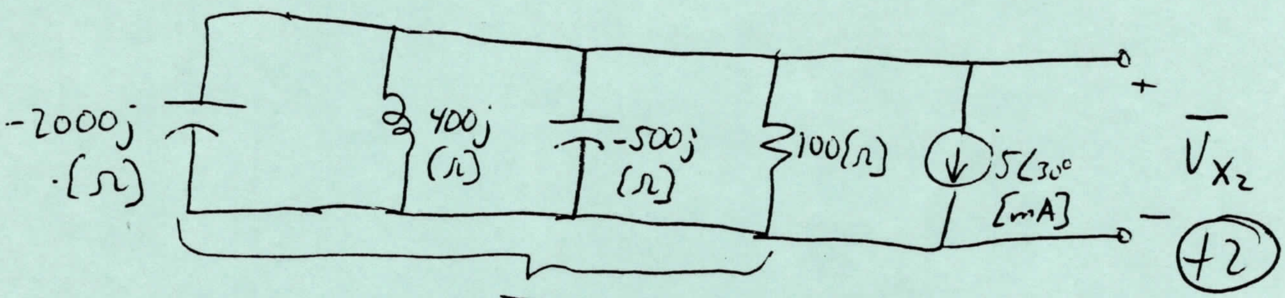
$$\bar{V}_{x_1} = \frac{.300 \angle -125^\circ \ (92.85 \angle +21.8^\circ)}{86.21 + 34.48j - 4000j} = \frac{27.87 \angle -103.2^\circ}{3{,}966 \angle -88.75^\circ}\ [V]$$

$$\bar{V}_{x_1} = 7.0 \angle -14.45^\circ\ [mV]$$

$$v_{x_1}(t) = 7.0 \cos(200t - 14.45^\circ)\ [mV]$$ (+2)

ROOM FOR EXTRA WORK

Then take is ⇒



+2

$$\overline{Z}_{eq_2} = \left(\frac{1}{100} + \frac{1}{-500j} + \frac{1}{400j} + \frac{1}{-2000j}\right)^{-1}$$

$$= (.01 + .002j - .0025j + .0005j)^{-1} = 100$$

+2

$$\overline{V}_{x_2} = -5\angle 30°(mA)\ 100\ [\Omega] = -500\angle 30°[mV]$$

$$v_{x_2}(t) = -500\cos(400t + 30°)\ [mV]$$

+2

$$\boxed{v(t) = 7.0\cos(200t - 14.45°) - 500\cos(400t + 30°)[mV]}$$

+2

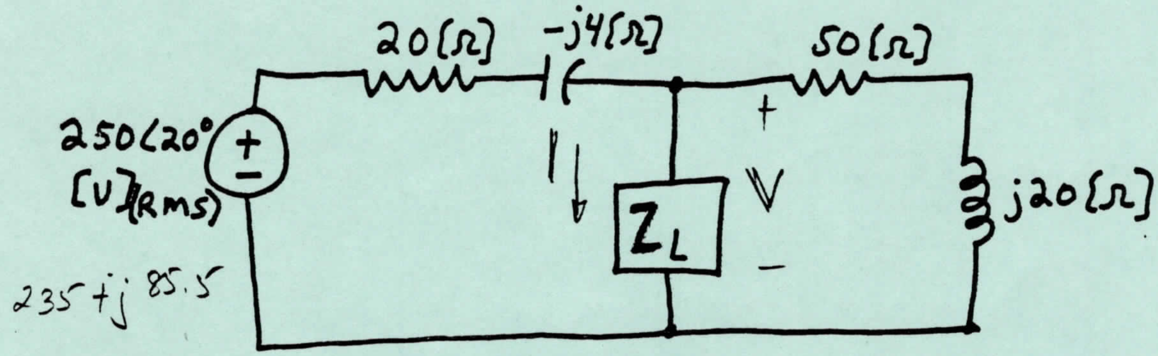
mixed domains −5

no units −2

adding in phasor domain −4

math error −1

6. (18 Points) a) If the load impedance $Z_L = 20 - j20$ [Ω], find the complex power delivered to the load. The angular frequency is 10 [krad/sec].



NODE VOLTAGE

$$\frac{V - 250\angle 20}{20 - j4} + \frac{V}{20 - j20} + \frac{V}{50 + j20} = 0$$

$$V(0.049\angle 11.3 + 0.035\angle 45 + 0.018\angle -21.8) = 12.25\angle 31.3$$

$$V = \frac{12.25\angle 31.3}{0.0924\angle -13.84} = 132.57\angle 45.14 \quad (93.5 + j93.4)$$

+4

$$\therefore I = \frac{132.57\angle 45.14}{28.28\angle -45} = 4.69\angle 90.14$$

+3

$$S = VI^{*} = (132.57)(4.69)\angle 45.14 - 90.14$$

$$S = 621.8\angle -45 \text{ [VA]}$$

$$\boxed{S = 439.6 + j\,{-439.6} \text{ [VA]} = P + jQ}$$

+6

6. (continued) b) What series circuit element (with what value) must be added to the load to reduce the reactive power in the new load to zero?

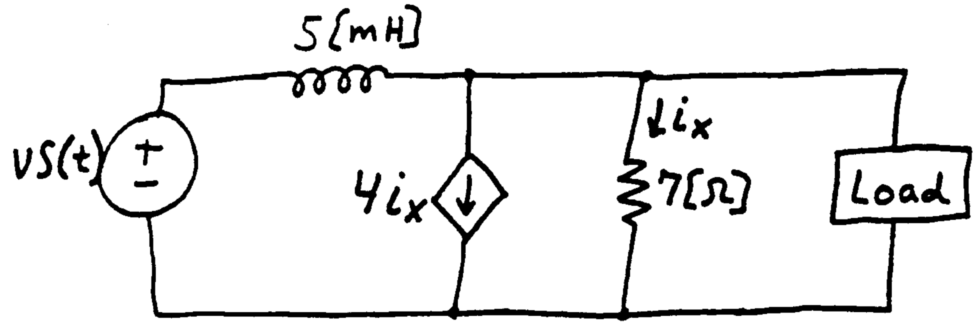
We need phase angle of 0

$\Rightarrow \quad Z \rightarrow Z + j20 = 20\,[\Omega]$ $\quad +4$

So new element is an inductor and

$\omega L = 20 \Rightarrow \boxed{L = 2\,[mH]}$ $\quad +1$

7. (10 Points) The source vS(t) is a sinusoidal source with a frequency of 100[Hz] and an amplitude of 30[V]. Find the value of impedance for the load that will maximize the power transfer to that load.



-4 wrong method for Z_{TH}

no units -2

mixed domains ~~-2~~

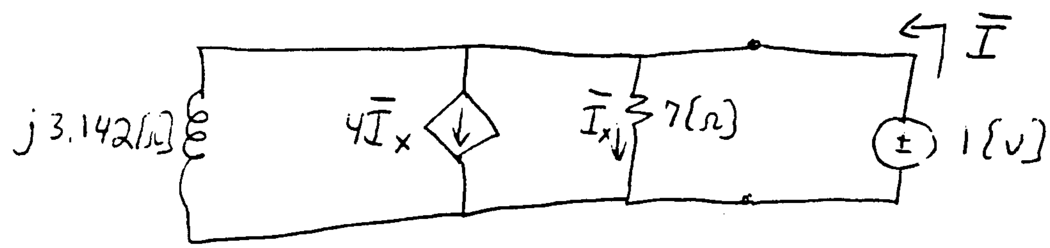
math error -1

Solution: We wish to obtain $\bar{Z}_{TH}$ as seen by the load, and then set

+2
$$\bar{Z}_{LOAD} = \bar{Z}_{TH}^{*}$$

Apply a phasor test source, w/ $\omega = 2\pi f = 628.3 \frac{rad}{sec}$

+2



+2
$$\bar{I} = \bar{I}_x + 4\bar{I}_x + \frac{1[V]}{j\,3.142[\Omega]}$$

$$\bar{I} = \left(5\left(\frac{1}{7}\right) + \frac{1}{j\,3.142}\right)[A]$$

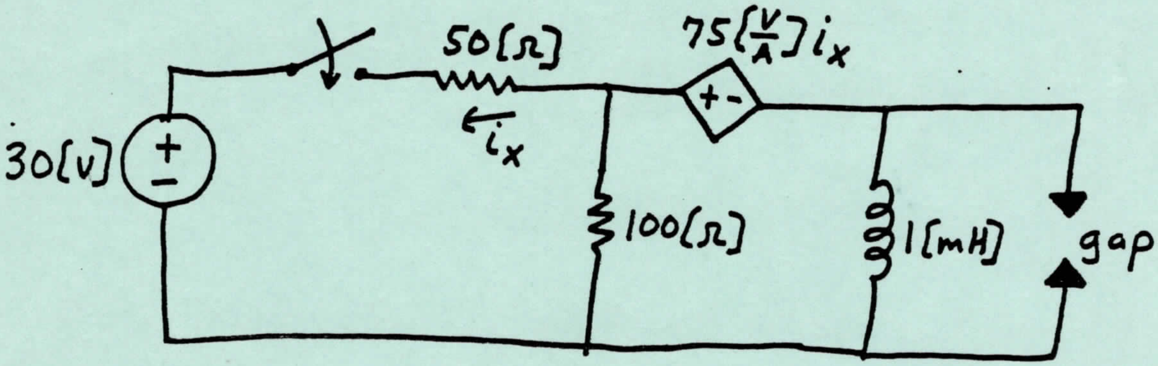
+2
$$\frac{\bar{V}}{\bar{I}} = \bar{Z}_{TH} = \frac{1[V]}{\left(\frac{5}{7} - j.318\right)[A]} = \left(\frac{1}{.782\angle -24.0^\circ}\right)[\Omega]$$

$$\bar{Z}_{TH} = 1.279\angle 24.0^\circ\ [\Omega]$$

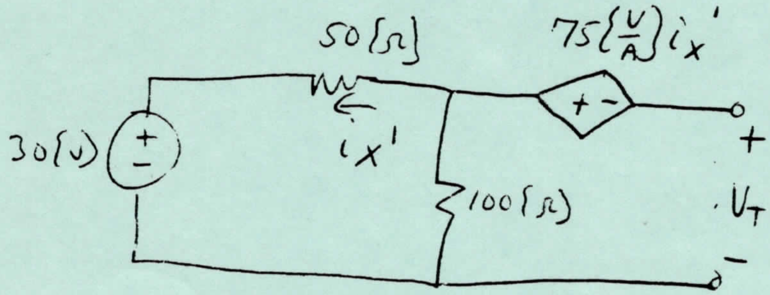
+2
$$\boxed{\bar{Z}_{LOAD} = 1.279\angle -24.0^\circ\ [\Omega] = (1.168 - 0.521j)[\Omega]}$$

OPTIONAL BONUS QUESTION (20 Points) YOU DO NOT NEED TO SOLVE THIS PROBLEM TO GET 100 ON THE EXAM.

The gap in the circuit below will arc over whenever the voltage across the gap reaches 1000[V]. The initial current in the inductor is zero. How long after the switch has been closed will the gap arc over?



Solution. We will start by finding the Thev. equiv. seen by the inductor, for $t>0$

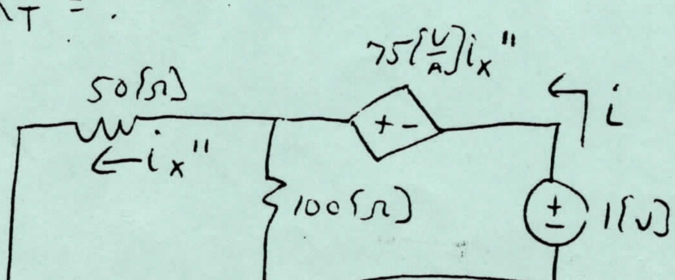


$$i_x' = -\frac{30[V]}{150[\Omega]}$$

$$i_x' = -0.2[A]$$

$$V_T = -i_x'(100[\Omega]) + (-75[\tfrac{V}{A}]\, i_x') = 20 + 15 = 35[V]$$

$R_T = ?$



$$i = i_x'' + \frac{i_x''}{2} = \frac{3}{2} i_x''$$

$$i_x'' = \frac{1 + 75 i_x''}{50}$$

$$i_x'' = 0.02 + 1.5 i_x''$$

$$-0.5 i_x'' = 0.02$$

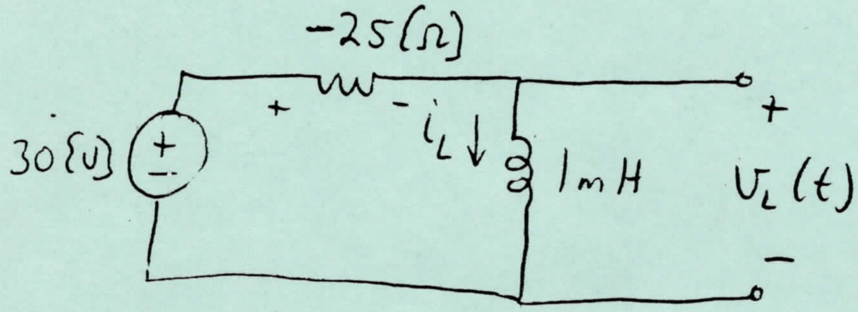
$$i_x'' = -0.04[A]$$

$$i = -0.04 + -0.02 = -.06$$

$$R_T = \frac{1[V]}{-0.06[A]} = -16.67[S]$$

ROOM FOR EXTRA WORK

So, we have



Now, for $V_L(t)$ we know

$$V_L(t) = 35\{V\}\, e^{-t/\tau} \quad ; \quad t > 0$$

where $\tau = \frac{L}{R} = \frac{10^{-3}}{-16.67} = -60 \times 10^{-6}\{s\}$

$$V_L(t) = 35\{V\}\, e^{16{,}667 t} \quad ; \quad t > 0$$

$$1000\{V\} = 35\{V\}\, e^{16{,}667 t}$$

$$\ln\left(\frac{1000}{35}\right) = 16{,}667 t$$

$$\boxed{t = 200\,\{\mu sec\}}$$

Limited partial credit.

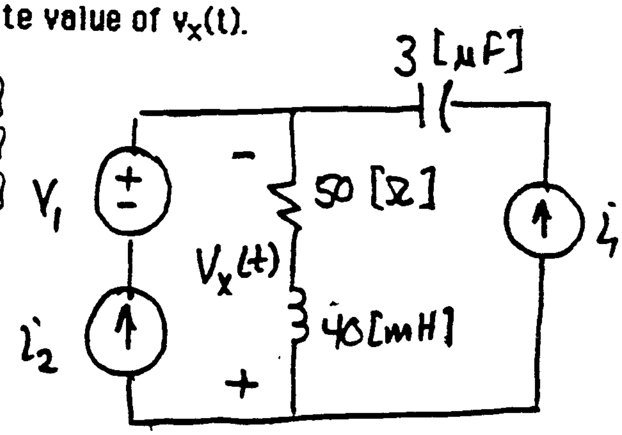
$i_L(0) = 0 \rightarrow$ +2 $\quad V_L(0^+) \rightarrow$ +5

nes $\tau \rightarrow$ +5

soln $\rightarrow$ +20

6. 14 points Find the steady-state value of $v_x(t)$.

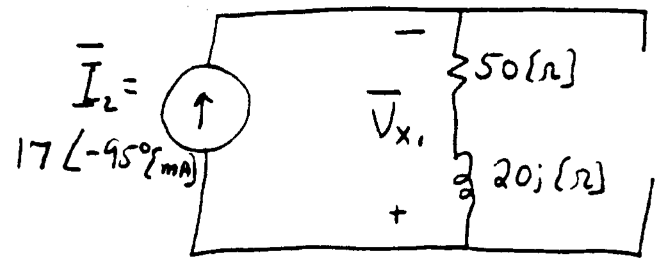
$v_1 = 90\cos(300t - 45^\circ)$ [V]
$i_1 = 6\sin(600t + 45^\circ)$ [mA]
$i_2 = 17\cos(500t - 95^\circ)$ [mA]



Soln

Neglect V_1 + capac., since they are in series w/ current sources.

Apply superposition: (+2) then take phasor transform

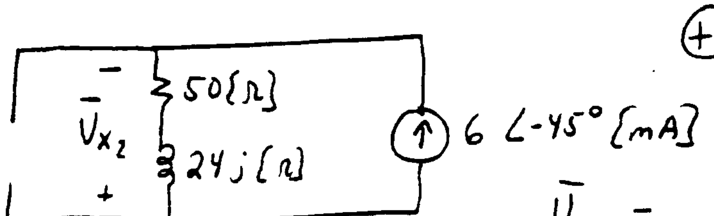


(+2) $\bar{V}_{x_1} = -(17\angle-95^\circ)[mA](50+j20)[\Omega]$

$\bar{V}_{x_1} = -(17\angle-95^\circ)[mA](53.9\angle21.8^\circ)$

$\bar{V}_{x_1} = -915\angle-73.2^\circ$ [mV]

(+2) $V_{x_1}(t) = -915\cos(500t - 73.2^\circ)$[mV]
$V_{x_1}(t) = 915\cos(500t + 107^\circ)$ [mV]



(+2) $\bar{V}_{x_2} = (-6\angle-45^\circ)(50+j24)$ [mV]

$\bar{V}_{x_2} = (6\angle135^\circ)(55.5\angle25.6^\circ)$ [mV]

$\bar{V}_{x_2} = 333\angle161^\circ$ [mV]

$V_{x_2}(t) = 333\cos(600t + 161^\circ)$ [mV] (+2)

$$\boxed{V_x(t) = \{915\cos(500t + 107^\circ) + 333\cos(600t + 161^\circ)\}\ [mV]}$$ (+4)

No superposition	-8
Incorrect Superposition-	
Adding in phasor domain	-6
Short/open errors	-6 → -3 2nd tim
mixed domains	-6
major ckt error	-4
Answer left in phasor domain	-4