Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Exam 2

November 2, 2024

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam.

1. _______________/40

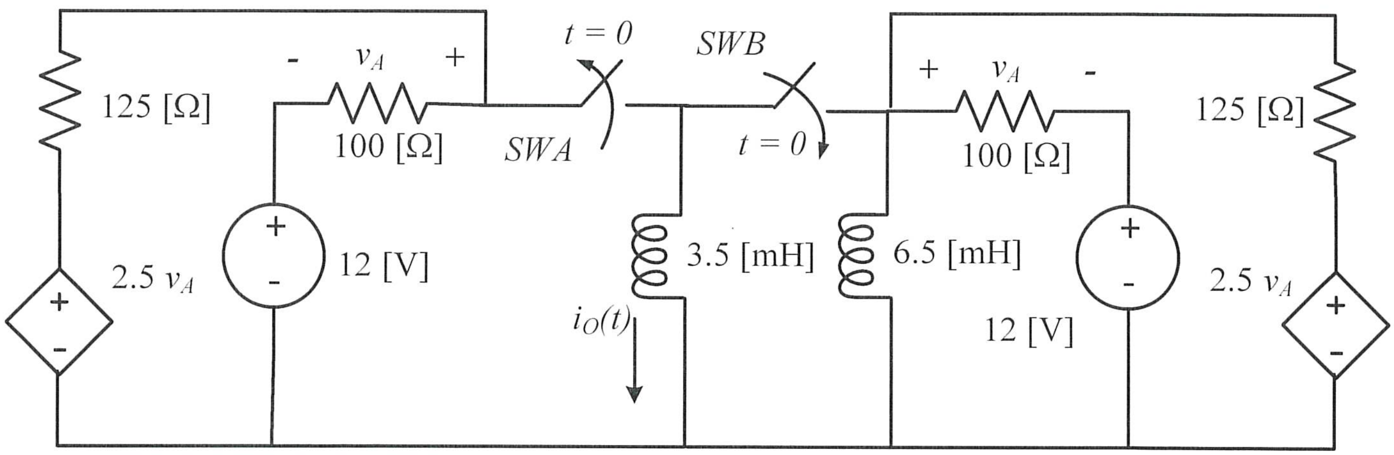
2. _______________/25

3. _______________/35

Total = 100

Room for extra work

1. {40 Points}In the circuit below, switch SWA was closed for a long time and switch SWB was open for a long time. Then, SWA opened at t = 0, and simultaneously, SWB closed. Find the current $i_O(t)$ for $t \geq 0$.

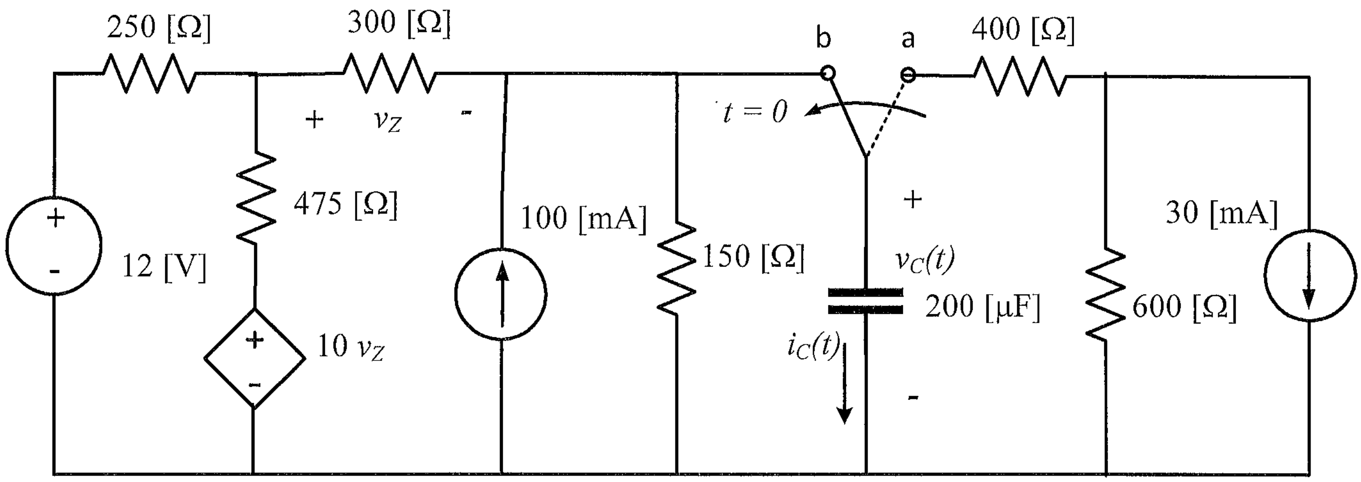


Room for extra work

2. {25 Points} In the circuit below, the switch was in position 'a' for a long time, and then moved to position 'b' at t = 0. The current $i_C(t)$ after the switch moved to 'b' is known to be as follows.

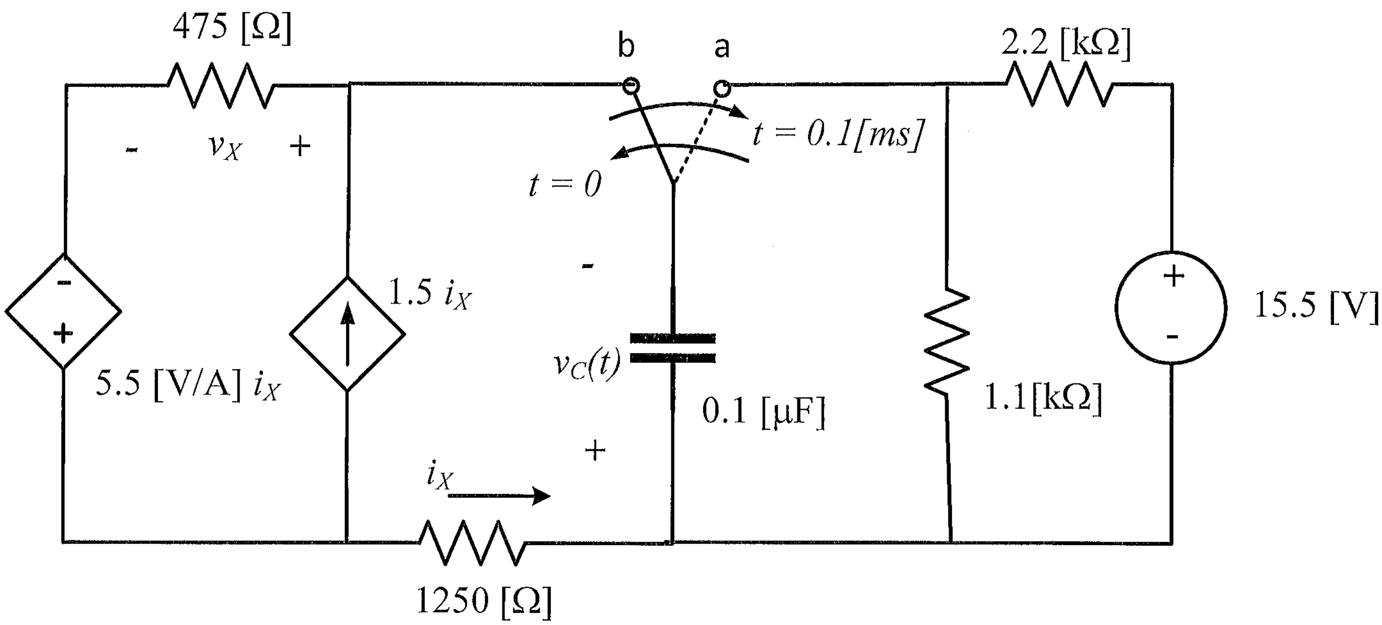
$$i_C(t) = C\frac{dv_C(t)}{dt} = 174.47[mA]e^{\frac{-t}{40.70[ms]}} \quad t > 0$$

Find the Thevenin equivalent of the circuit seen by the capacitor after the switch has moved to position 'b'. That circuit consists of the 150 [Ω] resistor, and everything to the left of it.



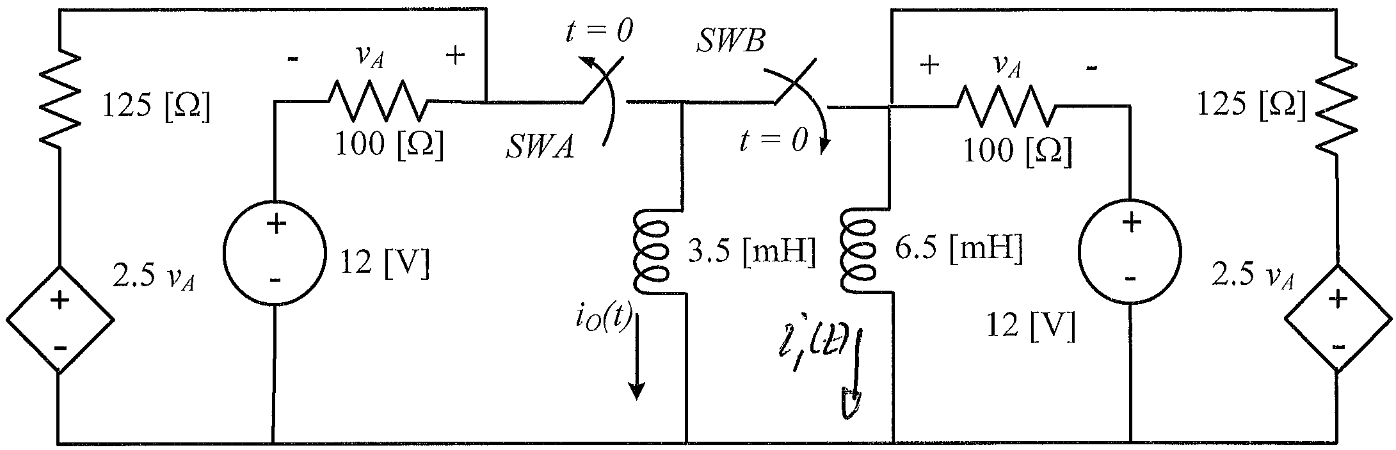
Room for extra work

3. {35 Points} The switch in the circuit below was at position 'a' for a long time, and moved to position 'b' at t = 0. At t = 0.1[ms] it moved back to position 'a'. Find $v_C(t)$ for $t \geq 0$.



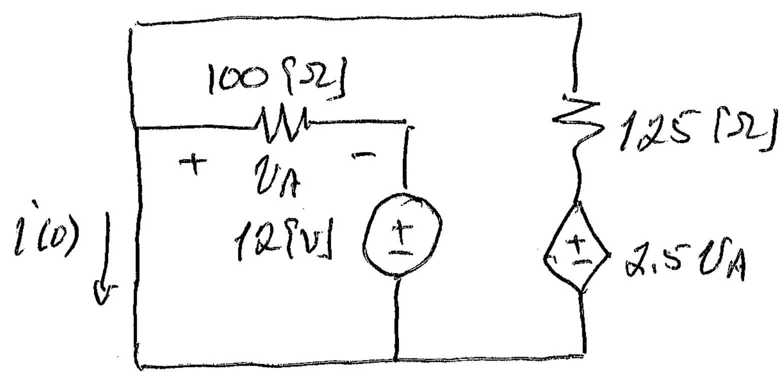
Room for extra work

1. {40 Points}In the circuit below, switch SWA was closed for a long time and switch SWB was open for a long time. Then, SWA opened at t = 0, and simultaneously, SWB closed. Find the current $i_O(t)$ for $t \geq 0$.



With SWA closed and SWB open, each inductor is connected to the same circuit, so the initial ($t<0$) current is the same in both. With $L \rightarrow$ short for steady state at $t<0$:

$t<0$



$$i(0) = \frac{12}{100} + \frac{2.5\, v_A}{125} \qquad i(0) = -120\ [mA]$$

$$v_A = -12\ [V]$$

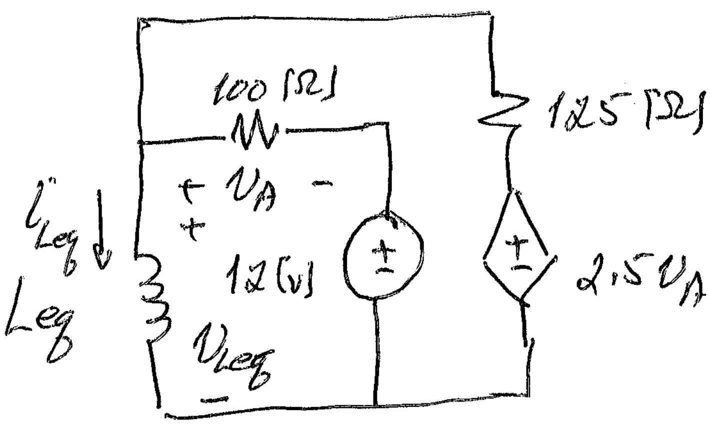
+5 $\quad \therefore\ i_o(0) = i_1(0) = -120\ [mA]$

Room for extra work

For $t>0$, the inductors are in parallel. We replace them with an equivalent inductor:

+2 $$L_{eq} = \left(\frac{1}{3.5[mH]} + \frac{1}{6.5[mH]}\right)^{-1} = 2.275\ [mH]$$

$t>0$

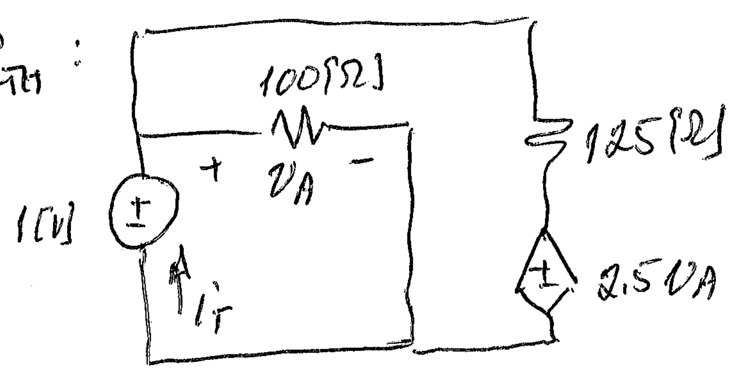


$$i_{Leq}(0) = i_0(0) + i_1(0) = -240\ [mA]$$

+3

+3 From the analysis previously, $i_{Leq,f} = -120\ [mA]$

R_{TH}:



$$i_T = \frac{1}{100} + \frac{1-2.5v_A}{125} = -2\ [mA]$$

$$R_{TH} = -500\ [\Omega]$$

+5 $v_A = 1\ [V]$

$$\tau_L = \frac{L_{eq}}{R_{TH}} = -4.55\ [\mu s]$$

+2

pg. 2

Room for extra work

$$i_{Leq}(t) = i_{Leq,f} + (i_{Leq}(0) - i_{Leq,f})e^{-t/\tau_L} \qquad t \geq 0$$

+5 $$i_{Leq}(t) = -0.120\,[A] + (-0.240 + 0.120)\,[A]\,e^{+t/4.55\,[\mu s]} \qquad t \geq 0$$

To find $i_0(t)$, we'll find $v_{Leq}(t)$ and integrate:

$$v_{Leq}(t) = L_{eq}\frac{d\,i_{Leq}(t)}{dt}$$

$$= (2.275\times 10^{-3})(-0.120)\left(\frac{1}{4.55\times 10^{-6}}\right)e^{t/4.55\,[\mu s]}$$

+5 $$= -60\,[V]\,e^{t/4.55\,[\mu s]} \qquad t > 0$$

+5 $$i_0(t) = \frac{1}{L_1}\int_0^t v_{Leq}(t)\,dt + i_0(0)$$

$$= \frac{1}{3.5\times 10^{-3}}\int_0^t (-60)e^{t/4.55\,[\mu s]}\,dt + (-0.120)$$

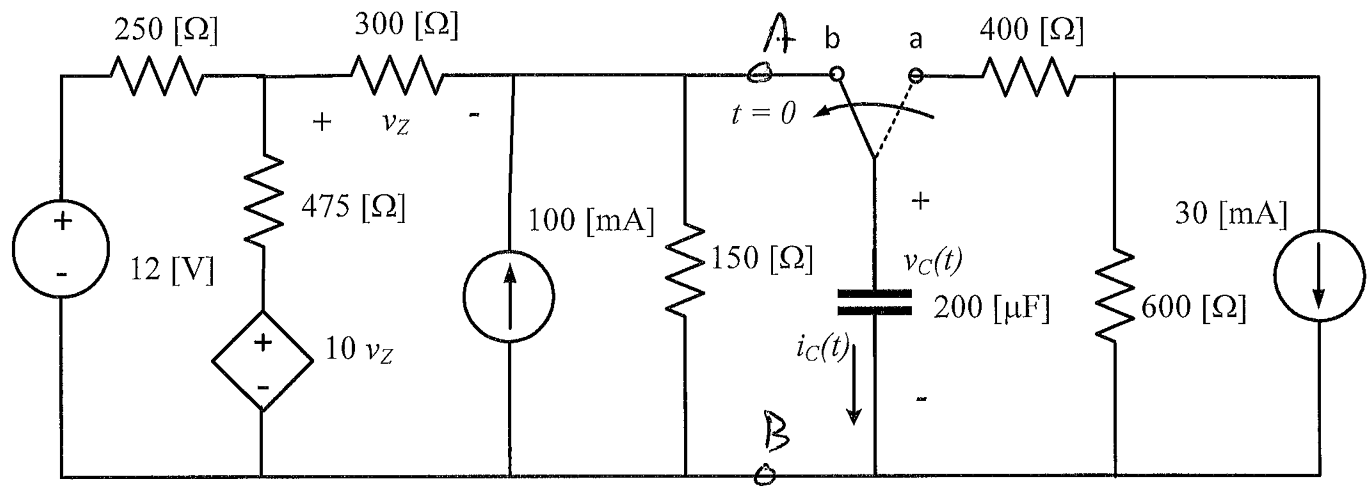
$$= -0.078\,[A]\left(e^{t/4.55\,[\mu s]} - 1\right) - 0.120\,[A]$$

+5 $$\boxed{i_0(t) = -0.078\,[A]\,e^{t/4.55\,[\mu s]} - 0.042\,[A]}$$

2. {25 Points} In the circuit below, the switch was in position 'a' for a long time, and then moved to position 'b' at t = 0. The current $i_C(t)$ after the switch moved to 'b' is known to be as follows.

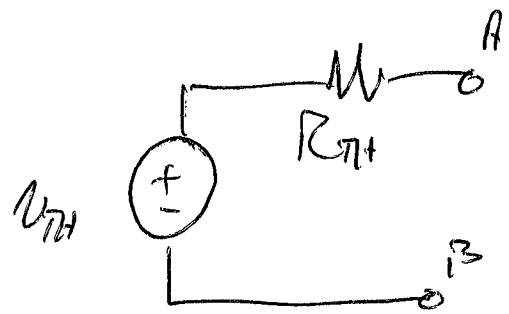
$$i_C(t) = C\frac{dv_C(t)}{dt} = 174.47[mA]e^{\frac{-t}{40.70[ms]}} \quad t > 0$$

Find the Thevenin equivalent of the circuit seen by the capacitor after the switch has moved to position 'b'. That circuit consists of the 150 [Ω] resistor, and everything to the left of it.



We could simply find V_{OC} and R_{TH} at A, B. But it's easier to note that $\tau_C = R_{TH} \cdot C$ and $V_{OC} = V_{C,f}$, and both of these are contained within $i_C(t)$.

+5



Room for extra work

$$v_c(t) = v_{C,f} + (v_c(0) - v_{C,f})\, e^{-t/\tau_c}$$

$$i_c(t) = C\,\frac{dv_c(t)}{dt}$$

$$= -C\,(v_c(0) - v_{C,f})\,\frac{1}{R_{TH}\cdot C}\, e^{-t/\tau_c}$$

$$= -\frac{(v_c(0) - v_{C,f})}{R_{TH}}\, e^{-t/\tau_c}$$

What we do not have yet is $v_c(0)$:

+2 $t<0 \Rightarrow v_c(0) = (-0.03)(600) = -18\,[V]$

From $\tau_c = R_{TH} C = 40.7\,[ms]$ and $C = 200\,[\mu F]$,

+6 we have $\boxed{R_{TH} = 203.5\,[\Omega]}$

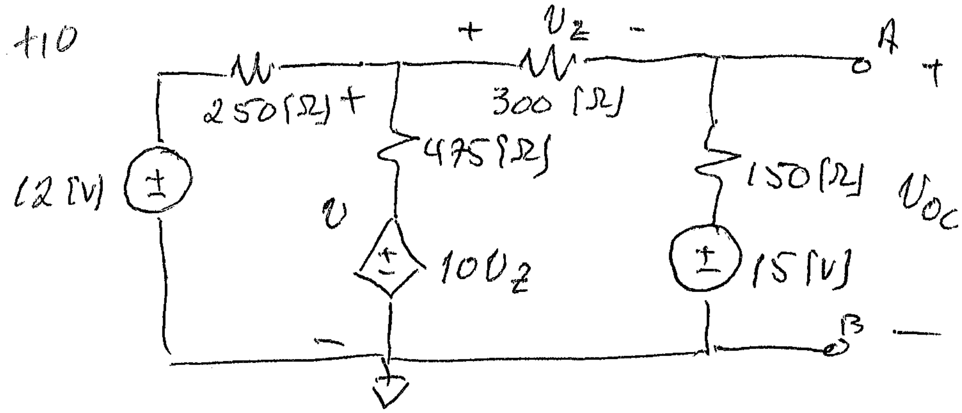
Then $-0.17447 = -\dfrac{(-18 - v_{C,f})}{203.5}$

+12 $\Rightarrow \boxed{v_{C,f} = 17.5\,[V] = v_{TH}.}$

On the next page we find the Thevenin Equivalent without using the information contained in $i_c(t)$.

Room for extra work

After a source transformation, we have...

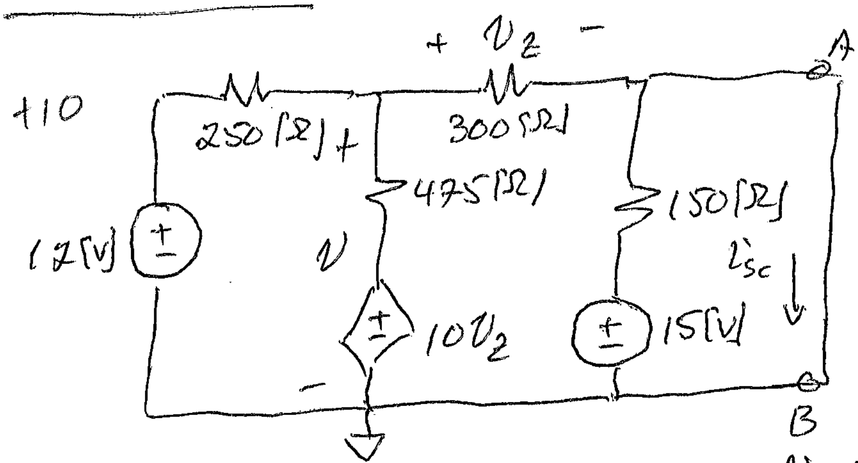


$$\frac{v - v_{oc}}{300} + \frac{v-12}{250} + \frac{v-10v_z}{475} = 0$$

$$\frac{v_{oc} - v}{300} + \frac{v_{oc} - 15}{150} = 0$$

$$v_z + v_{oc} - v = 0$$

$v_z = 5.0916$ [V] $\quad v = 22.635$ [V] $\quad \boxed{v_{oc} = 17.545 \text{ [V]}}$

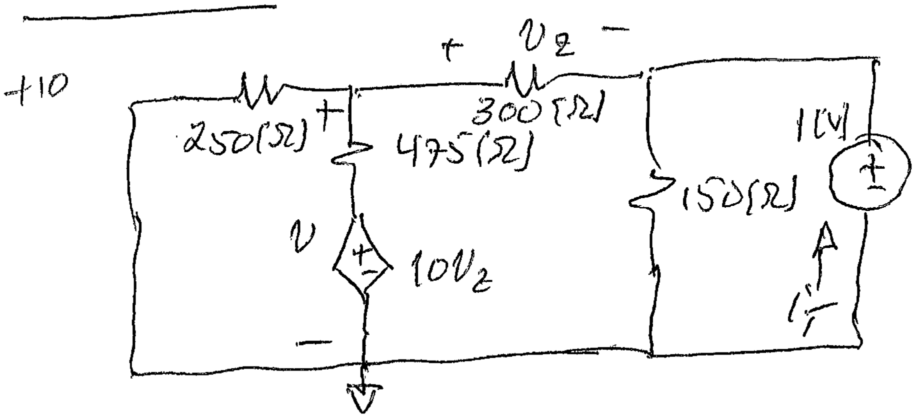


$$\frac{v-12}{250} + \frac{v}{300} + \frac{v - 10v_z}{475} = 0$$

$$v_z = v$$

$v = -4.1329$ [V] $= v_z$ $\quad i_{sc} = v_z/300 + 15/150 = 0.08622$ [A]

$\boxed{R_{TH} = v_{oc}/i_{sc} = 203.5 \text{ [Ω]}}$



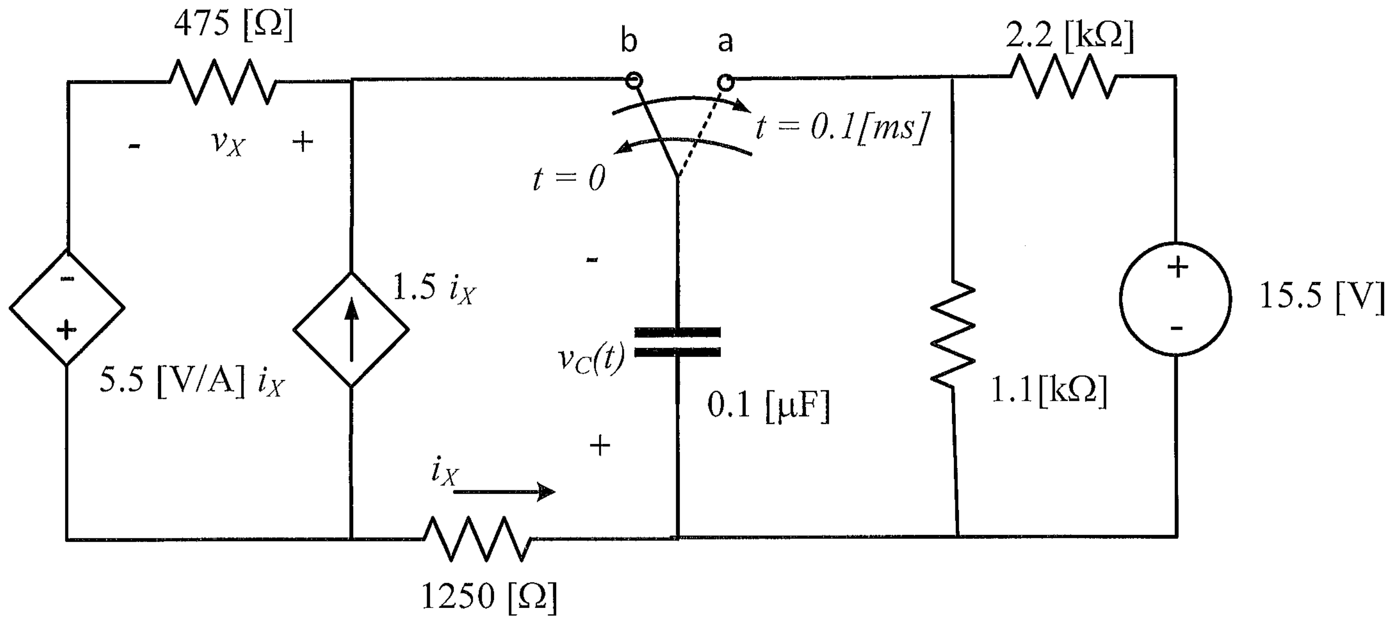
$$\frac{v - 10v_z}{475} + \frac{v}{250} + \frac{v-1}{300} = 0$$

$$v_z + 1 - v = 0$$

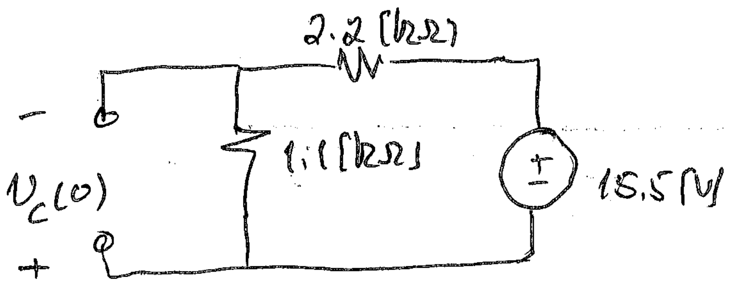
$$i_T = \frac{1}{150} - \frac{v_z}{300} = 4.914 \text{ [mA]}$$

$R_{TH} = 203.4$ [Ω] ✓

3. {35 Points} The switch in the circuit below was at position ‘a’ for a long time, and moved to position ‘b’ at t = 0. At t = 0.1[ms] it moved back to position ‘a’. Find $v_C(t)$ for $t \geq 0$.



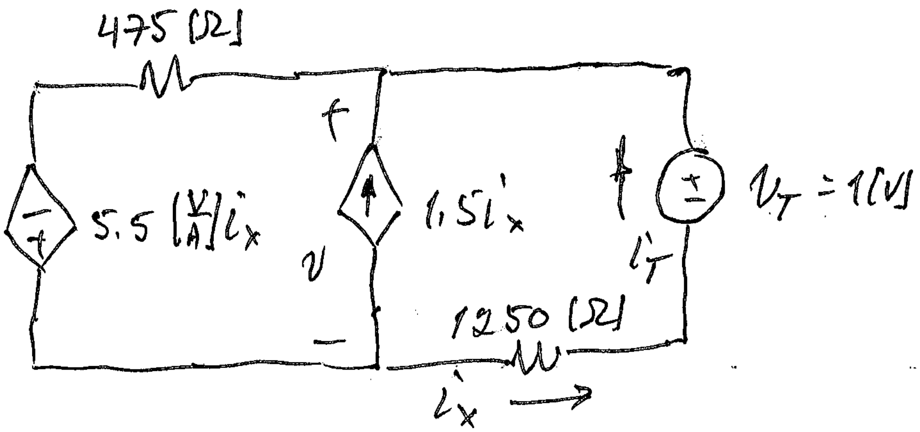
$t<0 \Rightarrow C \rightarrow$ open circuit



$$V_c(0) = -15.5 \frac{1.1}{1.1+2.2}$$

$$V_c(0) = -5.1667 \text{ [V]} \quad +3$$

$0<t<0.1$ [mS]



Room for extra work

$$\left.\begin{aligned} &\frac{v+5.5i_x}{475} - 1.5i_x + \frac{v-1}{1250} = 0 \\ &i_x = i_T = -\frac{v-1}{1250} \end{aligned}\right\} \quad \begin{aligned} &v = 0.4860\ [V] \\ &i_T = i_x = 0.4111\ [mA] \end{aligned}$$

+6 $R_{TH} = 2432\ [\Omega]$ $\tau_c = R_{TH}C = 0.2433\ [ms]$

+1

+3 $v_{c,f} = 0$ (no independent sources)

+5 $\therefore \boxed{v_c(t) = -5.1667\, e^{-t/0.2433\,[ms]}}$ $0 \le t \le 0.1\ [ms]$

(+2)

$t > 0.1\,[ms]$

For this time region, the initial voltage is

+5 $$v_c(0.1[ms]) = -5.1667\, e^{-0.1/0.2433} = -3.4254\ [V]$$

We have switched to the original configuration, so

+2 $$v_{c,f} = -5.1667\ [V]$$

+3 $R_{TH} = 1.1[k\Omega] \,//\, 2.2[k\Omega] = 733.3\ [\Omega] \Rightarrow \tau = 73.33\ (\mu s)$

+1

+6 $$\boxed{v_c(t) = -5.1667\ [V] + (-3.4254 + 5.1667)\ [V]\, e^{-\frac{(t-0.1\,[ms])}{0.07333\,[ms]}}}$$ $t \ge 0.1\ [ms]$