Signature: Solution Key

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

EXAM 3
ELEE 2335
~~November 22, 1986~~
April 25, 1987

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

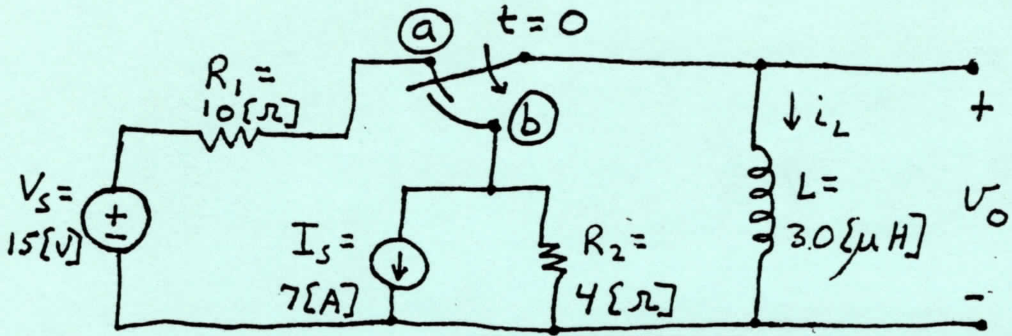
3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 25
2. 40
3. 10
4. 25

100

1. (25 Points) Assume that the circuit below has had the switch in position ⓐ for a long time. The switch is moved instantaneously to position ⓑ at $t = 0$. Find $v_0(t)$ for $t > 0$.

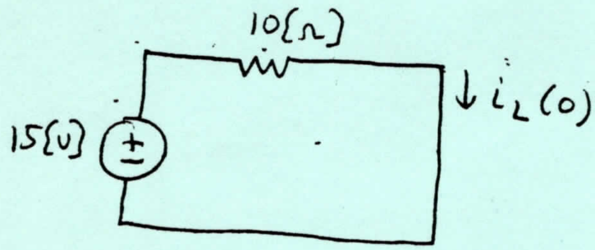


Solution: First find $i_L(t)$ for $t \geq 0$, the differentiate

$$i_L(t) = i_{L_{final}} + \left(i_L(0) - i_{L_{final}}\right) e^{-t/\tau} \quad \text{for } t \geq 0$$ +4

$i_L(0) = ?$

for $t < 0$

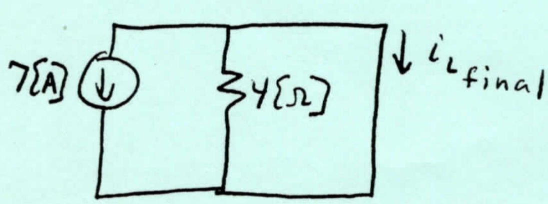


$$i_L(0) = \frac{15[V]}{10[\Omega]}$$

$$i_L(0) = 1.5[A]$$ +4

$i_{L_{final}} = ?$

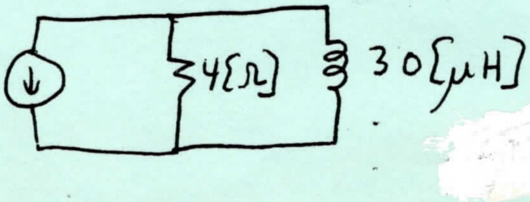
for $t = \infty$



$$i_{L\,final} = -7[A]$$ +4

$\tau = ?$

for $t > 0$



$$\tau = \frac{L}{R} = \frac{30 \times 10^{-6}}{4}[s]$$

$$= 7.5 \times 10^{-6}[s]$$ +4

See page 4

Room for Extra Work

1 con't

So

$$i_L(t) = -7[A] + 8.5[A]\, e^{-t/(7.5\times10^{-6})[s]}$$

$$v(t) = L\frac{di_L}{dt}$$

$$v(t) = -\frac{8.5[A]\; 30\times10^{-6}[H]}{7.5\times10^{-6}[s]}\, e^{-t/7.5\times10^{-6}[s]}$$ +4

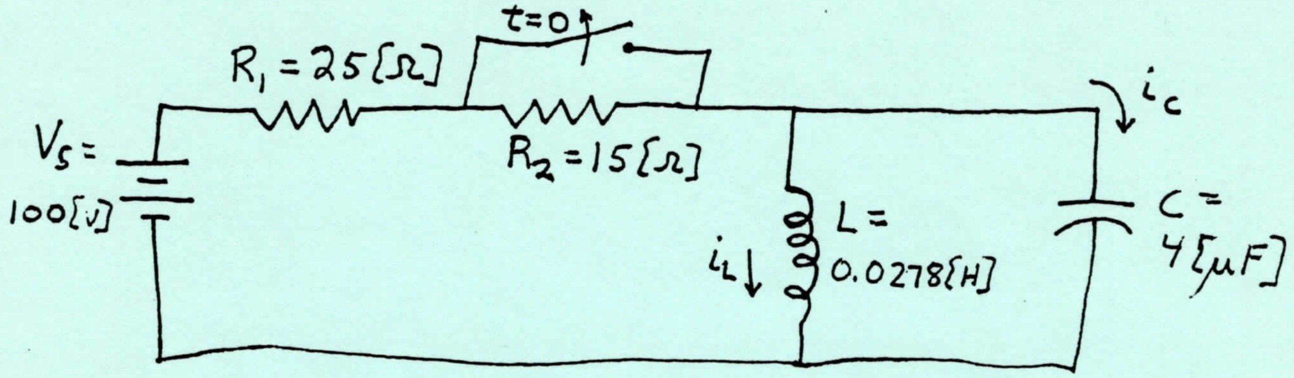
$$\boxed{\begin{array}{l} v(t) = -34[v]\, e^{-t/7.5\times10^{-6}[s]}\ ;\ t>0 \\ \text{or} \\ v(t) = -34[v]\, e^{-133{,}333t}\ ;\ t>0 \quad t \text{ in } [sec] \end{array}}$$ +5

-2 math error

-4 no units in soln.

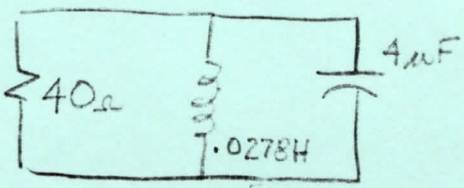
-4 each wrong method

2. (40 Points) The switch in the circuit below has been closed for a long time. At t = 0 the switch is opened.



a) Determine the expression for $i_L(t)$ for $t \geq 0$.

AFTER SWITCH IS OPEN, THE EQUIVALENT CKT TO DETERMINE R_{eq} IS



(PARALLEL COMBINATION)

$$\therefore \; S = -\frac{1}{2RC} \pm \left[\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}\right]^{1/2} = -3.125\times10^3 \pm (772.8\times10^3)^{1/2}$$

$$S = -3.125\times10^3 \pm 879.1 = -4{,}0004 \;;\; -2{,}245.9$$

$$\left.\begin{aligned} S_1 &= -4\times10^3 \\ S_2 &= -2.246\times10^3 \end{aligned}\right\} \text{ OVERDAMPED}$$

NOW IN GENERAL

$$i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$$

ALSO $i_L(0^-) = \dfrac{V_s}{R_1} = \dfrac{100}{25} = 4\,A$

$$i_L(t \to \infty) = \frac{V_s}{R_1+R_2} = \frac{100}{40} = 2.5\,A$$

(NOTE S_1 & S_2 ARE < 0)

$$\therefore \quad 4 = A_1 + A_2 + A_3 \quad ; \quad 2.5 = A_3$$

$$A_1 + A_2 = 4 - 2.5 = 1.5$$

(OVER)

Room for Extra Work

TO DETERMINE ADDITIONAL BOUNDARY CONDITIONS WE NOTE

$v_L(t) = v_C(t)$ AND $v_C(0^-) \equiv 0$

$\therefore \; v_C(t) = L\dfrac{di_L(t)}{dt} = L\left[s_1 A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t} \right]$ VOLTS

$v_C(0^-) = 0 = s_1 A_1 + s_2 A_2$

NOW, $A_1 + A_2 = 1.5 \longrightarrow s_1 A_1 + s_1 A_2 = 1.5 s_1$

$s_1 A_1 + s_2 A_2 = 0 \longrightarrow -s_1 A_1 - s_2 A_2 = 0$

$s_1 A_2 - s_2 A_2 = 1.5 s_1$

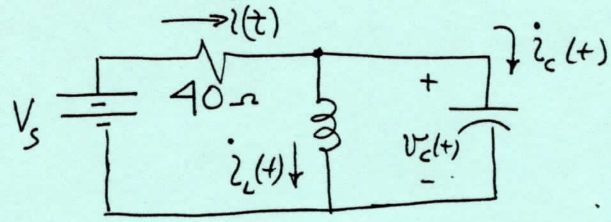
$\therefore \; A_2 = \dfrac{1.5 s_1}{s_1 - s_2} = \dfrac{-1.5 \times 4 \times 10^3}{-4 \times 10^3 + 2.246 \times 10^3} = \underline{\underline{3.42}}$

$A_1 = 1.5 - A_2 = 1.5 - 3.42 = \underline{\underline{-1.92}}$

$$\therefore \; \boxed{i_L(t) = \left(-1.92 e^{-4\times 10^3 t} + 3.42 e^{-2.246\times 10^3 t} + 2.5\right)[A]}$$

2. (continued) b) What is the value of i_C at the instant just after the switch is opened? In other words, find $i_C(0^+)$.

NOW IN GENERAL AT $t = 0^+$



$$i(0^+) - i_L(0^+) - i_c(0^+) = 0$$

$$i_L(0^+) = i_L(0^-) = \frac{V_s}{25} = \frac{100}{25} = 4A$$

$$i(0^+) = \frac{V_s}{40} = 2.5A \quad (\text{SINCE } V_c(0^-) = V_c(0^+) = 0)$$

$$\boxed{\therefore\ i_c(0^+) = 2.5 - 4 = -1.5[A]}$$

ALTERNATE

$$i_c(t) = C\frac{dv_c(t)}{dt} = CL\left[s_1^2 A_1 e^{s_1 t} + s_2^2 A_2 e^{s_2 t}\right] \text{ AMPS}$$

$$i_c(0^+) = CL\left[s_1^2 A_1 + s_2^2 A_2\right] = CL\left[(-4\times10^3)^2(-1.92) + (-2.246\times10^3)^2(3.42)\right]$$

$$= \underline{\underline{-1.4976[A]}}$$ CHECK

3. (10 Points) A 2 [kHz] sinuosoidal voltage has zero phase angle and a maximum amplitude of 8 [V]. When this voltage is applied across the terminals of the capacitor, the resulting steady-state current has a maximum amplitude of 1 [mA].

3

a) What is the frequency of the current in radians per second?

same as voltage which is:

$$2\pi f = 2\pi \times 2\times10^{3} = 12,566\,[rad/sec]$$

3

b) What is the capacitance of the capacitor in microfarads?

$$X_c = \frac{|V_c|}{|I_c|} = \frac{8}{1\times10^{-3}} = 8\times10^{3}$$

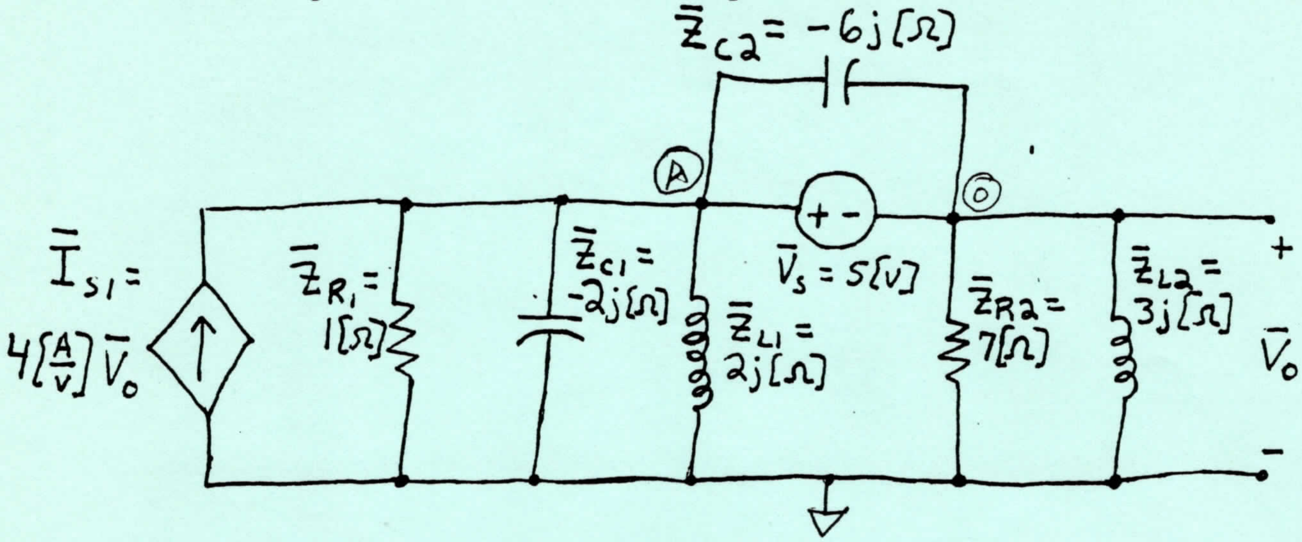
$$X_c = \frac{1}{\omega C}, \quad C = \frac{1}{X_c\omega} = \frac{1}{8\times10^{3}\times12.566\times10^{3}} = .009947\,\mu F$$

1

c) What is the impedance of the capacitor?

$$Z_c = -jX_c = -j\,8\times10^{3}\,\Omega = 8\times10^{3}\angle{-90^\circ}\,\Omega$$

4. (25 Points) This circuit has been drawn and labelled in the phasor domain. Use the node voltage method to solve for the phasor $\bar{V}_0$. Write the node voltage equations before starting to solve.



3 essen. nodes − 2 equs needed

Supernode Ⓐ + Ⓞ

$$-4\left[\tfrac{A}{V}\right]\bar{V}_o + \frac{\bar{V}_A}{1[\Omega]} + \underbrace{\frac{\bar{V}_A}{-2j[\Omega]} + \frac{\bar{V}_A}{2j[\Omega]}}_{\text{cancel}} + \underbrace{\frac{\bar{V}_A - \bar{V}_o}{-6j} + \frac{\bar{V}_o - \bar{V}_A}{-6j}}_{\text{Need not be included. Can include in supernode.}} +$$

$$+ \frac{\bar{V}_o}{7[\Omega]} + \frac{\bar{V}_o}{3j[\Omega]} = 0$$

+7

Supernode equ.

$$\bar{V}_A - \bar{V}_o = 5[V]$$

+5

Solving → $\bar{V}_A = 5[V] + \bar{V}_o$

$$-4\left[\tfrac{A}{V}\right]\bar{V}_o + \frac{5[V] + \bar{V}_o}{1[\Omega]} + \frac{\bar{V}_o}{7[\Omega]} + \frac{\bar{V}_o}{3j[\Omega]} = 0$$

+4

$$\left(-4 + 1 + \tfrac{1}{7} + \tfrac{1}{3j}\right)[\Omega^{-1}]\,\bar{V}_o = -5[A]$$

(See next page)

ROOM FOR EXTRA WORK

$$\bar{V}_o = \frac{-5[A]}{\left(-\frac{20}{7} - \frac{1}{3}j\right)[\Omega^{-1}]} = \frac{-5[A]}{2.877\angle 173^\circ [\Omega^{-1}]}$$

+4

$$\boxed{\begin{aligned} \bar{V}_o &= \frac{5\angle +180^\circ}{2.877\angle 173^\circ} = 1.738\angle 7^\circ [V] \\ &\text{- or -} \\ \bar{V}_o &= 1.725 - .212j\ [V] \end{aligned}}$$

+5

-4 no units

-2 math error

-4 major conceptual error

-2 not showing phasors

-6 not ~~eq~~ enough equs.