Signature: Solution Key

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

FINAL EXAM
ELEE 2335
May 8, 1987

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

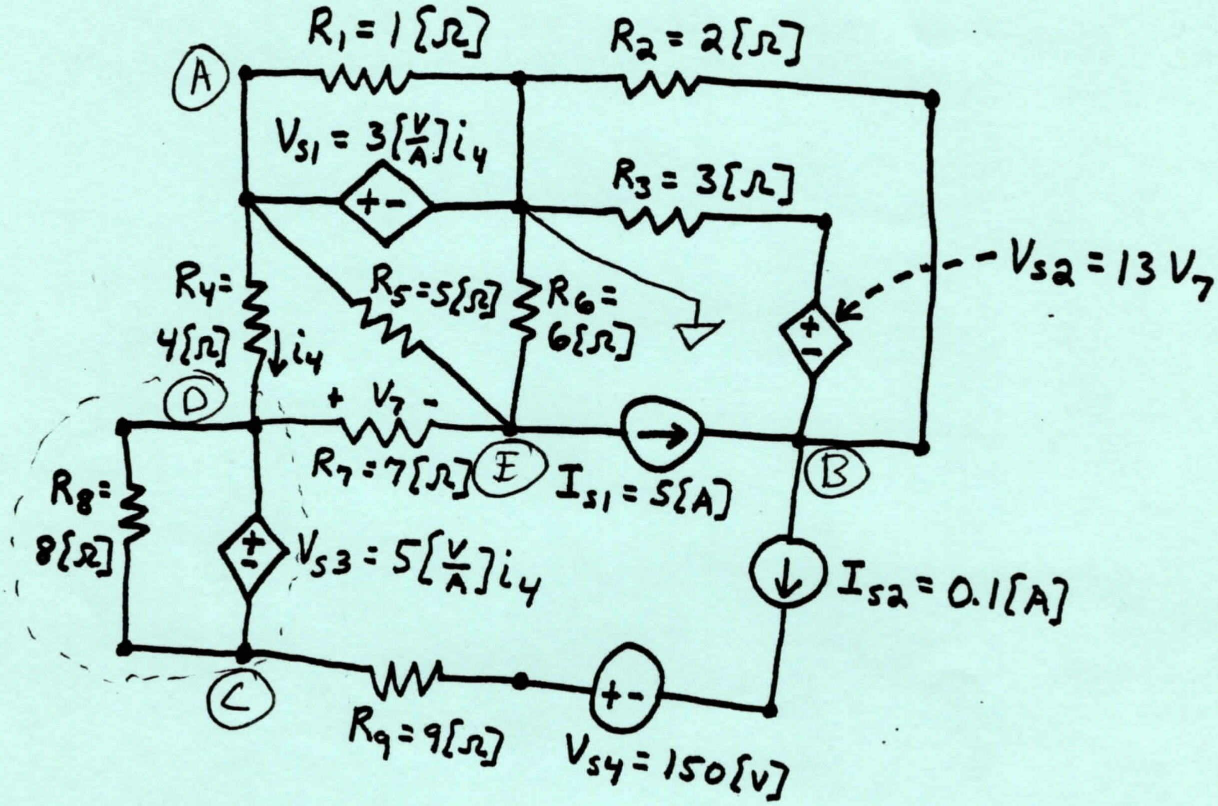
2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. Do not use red ink.

1. 25
2. 20
3. 20
4. 25
5. 30
6. 30
7. 25
8. 25

1. (25 Points) Write the node voltage equations that could be used to solve this circuit. Do not solve the equations. Do not simplify the circuit. No partial credit will be given on this problem.



Sample Soln:

(A) $$V_A = V_{s1} = 3\left[\tfrac{V}{A}\right] i_4$$

(B) $$\frac{V_B}{2[\Omega]} + 0.1[A] - 5[A] + \frac{V_B + 13V_7}{3[\Omega]} = 0$$

(C)+(D) Supernode

$$-0.1[A] + \frac{V_D - V_E}{7[\Omega]} + \frac{V_D - V_A}{4[\Omega]} = 0$$

$$V_D - V_C = 5\left[\tfrac{V}{A}\right] i_4$$

(E) $$5[A] + \frac{V_E}{6[\Omega]} + \frac{V_E - V_A}{5[\Omega]} + \frac{V_E - V_D}{7[\Omega]} = 0$$

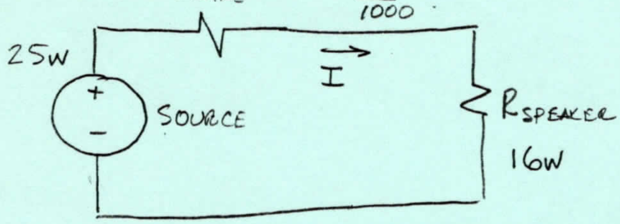
$$i_4 = \frac{V_A - V_D}{4[\Omega]} \qquad V_7 = V_D - V_E$$

No partial credit given.

2. (20 Points) A speaker is connected to an audio amplifier by 40 [ft] of 24-guage wire which is known to have a resistance of 25[Ω]/1000[ft]. As a result of the use of such small wire for the connection, the amplifier which is rated at 25 [Watts] delivers only 16 [Watts] to the speaker.

a) What is the value of the speaker resistance?

$R_{WIRE} = 40 \times \frac{25}{1000} = 1\,\Omega$



Power loss IN wire $= 25 - 16 = 9\,W = I_{rms}^2 \times 1$

$\therefore\ I_{rms} = \sqrt{9} = 3$ AMPS

$P_{SPEAKER} = I_{rms}^2\, R_{SPEAKER}$

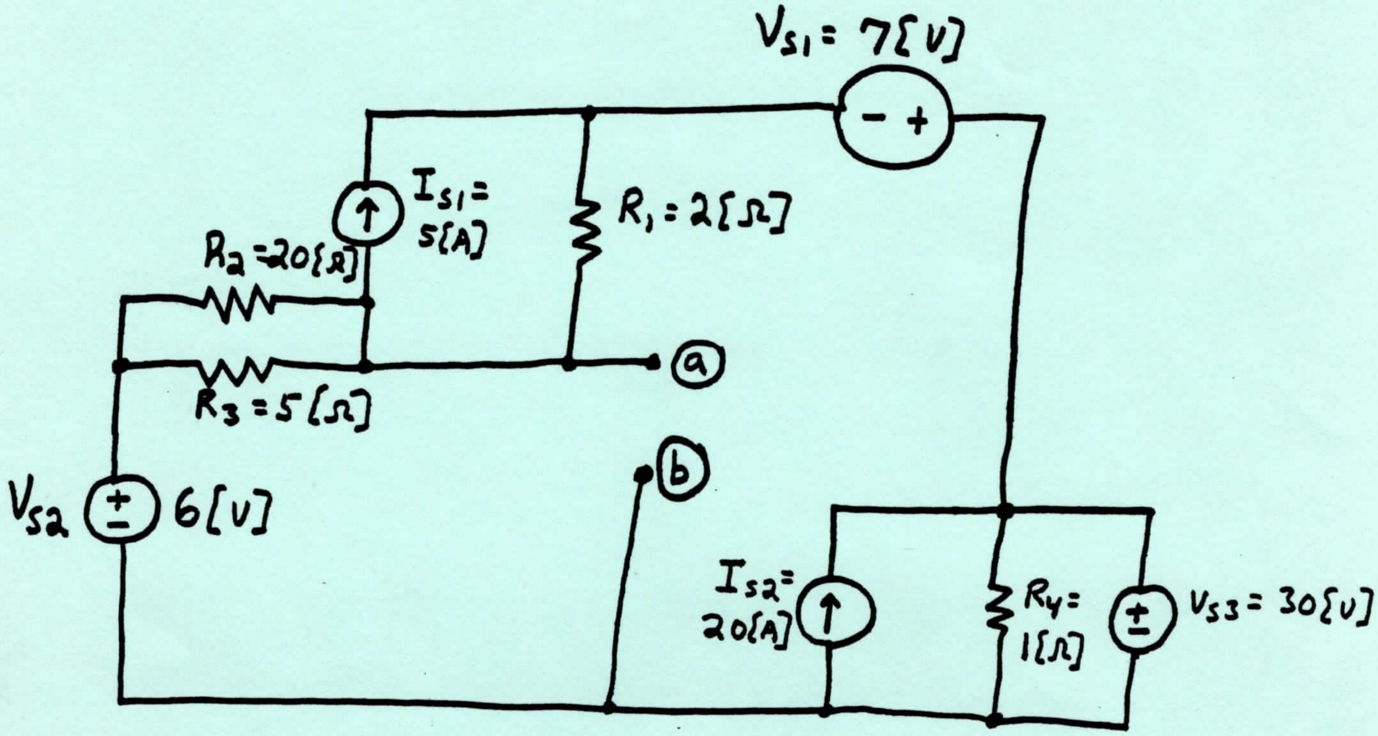
$R_{SPEAKER} = \frac{16}{9} = \underline{\underline{1.778\,\Omega}}$

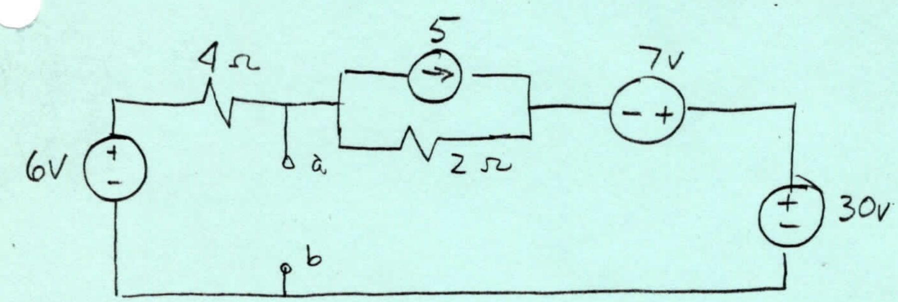
2 (continued) b) What is the value of the source voltage?

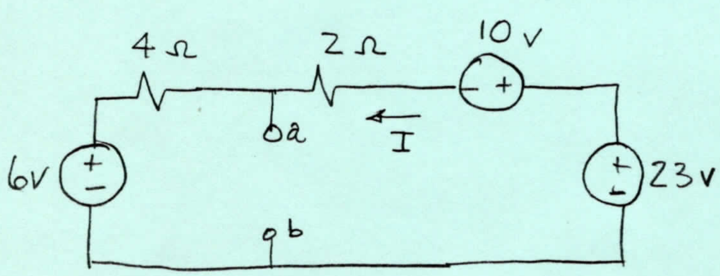
$V_{source(rms)}\ I_{rms} = 25$ watts

$\therefore V_{source\,(rms)} = \frac{25}{3} = \underline{\underline{8.33}}$ VOLTS

3. (20 Points) For the following circuit, determine the Thevenin equivalent with respect to terminals ⓐ and ⓑ. (Hint: you may wish to start by simplifying the circuit using source transformations.)



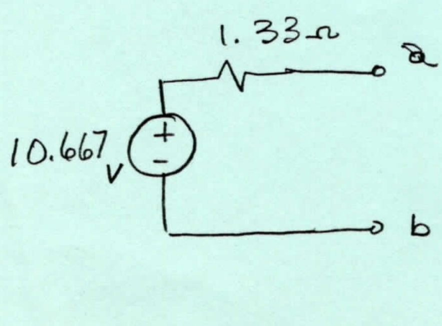




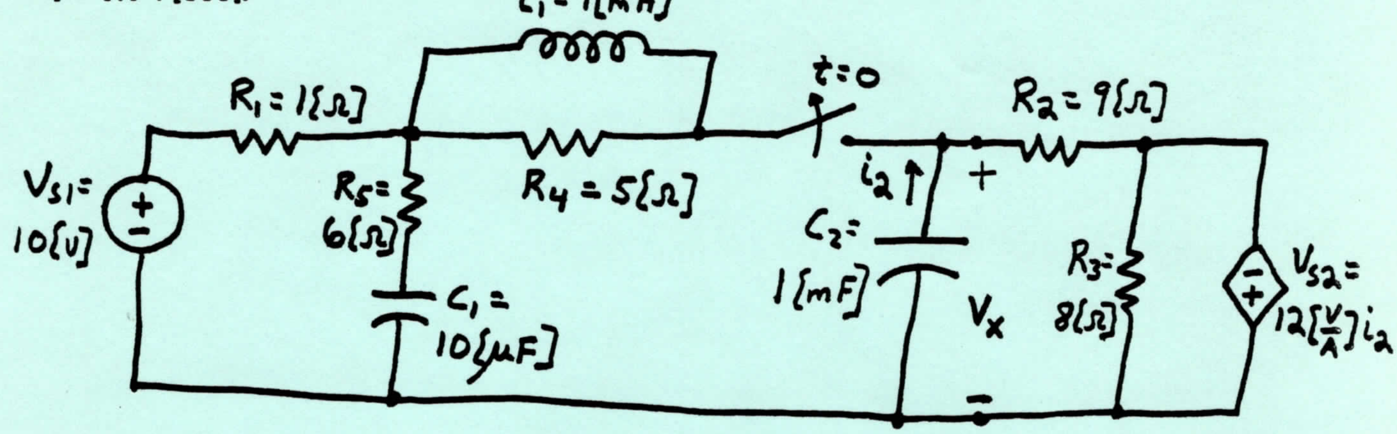
$$I = \frac{23-10-6}{6} = \frac{7}{6}\ A$$

$$V_{TH} = 6 + 4\left(\frac{7}{6}\right) = \underline{\underline{10.667\ V}}$$

$$R_{TH} = 2 \parallel 4 = \frac{8}{6} = \frac{4}{3} = \underline{\underline{1.33\ \Omega}}$$



4. (25 Points) The circuit shown was in a condition where no voltages or currents were changing. Then at t = 0, the switch was opened. Find v_X at t = 0.04 [sec].

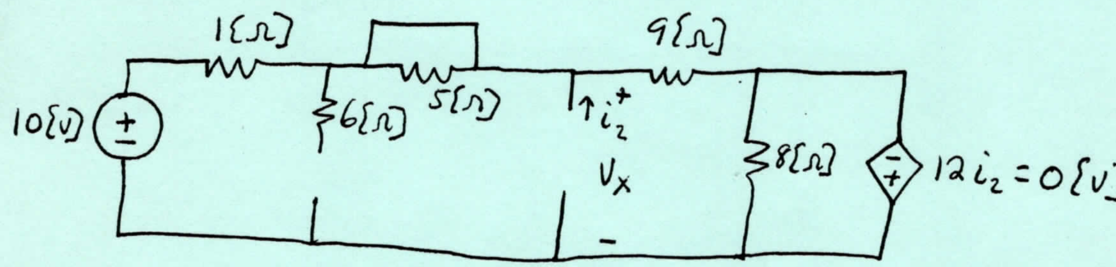


Soln: This is a natural response, so we need $V_x(0)$ and τ.

$$v_x(t) = V_x(0)\, e^{-t/\tau}$$

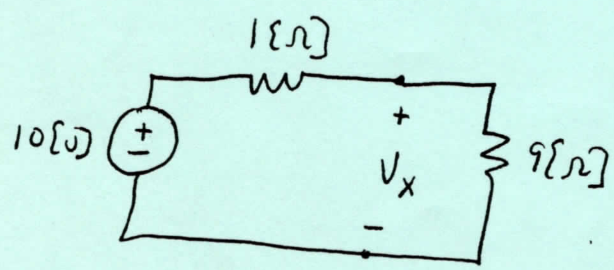
For $t \leq 0$ all inductors act like short circuits, capacitors as open ckts.

Redrawing



+5

Redraw again



$$V_x(0) = \frac{9}{9+1}\, 10[V] = 9[V]$$

+5

(con't on next page)

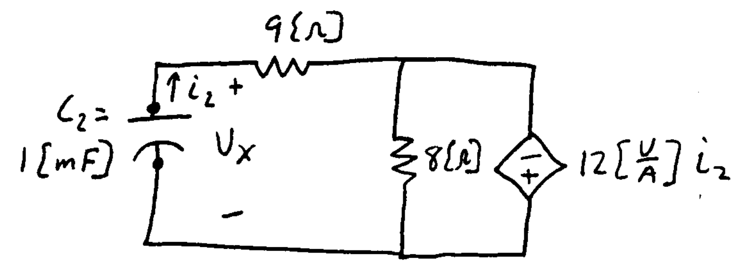
ROOM FOR EXTRA WORK

4. con't.

for $t \geq 0$.

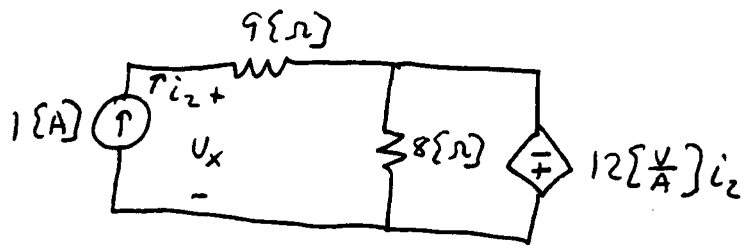
$\tau = R_{eq} C_2$

Grading Scheme.
10 pts total for $V_x(0)$
10 pts total for τ
5 pts total for problem solving techniques
-4 - no units on soln.
-2 - math error per occur.
-5 - conceptual error.
-10 - neglecting dep. source.



To find Req, grab two terminals of C_2, apply a test source of value 1[A], solve for V_x (+5)

$$\frac{V_x}{1[A]} = R_{eq}$$



$V_x = 9 i_2 - 12 i_2$

$V_x = 9[\Omega] 1[A] - 12[\Omega] 1[A]$

$V_x = -3[V]$

$R_{eq} = -3[\Omega]$

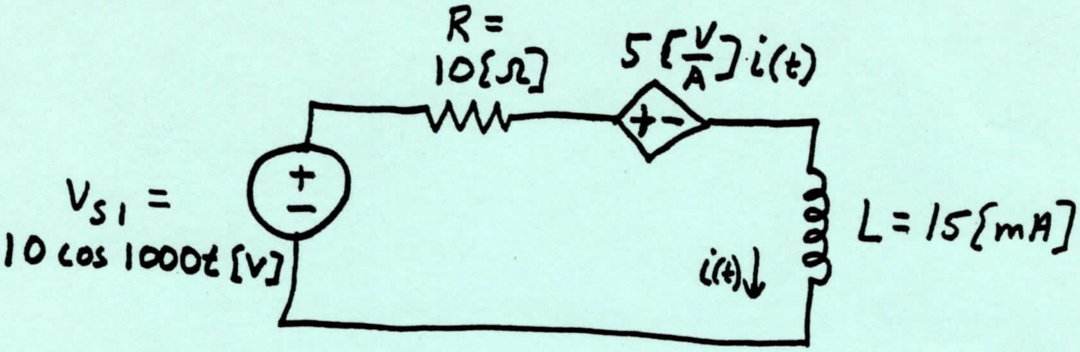
$\tau = -3[\Omega]\, 10^{-3}[F] = -3 \times 10^{-3}[s]$ (+5)

$V_x(t) = 9[V]\, e^{t/3\times10^{-3}[s]}$; for $t \geq 0$

at $t = 0.04[s]$

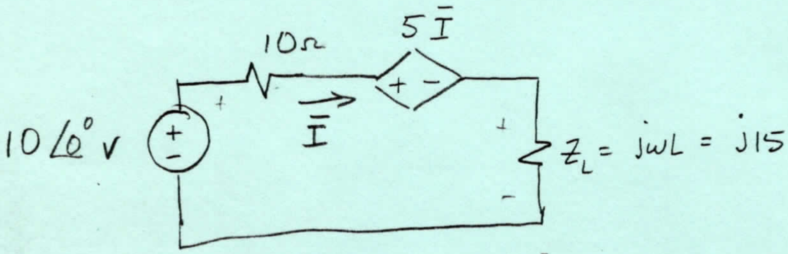
$$\boxed{V_x(0.04) = 9\, e^{.04/.003} = 5.557 \times 10^{+6}\ [V]}$$ (+5)

5. (30 Points) Use the circuit below for all parts of this problem.



a) For the circuit shown, determine the steady state value of i(t).

CONVERT TO PHASOR CKT.



$$10 - 10\bar{I} - 5\bar{I} - j15\bar{I} = 0$$

$$\therefore\ 10 = (15 + j15)\bar{I}$$

$$\bar{I} = \frac{10}{15+j15} = \frac{10}{15\sqrt{2}\angle 45^\circ} = \frac{10}{21.21}\angle -45^\circ = .471\angle -45^\circ \text{ A}$$

HENCE $i(t) = .471 \cos(1000t - 45^\circ)$ AMPS

5. (continued) b) What is the power absorbed by the 10[Ω] resistor?

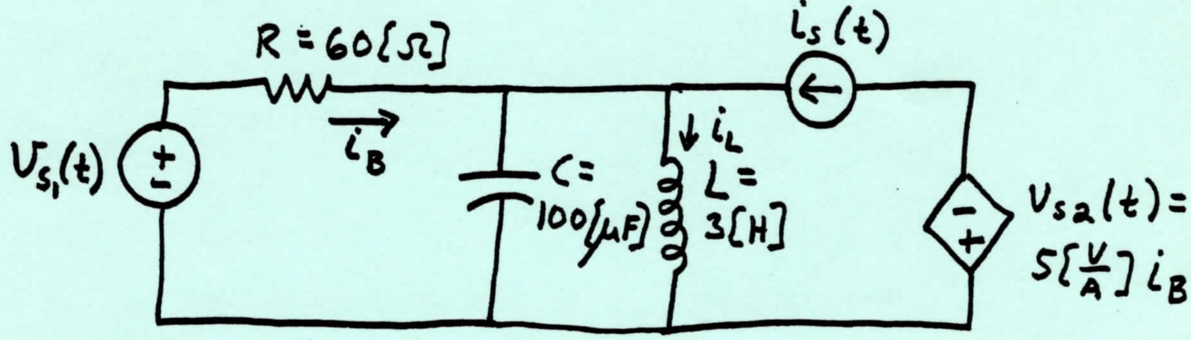
$$P_{10} = \frac{|\bar{I}|^2}{2} R = \frac{|.471|^2}{2} \times 10 = \underline{\underline{1.109 \text{ WATTS}}}$$

b) Determine the real power provided by the independent voltage source and by the dependent source.

$$P_{S1} = \frac{10 \times .471}{2} \cos(0+45^\circ) = \frac{4.71}{2} \times \frac{1}{\sqrt{2}} = \underline{\underline{1.665 \text{ WATTS}}}$$

$$P_5 = -\frac{.471 \times 5 \times .471}{2} \cos(-45^\circ+45^\circ) = \underline{\underline{-.555}} \text{ WATTS}$$

6. (30 Points) Note that the two independent sources given below have different frequencies. Solve for the steady state value for $i_L(t)$.

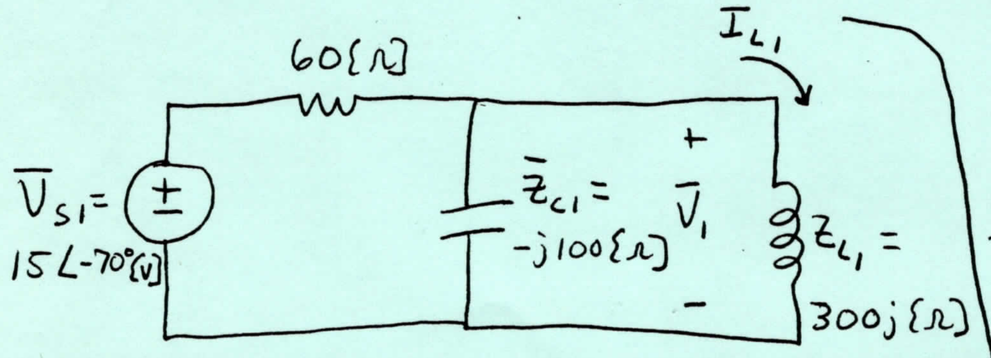


$$v_{s_1}(t) = 15[V]\sin(100t + 20°) = 15[V]\cos(100t - 70°)$$

$$i_s(t) = 0.3[A]\cos(50t - 60°)$$

+5 Solution- Need superposition. Take v_{s_1} first, set $i_s = 0$. Then transform to Phasor Domain →

+5



$$\bar{Z}_{C1} = \frac{1}{j100 \times 10^{-4}} = -j100[Ω]$$

$$Z_{L_1} = j100(3) = 300j[Ω]$$

$$\bar{Z}_{L_1} \parallel \bar{Z}_{C1} = \frac{(-j100)(\cancel{300j})^{3}}{\cancel{200j}_{2}} = -150j[Ω]$$

$$\bar{V}_1 = \frac{-150j}{-150j + 60}\bar{V}_{S1} = \frac{(150\angle -90°)(15\angle -70°)}{161\angle -68.2°}[V]$$

$$\bar{V}_1 = 13.93\angle -91.8°$$

$$\bar{I}_{L_1} = \frac{\bar{V}_1}{\bar{Z}_{L_1}} = \frac{13.93\angle -91.8°}{300\angle 90°} = 46.4\angle -181.8 [mA]$$

$$= 46.4\angle 178.2°$$

con't on page 13

Room for Extra Work

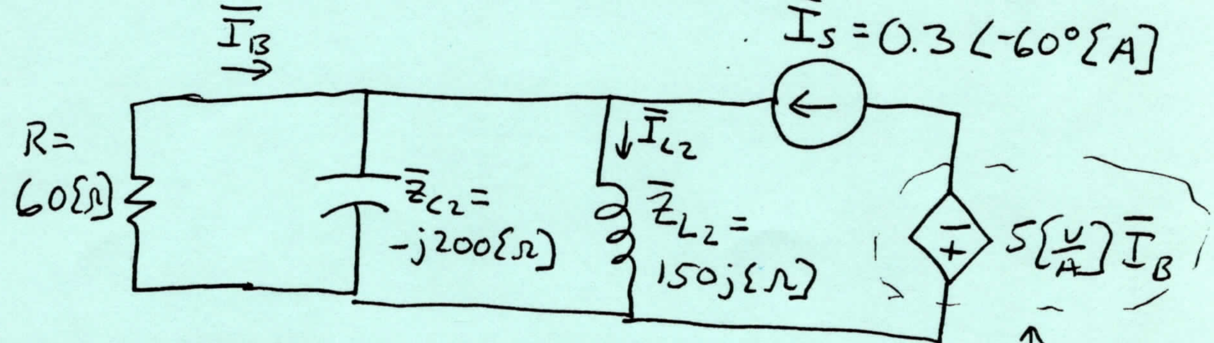
Grading Scheme:
- −10 not using superposition
- −10 mixed domains (1X Only)
- −4 no units in soln.
- −4 no clear dist. for phasors.

so

$$i_{L_1}(t) = 46.4 \cos(100t - 181.8^\circ)\ [mA]$$ +5

Next, Take i_s, set $v_{s_1} = 0$. Then transform.

+5



has no effect, in series with current source.

$$R \parallel \bar{Z}_{C2} = \frac{60(-200j)}{60-200j}\ [\Omega] = \frac{+12000\angle{-90^\circ}}{209\angle{-73.3^\circ}}\ [\Omega]$$

$$R \parallel \bar{Z}_{C2} = 57.5\angle{-16.7^\circ}\ [\Omega] = 55.1 - 16.5j$$

$$\bar{I}_{L_2} = \bar{I}_s \frac{57.5\angle{-16.7^\circ}}{150j + 55.1 - 16.5j} = \frac{((0.3)\angle{-60^\circ})(57.5\angle{-16.7^\circ})}{144\angle{67.6^\circ}}\ [A]$$

$$\bar{I}_{L_2} = (119\ \angle{-144^\circ})\ [mA]$$

+5 $$i_{L_2}(t) = 119 \cos(50t - 144^\circ)\ [mA]$$

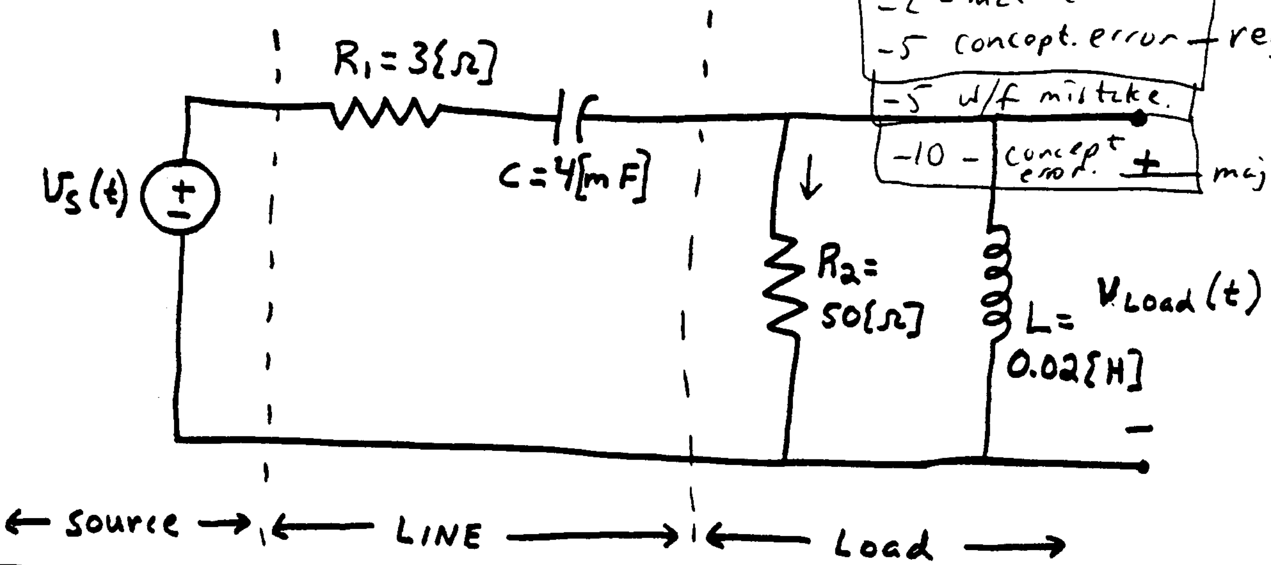
so

$$i_L(t) = \left(46.4 \cos(100t - 181.8^\circ) + 119 \cos(50t - 144^\circ)\right)\ [mA]$$

+5

7. (25 Points) For the circuit below find the real and reactive power dissipated in the line. Assume that the rms value of $v_{Load}(t)$ is 120[V]rms. The frequency of $v_S(t)$ is 100[Hz].

-4 - no or wrong units
-2 - math error
-5 concept. error — reg
-5 w/f mistake.
-10 - concept error + major



Solution:

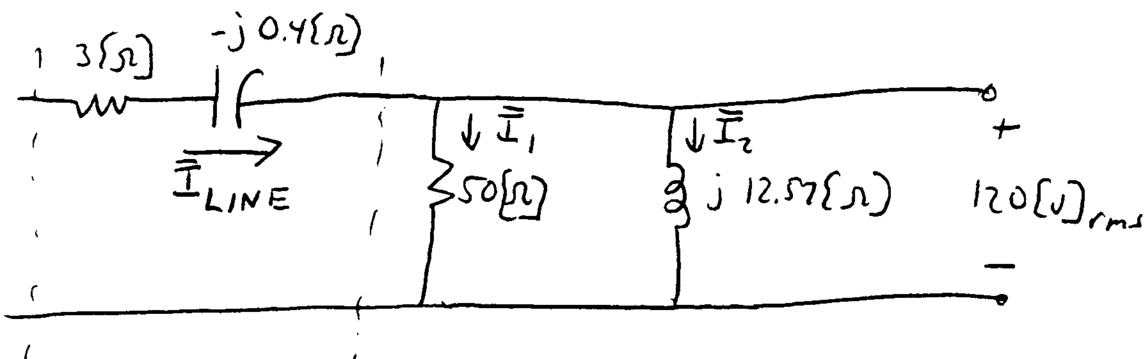
No phase ref. given, assume $V_{Load}(t)$ has zero phase shift.

(+5)

$\bar{V}_{Load} = 120[V]_{rms}$ | $\omega = 2\pi 100 = 200\pi \left[\frac{rad}{sec}\right]$

$\bar{Z}_L = j\omega L = j12.57[\Omega]$ | $\bar{Z}_C = \frac{1}{j\omega C} = -j0.4[\Omega]$

(+5)

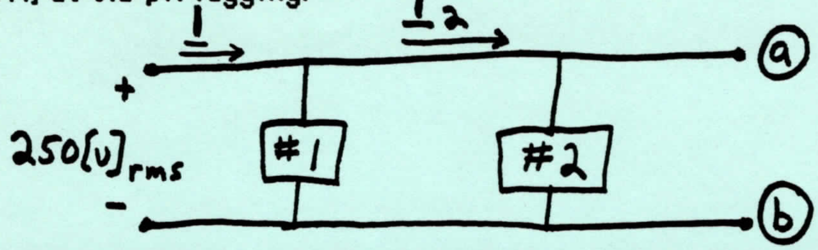


(+5) $\bar{I}_{LINE} = \bar{I}_1 + \bar{I}_2 = \left(\frac{120}{50} + \frac{120}{j12.57}\right)[A]_{rms} = 9.84\angle -75.90^\circ [A]_{rms}$

(+5) $P = |\bar{I}_{LINE}|^2 R = 96.9(3) = 290.69[W]$

(+5) $Q = |\bar{I}_{LINE}|^2 X = 96.9(-0.4) = -38.76[VAR]$

8. (25 Points) Two loads are connected in parallel across a 250[V]rms line as shown below. Load 1 absorbs 10[kW] and 5[kVAR]. Load 2 absorbs 10[kVA] at 0.8 pf. lagging.



a) Determine the value of a reactive load that should be connected to terminals a and b such that the current I will be in phase with the 250[V] line voltage.

$\mathcal{S}_1 = (10 + j5)$ KVA

$\mathcal{S}_2 = \left[10 \times .8 + j10 \sin(\cos^{-1}.8)\right]$ KVA $= (8 + j6)$ KVA

THE TOTAL LOAD, $\mathcal{S}'_T$, SEEN BY THE 250V SOURCE MUST HAVE UNITY POWER FACTOR.

$S_T = (10+8+ j5+j6 + jX)$ KVA $=$ 18 KW

$\therefore$ $jX = -j11$ KVARS

$\underline{\underline{\mathcal{S}_3 = -j11 \text{ KVARS}}}$; $\boxed{Z = -j5.682\ \Omega}$

8. (continued) b) With the reactive load connected to a and b, what is the value of **I**?

SINCE PF = 1

$$P = \bar{V}_{rms}\bar{I}_{rms} = 18\times10^{3}\ W$$

$$\bar{I}_{rms} = \frac{18\times10^{3}}{250} = \underline{\underline{72\ A\ rms}}$$

c) For the same conditions as in part b), what is the value of $\mathbf{I}_2$?

FOR LOAD #1

$$\bar{V}\bar{I}_1^* = (10 + j5)\ KVA$$

$$\bar{I}_1^* = \frac{1\times10^{4} + j5\times10^{3}}{250} = (40 + j20)\ A\ (rms)$$

$$\therefore\ \bar{I}_1 = (40 - j20)\ A(rms)$$

HENCE $\bar{I}_2 = \bar{I} - \bar{I}_1 = 72 - 40 + j20 = \underline{\underline{(32 + j20)\ A\ (rms)}}$

$$= \underline{\underline{37.74\angle 32^{\circ}}}\ A(rms).$$