Signature: Solution Key

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

FINAL EXAM
ELEE 2335
May 9, 1986

INSTRUCTIONS:

1. Sign your name on the upper left of this page.

2. All work is to be done in the spaces provided in this booklet. Use the backs if necessary. Indicate clearly where your work and answers may be found. Enclose your final answers in a box. No credit will be given unless the necessary work is shown. No crib sheets are allowed.

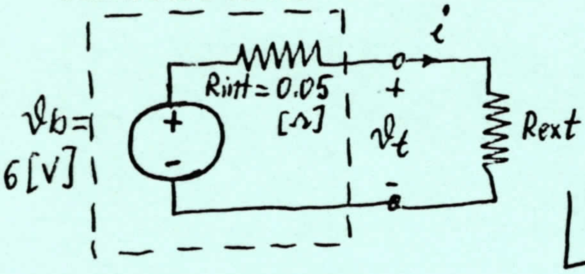
3. Make sure that some kind of clear distinction is obvious between complex quantities (phasors and impedances) and time domain functions. Underlined symbols, lines over symbols, or any other clear method will be acceptable. Show all of your units explicitly, both in your final answer and in your intermediate steps. Units in exam questions are placed within square brackets.

4. If your answers and work are not in ink, there will be no provision for changing your grade once the exam is returned to you. Do not use red ink.

1. 10
2. 20
3. 22
4.
5.
6.
7.
8. 20
9. 25
10. 20
B1.
B2. 20

1. (10 Points) A battery has an open circuit voltage of 6[V] and an internal resistance of 0.05[Ω].

a) What is the terminal voltage of the battery when a 0.2[Ω] load is connected to it?



$R_{ext} = 0.2[\Omega]$;

when $i' = 0$; $v_t = 6[V] = v_b$

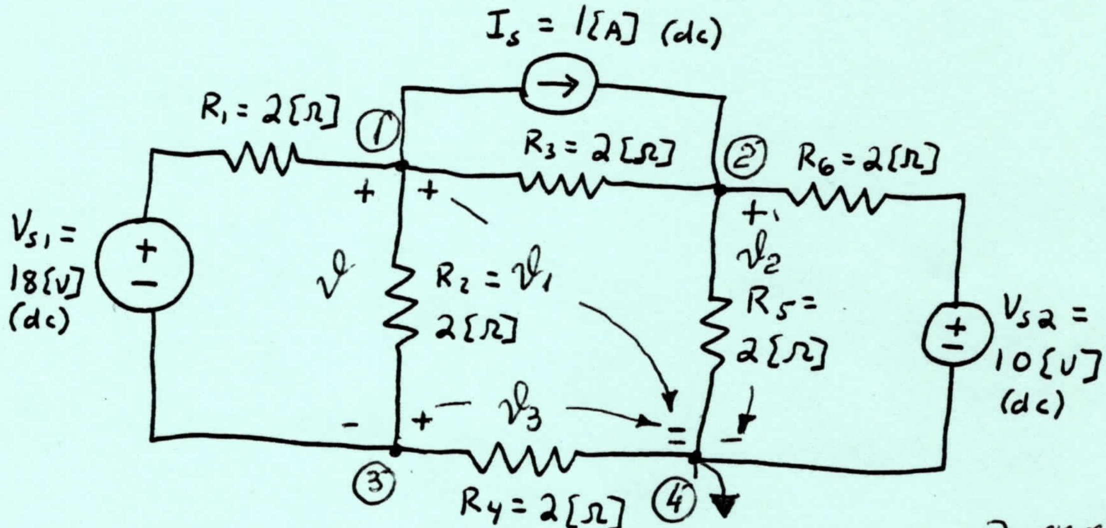
$$\boxed{v_t = v_b \frac{R_{ext}}{R_{int} + R_{ext}} = 6 \frac{0.2}{0.05 + 0.2} = 4.8\,[V]}$$

b) What power will be dissipated by a 0.5[Ω] load?

$R_{ext} = 0.5[\Omega]$; $\quad i' = \dfrac{v_b}{R_{int} + R_{ext}} = \dfrac{6}{0.55} = 10.909091\,[A]$.

$$\boxed{p_{ext} = R_{ext} \cdot i'^2 = 59.504132\,[W]}$$

2. (20 Points) Determine the voltage v in the following circuit using the node voltage method.



essential nodes: $n_e = 4$; 1 reference node: #4; 3 nonreference nodes: # 1, 2, 3 } or other combination

KCL:

$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 2[\Omega]$

as result:

node #1: $\dfrac{v_1 - v_{s1} - v_3}{R_1} + \dfrac{v_1 - v_3}{R_2} + \dfrac{v_1 - v_2}{R_3} + I_s = 0$

— " — #2: $\dfrac{v_2 - v_1}{R_3} - I_s + \dfrac{v_2}{R_5} + \dfrac{v_2 - v_{s2}}{R_6} = 0$

— " — #3: $\dfrac{v_3 + v_{s1} - v_1}{R_1} + \dfrac{v_3 - v_1}{R_2} + \dfrac{v_3}{R_4} = 0$

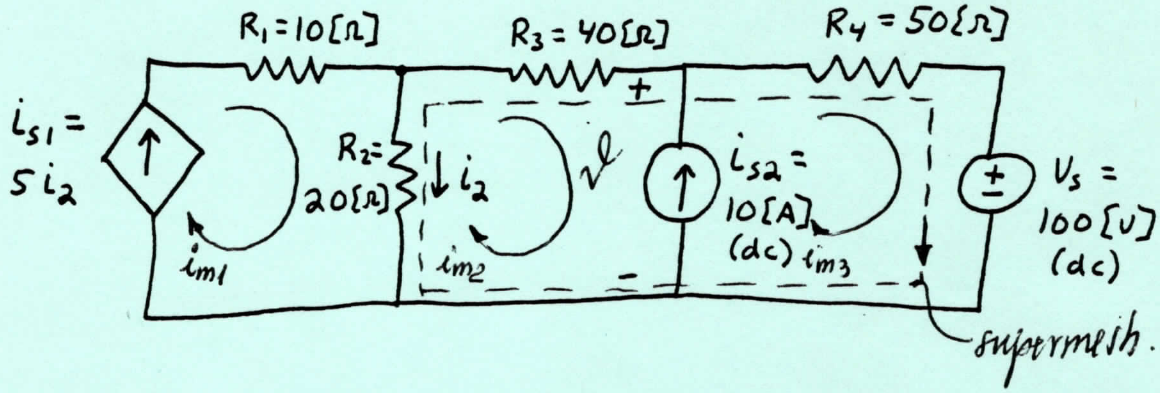
$$\begin{cases} 3v_1 - v_2 - 2v_3 = v_{s1} - 2I_s \\ -v_1 + 3v_2 = v_{s2} + 2I_s \\ -2v_1 + 3v_3 = -v_{s1} \end{cases} \Rightarrow \begin{cases} (1)\ 3v_1 - v_2 - 2v_3 = 16 \\ -v_1 + 3v_2 = 12 \rightarrow v_2 = \dfrac{12 + v_1}{3} \\ -2v_1 + 3v_3 = -18 \rightarrow v_3 = \dfrac{-18 + 2v_1}{3} \end{cases}$$

$$(1)\quad 3v_1 - \frac{12 + v_1}{3} - 2\,\frac{-18 + 2v_1}{3} = 16 ; \Rightarrow 4v_1 = 24 ; \Rightarrow v_1 = 6[V] ,$$

$$v_2 = \frac{12 + 6}{3} = 6[V] ; \quad v_3 = \frac{-18 + 12}{3} = -2[V] ;$$

$$\boxed{v = v_1 - v_3 = 6 - (-2) = 8[V]}$$

3. (22 Points) Determine the voltage v in the following circuit using the mesh current method.



KVL:

mesh #1: $i_{m1} = i_{s1} = 5 i_2 = 5(i_{m1} - i_{m2}) \Rightarrow -4 i_{m1} = -5 i_{m2} \Rightarrow i_{m1} = \frac{5}{4} i_{m2}$

supermesh 2+3: $R_3 i_{m2} + R_4 i_{m3} + v_s + R_2 (i_{m2} - i_{m1}) = 0$

indep. current source: $i_{m3} - i_{m2} = i_{s2} = 10\,[A] \Rightarrow i_{m3} = 10 + i_{m2}$

using the supermesh relation:

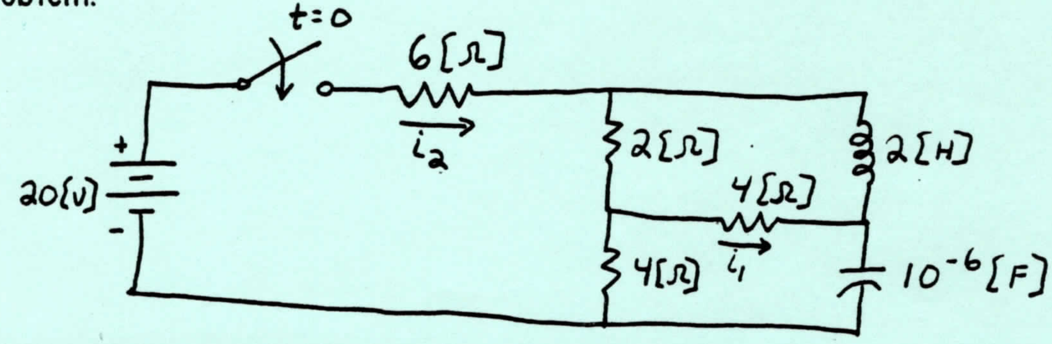
$$R_3 i_{m2} + R_4 (10 + i_{m2}) + v_s + R_2 \left(i_{m2} - \frac{5}{4} i_{m2}\right) = 0$$

$$i_{m2}\left(40 + 50 + 20 - \frac{5}{4} 20\right) = -10 * 50 - 100 \;;\; \Rightarrow i_{m2} = -7.0588235\,[A]$$

$i_{m1} = -8.8235294\,[A]$; $i_{m3} = 2.9411765\,[A]$

$$\boxed{v = R_4 i_{m3} + v_s = 50 * 2.9411765 + 100 = 247.05883\,[V]}$$

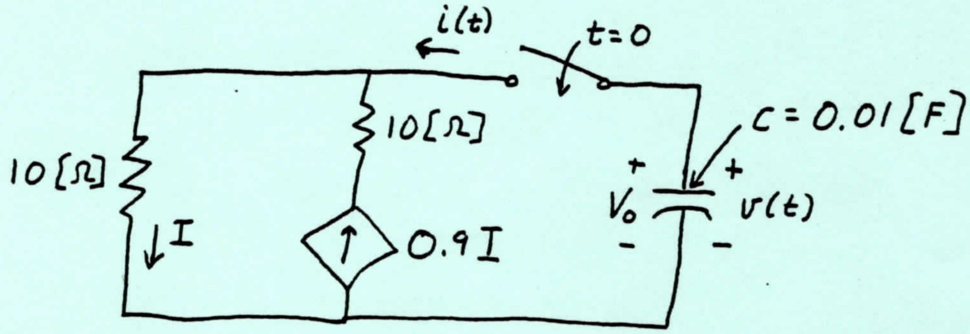
4. (5 Points) In the network shown, no energy is stored in the circuit before the switch is closed. Do not use Laplace techniques to solve this problem.



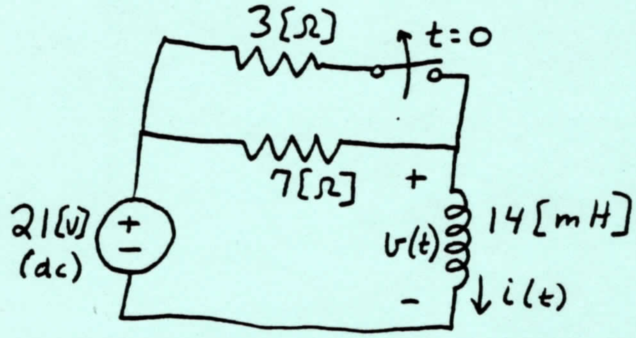
a) Determine the values of i_1 and i_2 at $t = 0^+$.

b) Determine the values of i_1 and i_2 as t goes to infinity.

5. (13 Points) In the circuit shown, the capacitor is charged so that it has an initial voltage V_o of 10[V]. At t = 0, the switch is closed. Determine the values of v(t) and i(t) for all time after the switch is closed. Do not use Laplace techniques to solve this problem.



6. (15 Points) In the circuit shown the switch has been closed for a long time. At t = 0, the switch is opened. Do not use Laplace techniques to solve this problem.

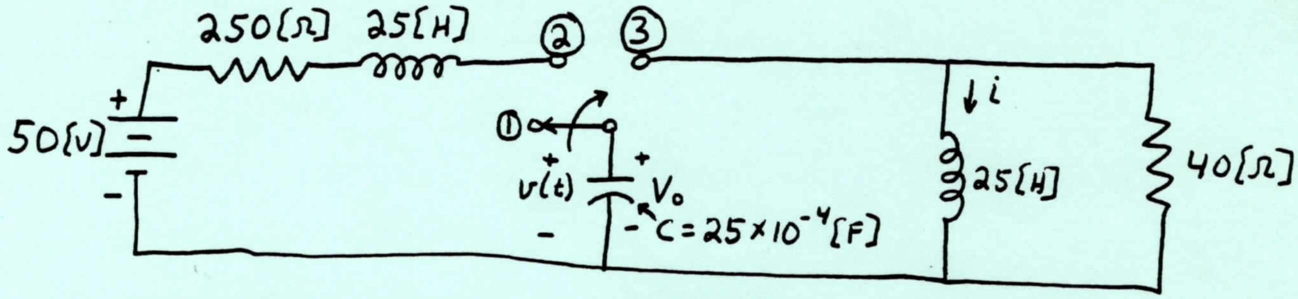


a) Determine the values of i(t) and v(t) for all time after the switch is opened.

6. continued

b) What is the value of the voltage across the switch immediately after the switch is opened?

7. (25 Points) In the circuit given, the switch has been in position 1 for a long time and the voltage V_0 across the capacitor is 10[V]. Do not use Laplace techniques to solve this problem.



a) At t = 0, the switch is moved to position 2. Determine v(t) after the switch is placed in position 2.

7. continued

b) After the switch has been at position 2 for a long time, the switch is moved to position 3 at $t_1 = 0$. Determine $i(t_1)$ after the switch is placed in position 3

8. (20 Points) For the Laplace Transform functions given below, find the inverse Laplace Transforms in terms of exponentials.

-3pts for leaving off $u(t)$

-3pts math error.

a) F(s) = (3s + 4) / (s² + 5s + 4)

+3 $$F(s) = \frac{3s+4}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

+2 $$A = \left.\frac{3s+4}{s+4}\right|_{s=-1} = \frac{1}{3}$$

+2 $$B = \left.\frac{3s+4}{s+1}\right|_{s=-4} = \frac{-8}{-3} = 2\tfrac{2}{3}$$

+3 $$\boxed{f(t) = \left(\frac{1}{3}e^{-t} + \frac{8}{3}e^{-4t}\right)u(t)}$$

b) F(s) = (3s + 4.5) / (s² + 2.25)

+3 $$F(s) = \frac{3s+4.5}{(s+1.5j)(s-1.5j)} = \frac{A}{s+1.5j} + \frac{B}{s-1.5j}$$

+2 $$A = \left.\frac{3s+4.5}{s-1.5j}\right|_{s=-1.5j} = \frac{-4.5j+4.5}{-3j} = 1.5 + j1.5$$

+2 $$B = A^* = 1.5 - 1.5j$$

+3 $$\boxed{f(t) = \left[(1.5+1.5j)\left(e^{-1.5jt}\right) + (1.5-1.5j)\left(e^{1.5jt}\right)\right]u(t)}$$

This can be simplified using Euler's identity to

$$f(t) = \left[3\cos(1.5t) + 3\sin(1.5t)\right]u(t)$$

or

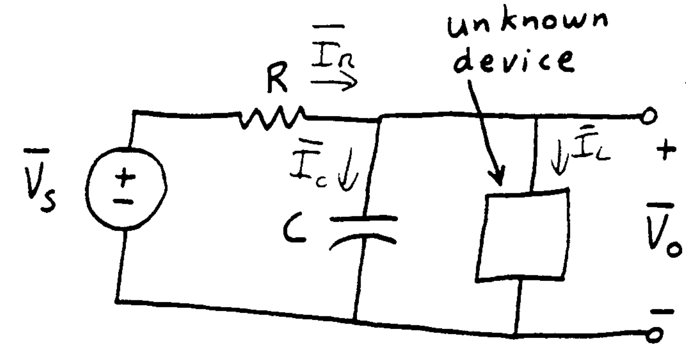
$$f(t) = \left[4.24\cos(1.5t - 45°)\right]u(t)$$

9. (25 Points) In this problem, phasors are indicated by lines over the function name. All phasor values given in this problem are rms values. Solve for the real power, the reactive power, and the complex power absorbed in the device shown.

$R = 100[\Omega]$
$C = 2.2[\mu F]$
$w = 1000$ [rad/sec]

$\overline{V_o}$(rms) $= 15\angle 0°$ [V]

$\overline{V_s}$(rms) $= 24\angle 60°$ [V]



First, we need to solve for $\overline{I_L}$

$$\overline{I_L} = \overline{I_R} - \overline{I_c}$$

$$\overline{I_R} = \frac{\overline{V_s} - \overline{V_o}}{R} = \frac{24\angle 60° - 15}{100}[A] = \frac{12 + 20.78j - 15}{100}[A]$$

(+3) $$\overline{I_R} = \frac{-3 + 20.78j}{100}[A]$$

(+3) $$\overline{I_c} = \frac{\overline{V_o}}{\frac{1}{jwC}} = 15j(10^3)(2.2\times10^{-6})[A] = j3.3\times10^{-2}[A]$$

$$\overline{I_L} = (-.03 + .2078j - j3.3\times10^{-2}) = (-.03 + .1748j)[A]$$

(+5) $$\overline{I_L} = .177\angle 99.7°[A]$$

(+5) $$\overline{S} = P + jQ = \overline{V_{eff}}\,\overline{I_{eff}}^* = \overline{V_o}\,\overline{I_L}^* = 15(.177\angle -99.7°) =$$

(+3) $$\overline{S} = 2.66\angle -99.7°\ [\text{Volt Amp.}]$$

$$\boxed{\overline{S} = -.45 - 2.62j\ [VA]}$$

(over)

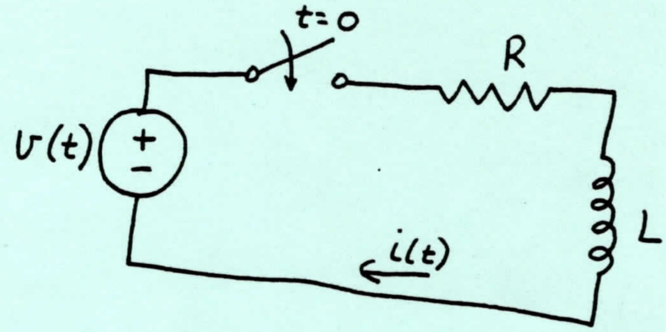
ROOM FOR EXTRA WORK

Thus $P = -.45$ [watts] +3

$Q = -2.62$ [VARs] +3

10. (20 Points) For the circuit below, use Laplace Transform methods.
a) Find i(t) for t ≥ 0.

$v(t) = 6e^{-2t}$ [V]; for t ≥ 0
R = 10[Ω]
L = 3[H]
i(0) = 1[A]



$$v(t) = i(t)R + L\frac{di(t)}{dt}$$

Laplace transform this equation to obtain:

$$V(s) = I(s)R + L(sI(s) - i(0))$$ 5pts

$$\frac{6}{s+2} = I(s)\,10 + 3(sI(s) - 1)$$

$$\frac{6}{s+2} = I(s)(10+3s) - 3$$

$$\frac{6}{s+2} + 3 = \frac{6+3s+6}{s+2} = I(s)(10+3s)$$

3pts $$I(s) = \frac{3s+12}{(s+2)(3s+10)} = \frac{s+4}{(s+2)(s+\frac{10}{3})} = \frac{A}{s+2} + \frac{B}{s+\frac{10}{3}}$$

1pt $$A = \left.\frac{(s+4)}{(s+\frac{10}{3})}\right|_{s=-2} = \frac{2}{4/3} = 1.5$$

1pt $$B = \left.\frac{s+4}{s+2}\right|_{s=-\frac{10}{3}} = \frac{2/3}{-4/3} = -0.5$$

4pts $$\boxed{i(t) = \left(1.5e^{-2t} - 0.5e^{-(10/3)t}\right)\text{ [A] for } t \geq 0}$$

10. continued

b) Check your solution using the initial and final value theorems.

Initial Value Theorem:

$$i(0) = \lim_{s\to\infty} sI(s) = \lim_{s\to\infty} \frac{s(s+4)}{(s+2)(s+\frac{10}{3})}$$

+3pts

$$= \lim_{s\to\infty} \frac{s^2+4s}{s^2+\frac{16}{3}s+\frac{20}{3}} = 1$$

This checks ✓

Final Value Theorem:

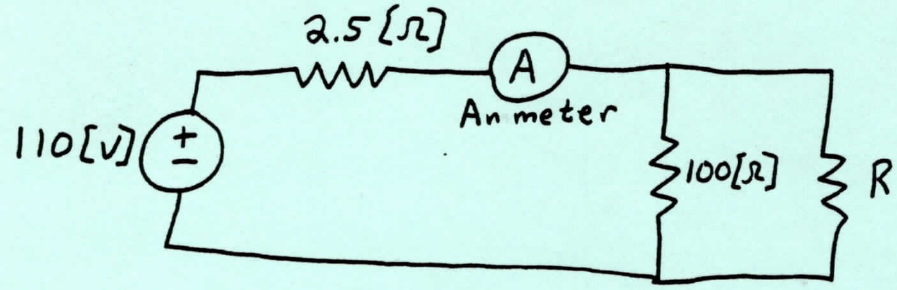
$$i(\infty) = \lim_{s\to 0} s(I(s)) = \lim_{s\to 0} \frac{s(s+4)}{(s+2)(s+\frac{10}{3})} = 0$$

3pts

This also checks, at

$t=\infty$, $v(t) = 0$, thus $i(t)=0$. ✓

Bonus Question 1. (10 Points) An ammeter with an internal resistance of 0.5[Ω] is inserted in an electrical network as shown and reads 6.25[A].



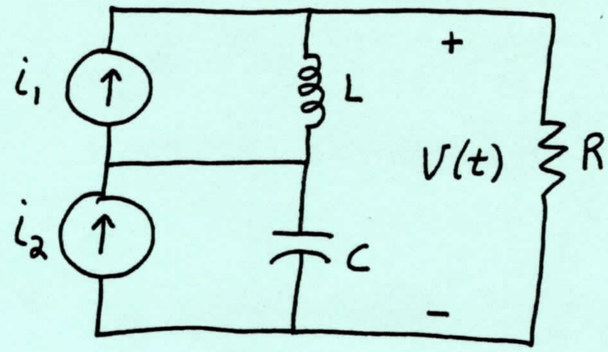
a) What is the value of the resistor R?

Bonus Question 1 continued.

b) What would be the percent error in the calculated value of the resistance R if the ammeter resistance was neglected?

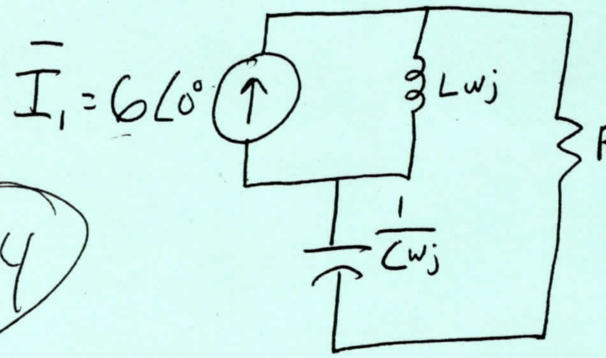
Bonus question 2. (20 Points) Find v(t) using phasor techniques.

$i_1(t) = 6\cos(30t)$ [A]
$i_2(t) = 5\cos(90t + 30°)$ [A]
L = 1[H]
C = 100[µF]
R = 100[Ω]



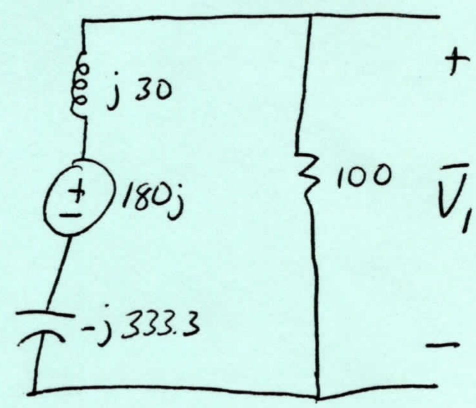
Need to solve using superposition. ($\omega_1 \neq \omega_2$)

Take i_1 first., set $i_2 = 0$



+4

$\Rightarrow$



+2 $\bar{V}_1 = \dfrac{100}{100+30j-j333.3}\,180j$ [V]

$\bar{V}_1 = \dfrac{18000j}{100-303.3j}$ [V] $= \dfrac{18000\angle 90°}{319.4\angle -71.8°}$ [V] $=$

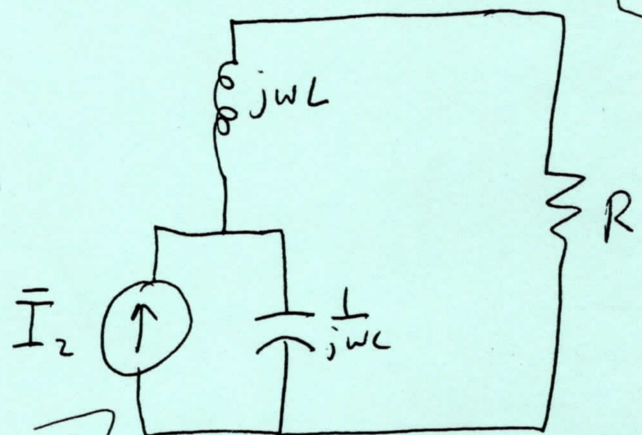
$\bar{V}_1 = 56.4\angle +161.8°$ [V]

+2 $v_1(t) = 56.4\cos(30t + 161.8°)$ [volts]

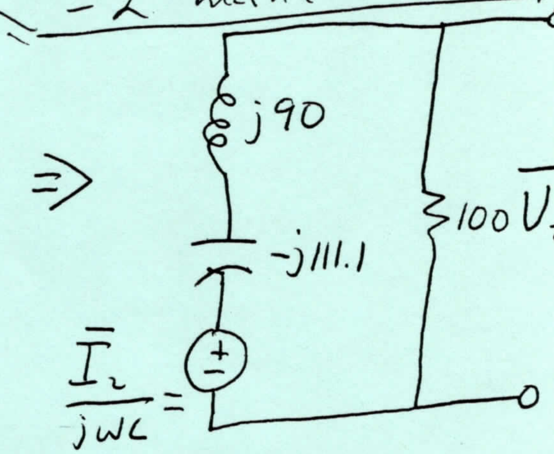
ROOM FOR EXTRA WORK

−5 mixed domains.
−~~8~~10 no superpos.
−5 wrong applic. of superposition
−2 math error

Now for $i_2(t)$



$\Rightarrow$



$555\angle -90° + 30°$

+4 $\bar{V}_2 = \dfrac{100}{100 + 90j - 111j} \; 555\angle -60° \; [V]$

+2 $\bar{V}_2 = \dfrac{55500\angle -60°}{100 - 21.1j} = \dfrac{55500\angle -60°}{102.2\angle -11.9°} = 543\angle -48.1° [V]$

+2 $V_2(t) = 543\cos(90t - 48.1°)\,[V]$

so $v(t) = v_1(t) + v_2(t)$

+4 $\boxed{v(t) = 56.4\cos(30t + 161.8°)\,[V] + 543\cos(90t - 48.1°)\,[V]}$