The circuit shown below has a switch which closed at t = 0. The voltages *v1* and *v2* were measured before the switch was closed, and it was found that





In addition, for time greater than zero, it was determined that



Explore the energy stored in the capacitors for t < 0, and for t = .



Solution: For *t*  0, we have no current through the capacitors, and therefore no change in the voltages across the capacitors. Thus, we can write





So, the total stored energy, in the combination of the two capacitors, is 748.5[J].

 Now, once the switch closes, current begins to flow. This makes sense, since when the switch closes, we now have 22[V] across the resistor, so current must flow. If you think about it, it makes sense that the current will decrease with time. As the charges move, the voltages across the capacitors will increase and decrease, so that the voltage across the resistor will decrease. When the current decreases to zero, the two voltages *v1* and *v2* will be equal.

 What value will these voltages have? We can solve this by integrating the current *iR(t)*, with the defining equation for the capacitor, that is





You can check the integrals above to make sure this is correct.

 Now, let’s look at the energy after a long time. We get





So, the total stored energy, in the combination of the two capacitors, after a long time, is 264.5[J].

 Clearly, there has been a loss of energy. Where did the energy go? It was dissipated in the resistor. If you were to find the power in the resistor, and integrate it over time, you would get the same amount of energy dissipated in the resistor, as that which has been lost from the capacitors.

 Now, we note that we had two series capacitors in the original circuit. Let us consider the possibility of using equivalent circuits here, and what happens when we do so. The equivalent capacitance for the two series capacitors would be



This equivalent capacitance would have a voltage across it, at t = 0, of {15 – (-7)}[V]. Thus, this equivalent capacitance would have energy stored in it of



 This number should look familiar. The total stored energy, in the combination of the two capacitors, was 748.5[J]. The total stored energy, in the combination of the two capacitors, after a long time, was 264.5[J]. The difference in energy between these two conditions is 484[J]. Does this seem like a coincidence? It is not. It is exactly what we should have expected.

 This must be the result that must occur, for our equivalent circuits to be equivalent with respect to the outside world. We found that this much energy was lost in the circuit, and it must have been lost in the resistor. Therefore, if we find an equivalent circuit with respect to the resistor, the same thing must happen to the resistor in each case; the same thing must happen in the actual circuit, and in the equivalent circuit, for the resistor.

 However, the behavior inside the equivalent is not the same. In the original circuit, there was still some energy stored after a long time. In the equivalent, there is no energy stored after a long time. Equivalent circuits means that the two circuits are equivalent, with respect to the outside world.

 The energy stored in the two series capacitors after a long time, in this case, is a minimum. You can’t get the last 264.5[J] out of these capacitors, no matter what you do, as long as they are connected together in series. You can drive the energy in one capacitor to zero, but in doing so, you will increase the energy in the other. The total will not go below this minimum energy level. A similar situation exists for parallel inductors.

The circuit shown below has a switch which closed at
t = 0. The voltages *v1* and *v2* were measured before the switch was closed, and it was found that





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Explore the energy stored in the capacitors for t < 0, and for t = .



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