Chapter 5
Waveguides and Resonators

ECE 3317

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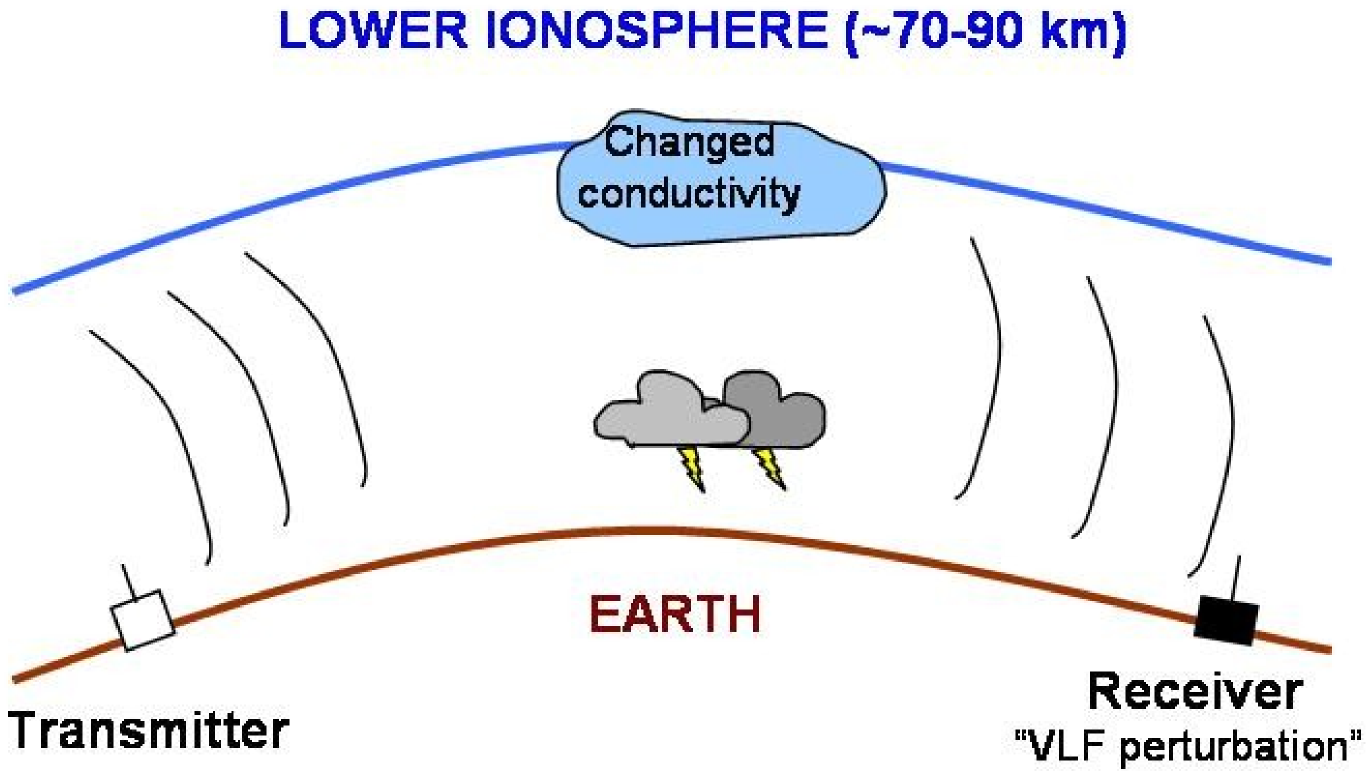
What is a "waveguide" (or transmission line) ?

Structure that transmits electromagnetic waves in such a way that the wave intensity is limited to a finite cross-sectional area

In this chapter we will focus on three types of waveguides:

1. **Parallel-Plate Waveguides**
2. **Rectangular Waveguides**
3. **Coaxial Lines**



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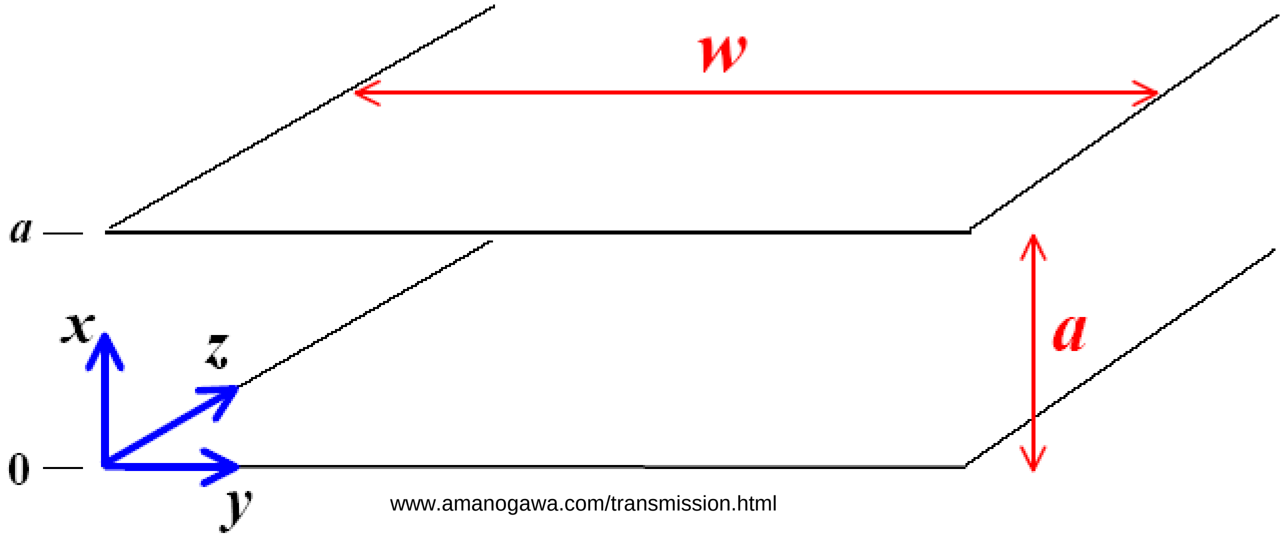
Parallel Plate Waveguide



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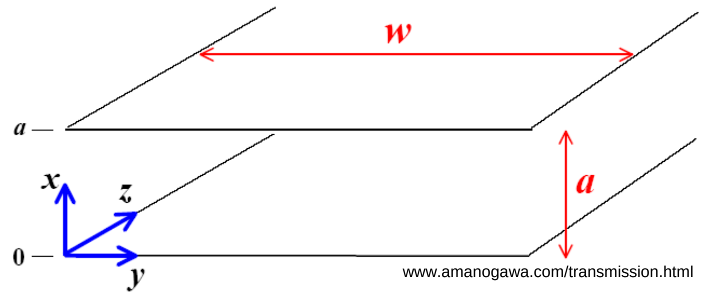
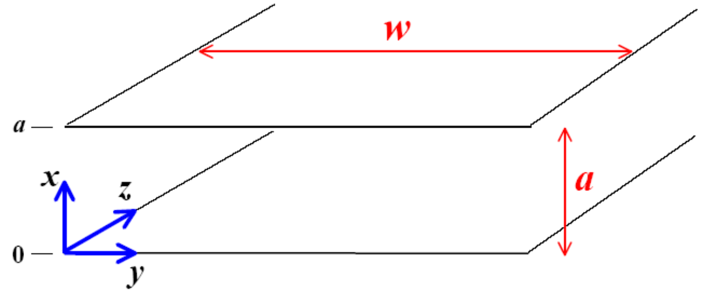
Parallel Plate Waveguide



- Assume both plates to be perfect conductors
- Assume waveguide to be very large in $\hat{y}$ direction $(w >> \lambda)$
- Field vectors have no y-dependence, $\frac{\partial}{\partial y} = 0$
- Neglect any fringing fields at $y = 0$ and $y = w$
- Propagation along the $+\hat{z}$ direction

Parallel Plate Waveguide



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From Maxwell Equations

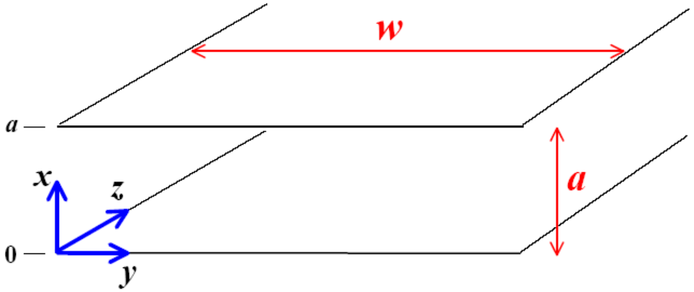
$$(E_y, H_x, H_z) \text{ TE (Transverse Electric) } E_z = 0$$

or

$$(H_y, E_x, E_z) \text{ TM (Transverse Magnetic) } H_z = 0$$

(Sect. 5.1)

TE Waves in Parallel Plate Waveguides



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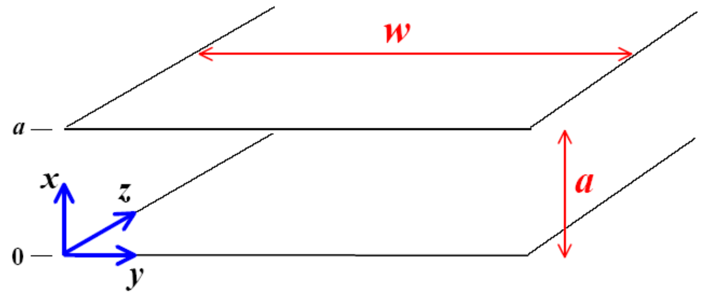
$\left(E_y, H_x, H_z\right)$ B.C. at $x = 0$ and $x = a \Rightarrow E_y = 0$

Remember: $\dfrac{\partial}{\partial y} = 0$

The wave equation for the TE case derived from Maxwell's Equations is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu\varepsilon\right) E_y = 0$$

TE Waves in Parallel Plate Waveguides



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For ppg. in $+\hat{z}$ direction

$$\mathrm{E_y} = E_0 \sin(k_x x) e^{-jk_z z}$$ (satisfies B.C at $x = 0$) (5.5)

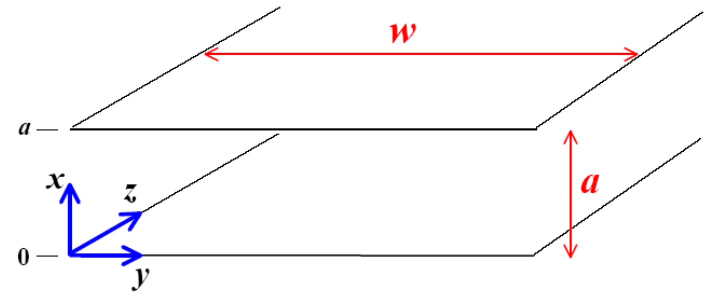
with

$$k_x^2 + k_z^2 = \omega^2 \mu \varepsilon = k^2$$ (5.6)

To satisfy B.C. ($\mathrm{E_y} = 0$) at $x = a$

$$k_x a = m\pi$$ (m is any integer except 0) (5.7)

TE Waves in Parallel Plate Waveguides



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The Electric field can be expressed in another form by substituting in for k_x

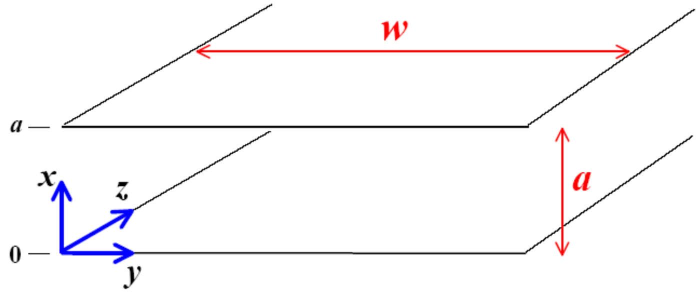
$$\mathrm{E_y} = E_0 \sin\left(\frac{m\pi}{a}x\right)e^{-jk_z z}$$

with

$$k_z = \left[\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2\right]^{\frac{1}{2}} = \omega\sqrt{\mu\varepsilon}\left[1 - \left(\frac{m\lambda}{2a}\right)^2\right]^{\frac{1}{2}}$$

$$\left(\text{where } \lambda = \frac{2\pi}{k}\right)$$

TE Waves in Parallel Plate Waveguides



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The guided wave propagates with the phase velocity

$$v_p = \frac{\omega}{k_z} = \frac{1}{\sqrt{\mu\varepsilon}}\left[1 - \left(\frac{m\lambda}{2a}\right)^2\right]^{-\frac{1}{2}}$$

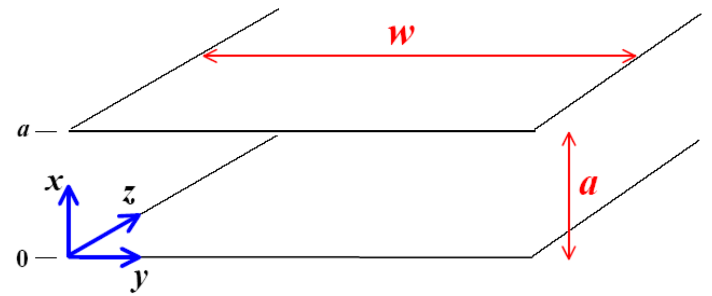
Note: k_z is imaginary for

$$k < k_x \quad \text{or} \quad \omega\sqrt{\mu\varepsilon} < \frac{m\pi}{a} \quad \text{or} \quad \lambda > \frac{2a}{m}$$

If k_z becomes imaginary, the wave will attenuate exponentially, and the velocity for the guided wave will also become undefined.

TE Waves in Parallel Plate Waveguides

$$\omega_c = \frac{\pi m}{a\sqrt{\mu\varepsilon}}$$



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$$f_c = \frac{\omega_c}{2\pi} = \frac{m}{2a\sqrt{\mu\varepsilon}}$$

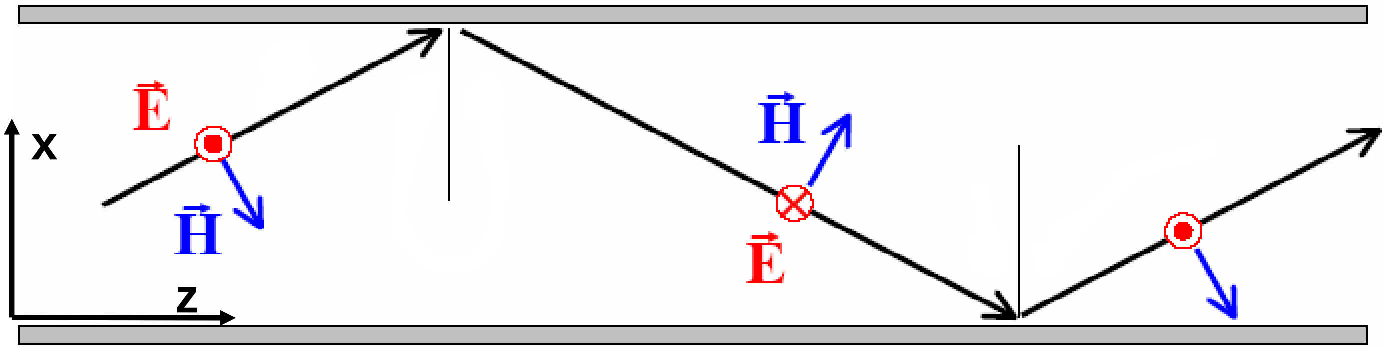
(**cutoff frequency** of TE_m mode)
frequency at which k_z becomes imaginary

exponential attenuation when $f < fc$

$$\lambda_c = \frac{2a}{m}$$

(**cutoff wavelength** of TE_m mode)
wavelength at which k_z becomes imaginary

Physical Interpretation TE mode



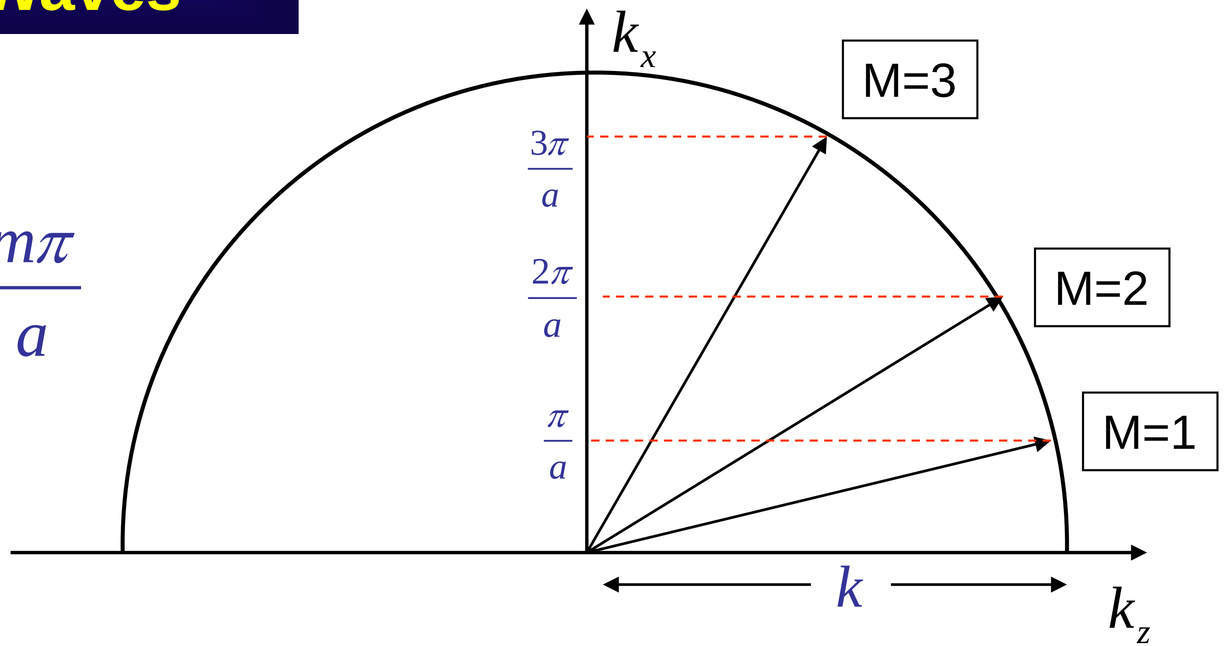
$$E_y = E_0 \sin(k_x x) e^{-jk_z z} = E_0 \left[\frac{j}{2} \left(e^{-jk_x x} - e^{+jk_x x} \right) \right] e^{-jk_z z}$$

$$E_y = \frac{jE_0}{2} \left[\underbrace{e^{-jk_x x - jk_z z}} - \underbrace{e^{jk_x x - jk_z z}} \right] \qquad (5.8)$$

wave trav. in $+\hat{x}$ and $+\hat{z}$ dir. $\left(\text{coeff } \frac{jE_0}{2}\right)$	wave trav. in $-\hat{x}$ and $+\hat{z}$ dir. $\left(\text{coeff } -\frac{jE_0}{2}\right)$

Physical Interpretation of Guided Waves

$$k_x = \frac{m\pi}{a}$$



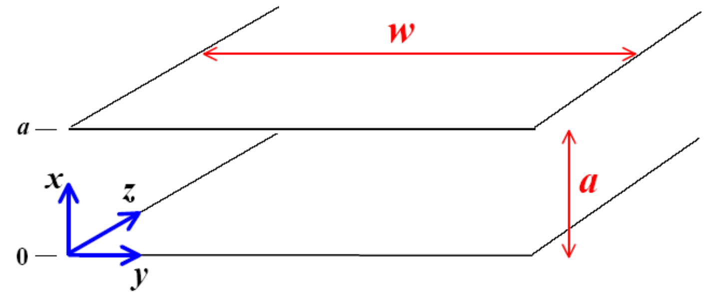
(fig 5.3)

For this case $k \doteq \frac{3.5\pi}{a}$

so modes $m = 1, 2, 3$ will ppg. but for $m \geq 4$, k_z is imaginary.

$$k_z = \left[k^2 - \left(\frac{m\pi}{a} \right)^2 \right]^{\frac{1}{2}} \qquad (5.9)$$

TM Waves in Parallel Plate Waveguides



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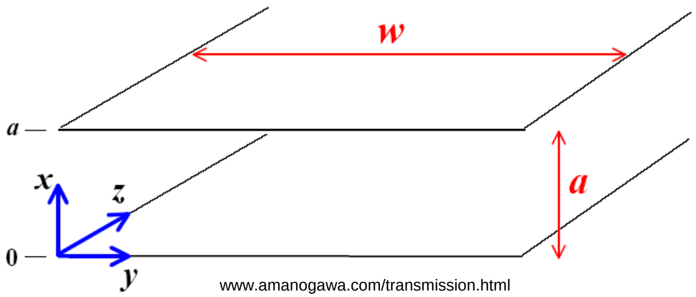
$\left(\mathrm{H_y}, \mathrm{E_x}, \mathrm{E_z}\right)$ can find $\mathrm{H_y} = H_0 \cos k_x x e^{-jk_z z}$

$$\mathrm{E_z} \sim \frac{\partial}{\partial x} \mathrm{H_y} \Rightarrow \mathrm{E_z} \sim \sin k_x x$$

For B.C. at $x = a$ $\left(\mathrm{E_z}\big|_{x=0,a} = 0\right)$

$$\boxed{k_x = \frac{m\pi}{a}}$$

TM Waves in Parallel Plate Waveguides



For ppg. in $+\hat{z}$ direction

$$\mathrm{H_y} = H_0 \cos(k_x x) e^{-jk_z z}$$ (satisfies B.C at $x = 0$)

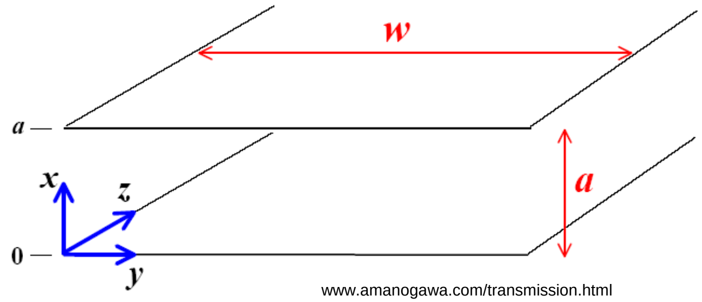
with

$$k_x^{\,2} + k_z^{\,2} = \omega^2 \mu \varepsilon = k^2$$

To satisfy B.C. ($\mathrm{E_z} = 0$) at $x = a$

$$k_x = \frac{m\pi}{a}$$ (m is any integer including 0)

TM Waves in Parallel Plate Waveguides



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The field solutions can be expressed in another form by substituting in for k_x

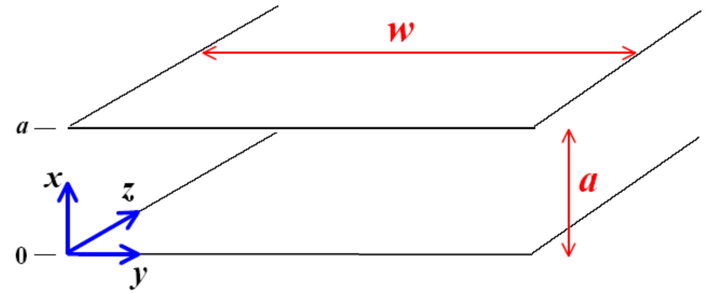
$$H_y = H_0 \cos\left(\frac{m\pi}{a} x\right) e^{-jk_z z}$$

with

$$k_z = \left[\omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2\right]^{\frac{1}{2}} = \omega\sqrt{\mu\varepsilon}\left[1 - \left(\frac{m\lambda}{2a}\right)^2\right]^{\frac{1}{2}}$$

$$\left(\text{where } \lambda = \frac{2\pi}{k}\right)$$

TM Waves in Parallel Plate Waveguides



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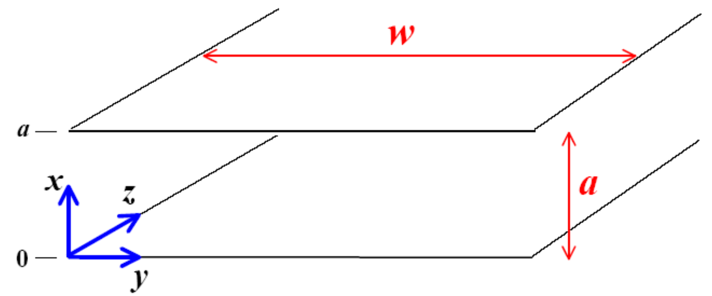
since $\mathrm{E_z} \sim \frac{\partial}{\partial x}\mathrm{H_y}$

$$\mathrm{E_z} = -\frac{k_x}{j\omega\varepsilon} H_0 \sin\left(\frac{m\pi}{a}x\right) e^{-jk_z z}$$

$$\mathrm{E_x} = \frac{k_z}{\omega\varepsilon} H_0 \cos\left(\frac{m\pi}{a}x\right) e^{-jk_z z}$$

TM Waves in Parallel Plate Waveguides



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Remember that for TM waves we can now have $m = 0$.

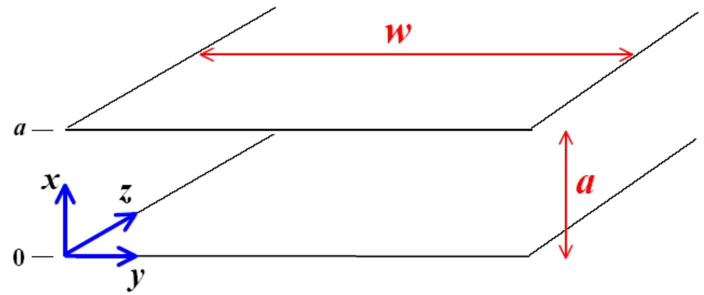
For TM_0:

$$\begin{cases} m = 0. \\ k_x = 0 \\ k_z = \omega\sqrt{\mu\varepsilon} = k \end{cases}$$

For this case **E** and **H** are both perpendicular to the direction of propagation ($\hat{z}$). We refer to this case as

TEM mode (Transverse Electromagnetic)

TM$_0$ Mode



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The field solutions for the TM$_0$ or TEM mode are given by

$$\mathrm{H_y} = H_0 e^{-jk_z z} \qquad (5.13a)$$

$$\mathrm{E_x} = \eta H_0 e^{-jk_z z} \qquad (5.13b)$$

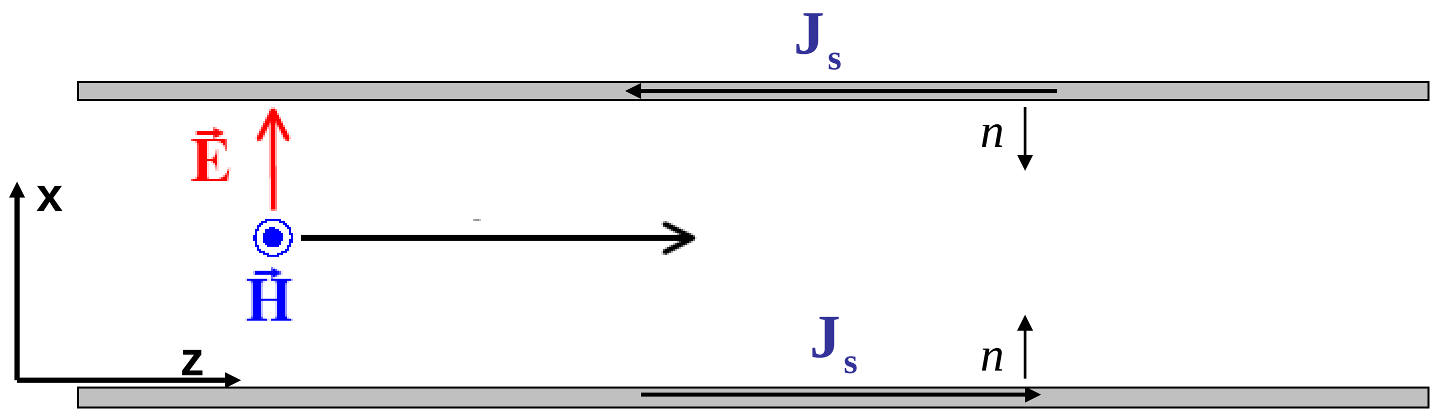
$$\mathrm{E_z} = 0$$

or equivalently

$$\mathrm{E_x} = E_0 e^{-jk_z z}$$

$$\mathrm{H_y} = \frac{E_0}{\eta} e^{-jk_z z}$$

Physical Interpretation TEM mode



(on bottom plate) $\mathbf{J}_s = \hat{\boldsymbol{n}} \times \mathbf{H}\big|_{x=0} = \hat{\boldsymbol{x}} \times \hat{\boldsymbol{y}} \dfrac{E_0}{\eta} e^{-jkz} = \hat{\boldsymbol{z}} \dfrac{E_0}{\eta} e^{-jkz}$ (5.13c)

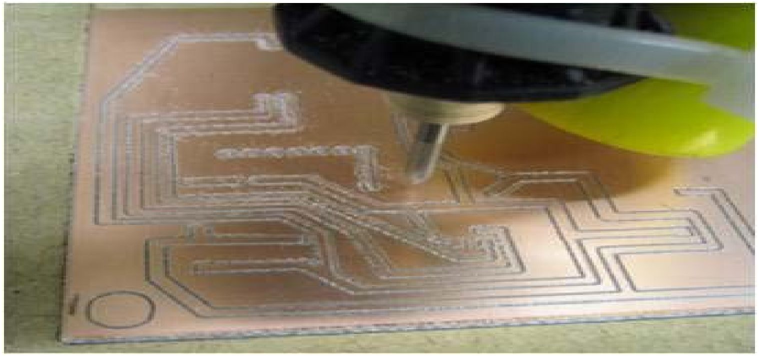
$\rho_s = \hat{\boldsymbol{n}} \bullet \mathbf{D} = \hat{\boldsymbol{x}} \bullet \varepsilon \mathbf{E} = \varepsilon E_0 \, e^{-jkz}$ (5.11d)

(on top plate $n = -\hat{x}$) $\mathbf{J}_s = -\hat{\boldsymbol{z}} \dfrac{E_0}{\eta} e^{-jkz}$

$\rho_s = -\varepsilon E_0 \, e^{-jkz}$

$\mathbf{J}_s$ for TE and TM modes has both $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$ components.

Microstrip Lines



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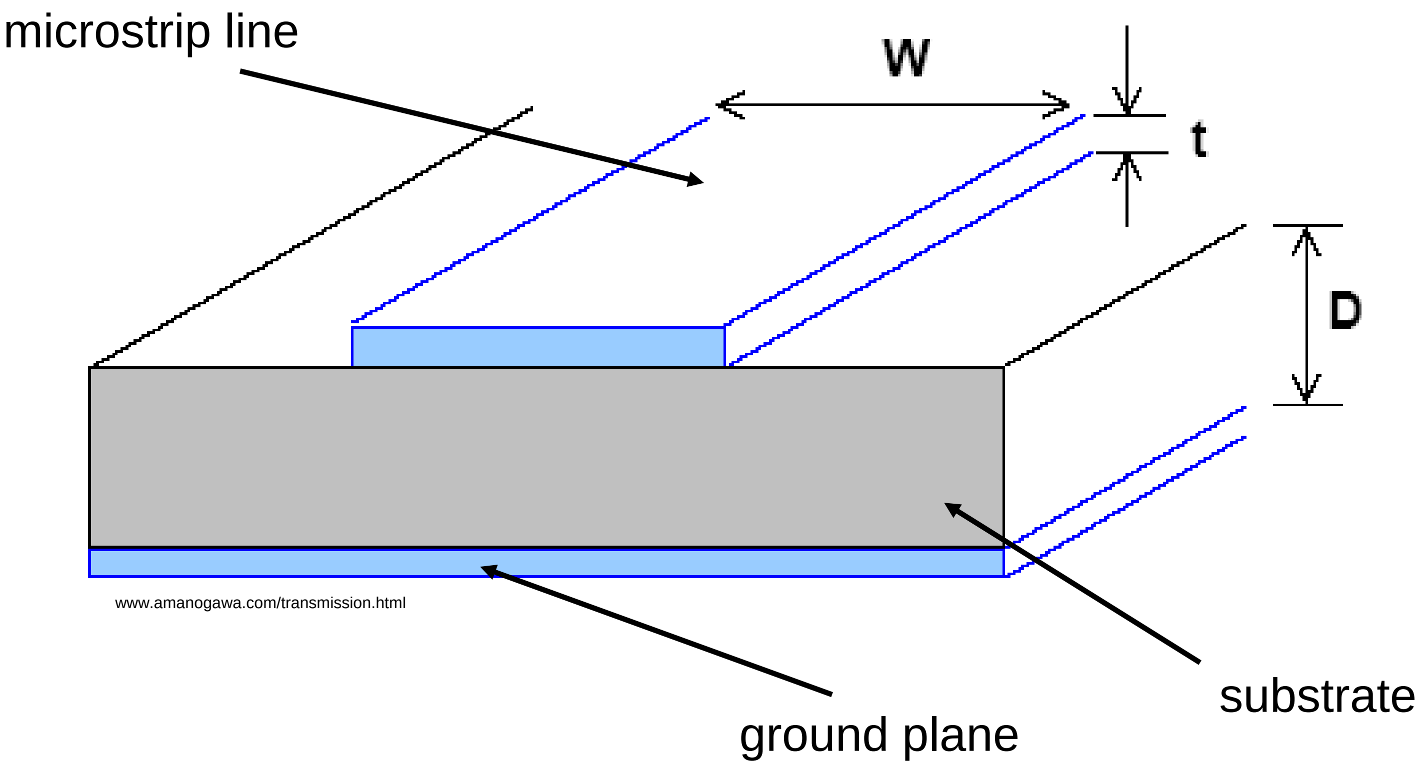

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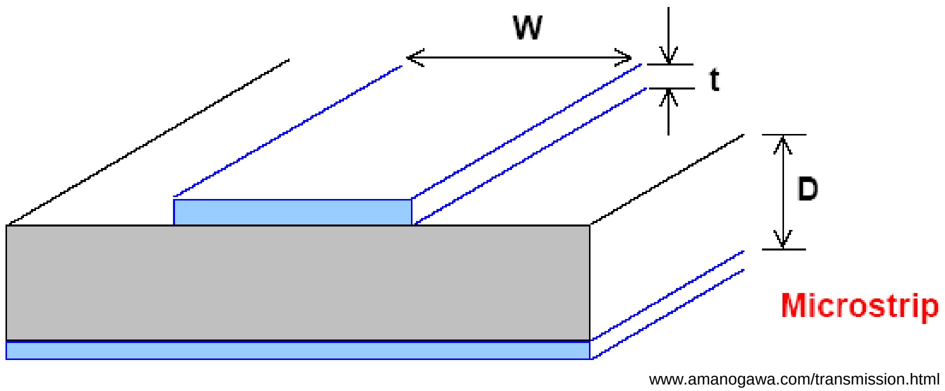


The time-average Poynting power density can be found by:

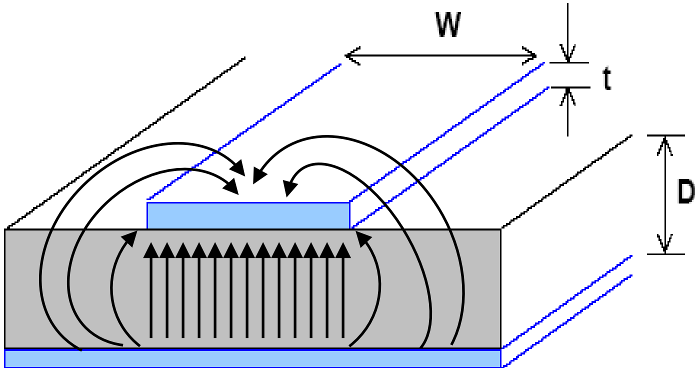
$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$$

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[E_0 e^{-jkz}\frac{E_0}{\eta}e^{+jkz}(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{y}})\right]$$

$$\langle \mathbf{S} \rangle = \hat{\boldsymbol{z}}\frac{E_0{}^2}{2\eta} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

$$\boxed{\text{Total Power} = \mathrm{P} = \frac{E_0^2}{2\eta}wD} \quad [\mathrm{W}]$$

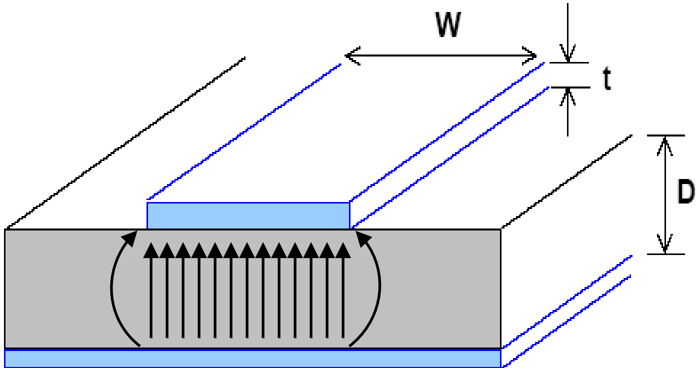
Microstrip Lines



$\varepsilon = \varepsilon_0$

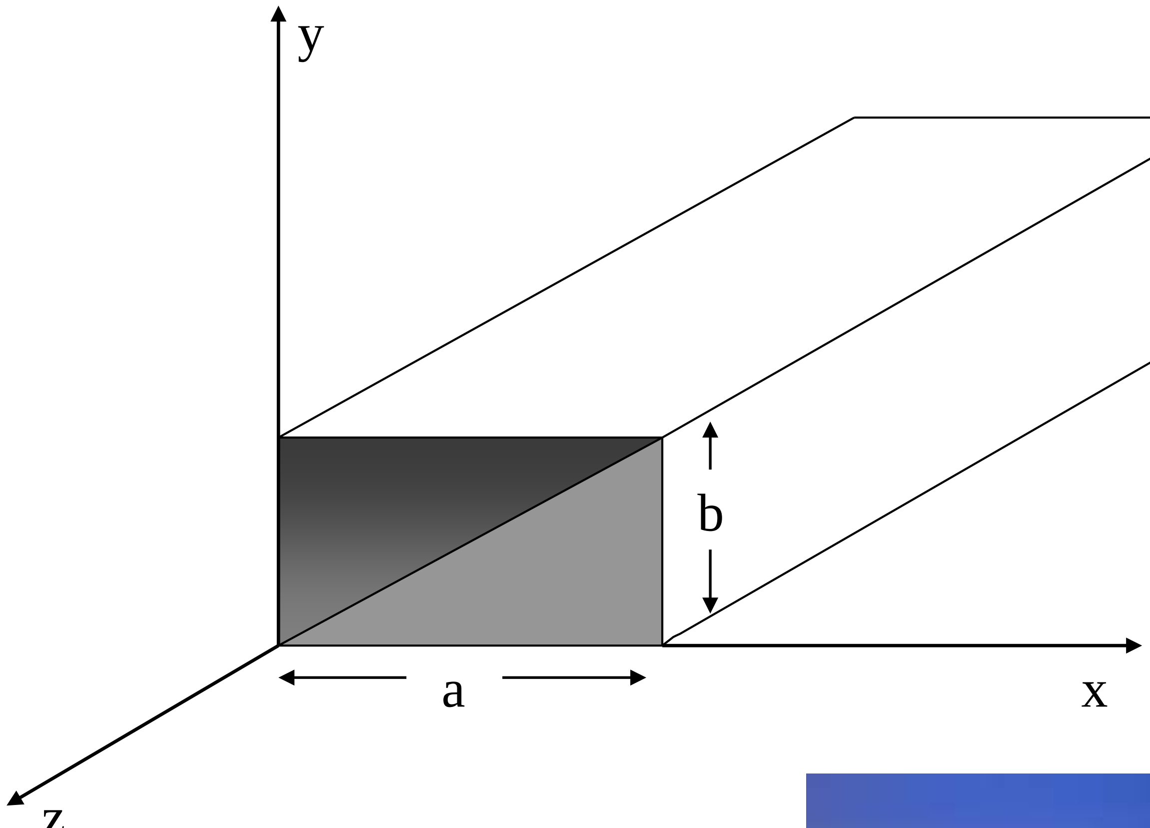
quasi TEM-fields

$k \doteq \omega\sqrt{\mu\varepsilon}$



$\varepsilon > \varepsilon_0$

Rectangular Waveguide



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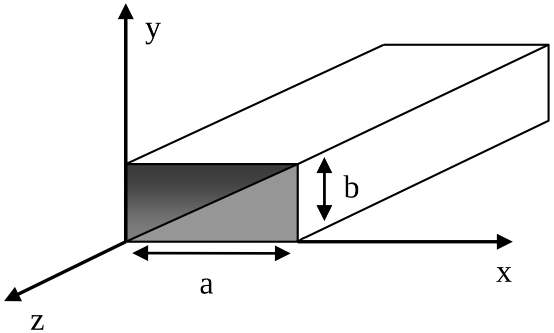
Rectangular Waveguide



MAX equations ⇒ HELMHOLTZ equation

(wave equation)

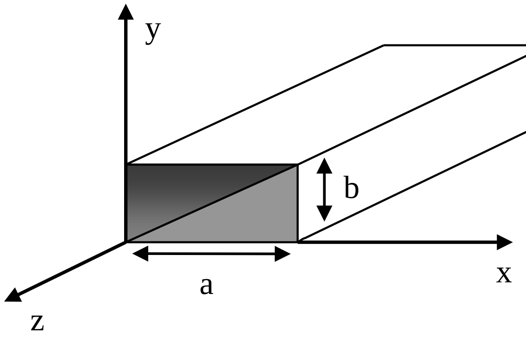
$$\nabla^2 \mathbf{E} + \omega^2 \mu \varepsilon \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + \omega^2 \mu \varepsilon \mathbf{H} = 0$$

6 equation like

$$\frac{\partial^2 \mathrm{H}_z}{\partial x^2} + \frac{\partial^2 \mathrm{H}_z}{\partial y^2} + \frac{\partial^2 \mathrm{H}_z}{\partial z^2} + \omega^2 \mu \varepsilon \mathrm{H}_z = 0$$

Rectangular Waveguide



For a separable solution assume

$$H_z(x,y,z) = X(x)Y(y)Z(z)$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\omega^2 \mu\varepsilon$$

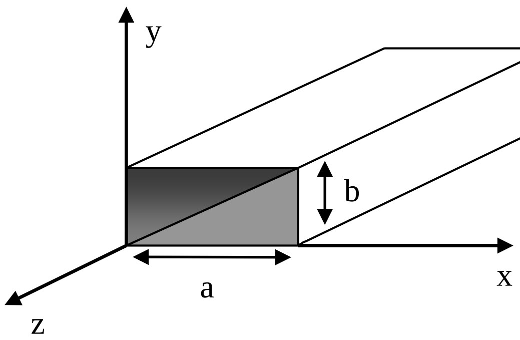
each term is a constant

$$-k_x^2 - k_y^2 - k_z^2 = -\omega^2 \mu\varepsilon$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

$$\Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Rectangular Waveguide



$$X = c_1 \sin k_x x + c_2 \cos k_x x$$

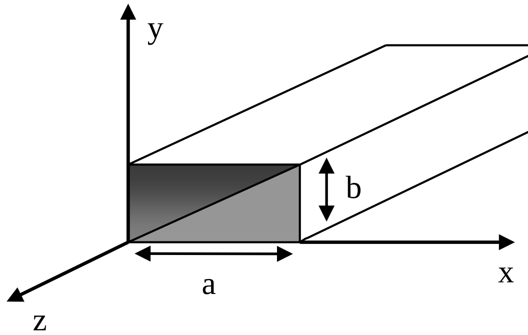
$$Y = c_3 \sin k_y y + c_4 \cos k_y y$$

$$Z = c_5 e^{-jk_z z} + c_6 e^{+jk_z z}$$

$$H_z = X\,Y\,Z$$

Using Maxell's equations we can solve for the transverse fields (E_x, E_y, H_x, H_y)in terms of the longtitudinal fields (E_z, H_z)

Rectangular Waveguide

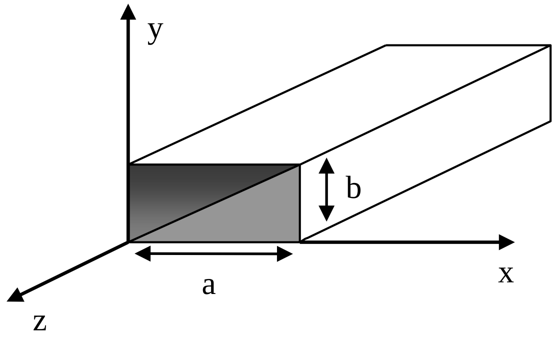


$$E_x = \frac{-jk_z}{k_c^{\ 2}}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{k_c^{\ 2}}\frac{\partial H_z}{\partial y} \quad ; \quad E_y = \frac{-jk_z}{k_c^{\ 2}}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{k_c^{\ 2}}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-jk_z}{k_c^{\ 2}}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{k_c^{\ 2}}\frac{\partial E_z}{\partial y} \quad ; \quad H_y = \frac{-jk_z}{k_c^{\ 2}}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{k_c^{\ 2}}\frac{\partial E_z}{\partial x}$$

where $k_c^{\ 2} = k_x^{\ 2} + k_y^{\ 2} = \omega^2\mu\varepsilon - k_z^{\ 2}$

Rectangular Waveguide



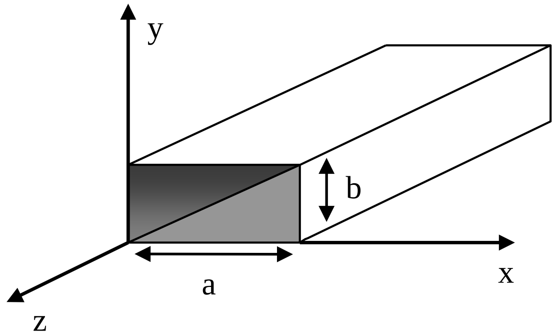
Special cases:

1) when $E_z = 0$; $H_z \neq 0 \Rightarrow$ Transverse Electric mode (**TE**)

2) when $H_z = 0$; $E_z \neq 0 \Rightarrow$ Transverse Magnetic mode (**TM**)

If $E_z = H_z = 0 \Rightarrow$ all comp. $= 0$
Transverse ElectricMagnetic mode (TEM)
can not exist

Rectangular Waveguide TE: Case (E_z=0)



B.C. tangential $\mathbf{E} \rightarrow 0$ at $x = 0, a$ and $y = 0, b$

Side walls $E_y \sim \dfrac{\partial H_z}{\partial x} = 0 \quad (\text{at } x = 0, a)$

$$E_y \sim k_x c_1 \cos k_x x - k_x c_2 \sin k_x x \Rightarrow c_1 = 0 \text{ and } k_x = \frac{m\pi}{a}$$

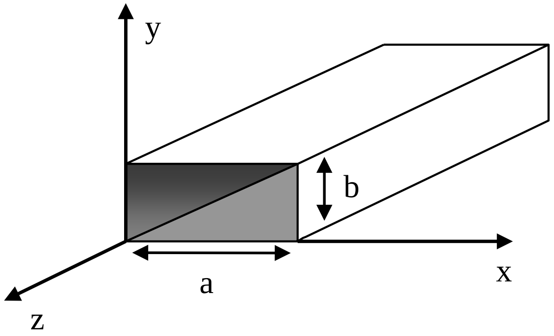
Top and bottom walls $E_x \sim \dfrac{\partial H_z}{\partial y} = 0 \quad (\text{at } y = 0, b)$

$$E_x \sim k_y c_3 \cos k_y y - k_y c_4 \sin k_y y \Rightarrow c_3 = 0 \text{ and } k_y = \frac{n\pi}{b}$$

Let $c_5 c_2 c_4 = H_0 \quad \Rightarrow \quad H_z = X\,Y\,Z$

$$H_z = H_0 \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-jk_z z}$$

Rectangular Waveguide TE: Case (E_z=0)



Propagation constant $k_z = \sqrt{k^2 - k_c^2}$

where $k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

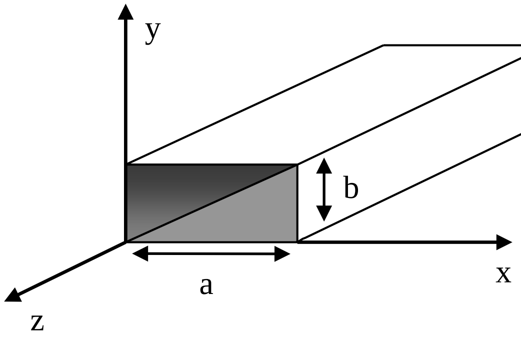
substituting in for k_c^2

$$k_z = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad (5.19)$$

Note: propagation (k_z real) for $k > k_c$

exponential attenuation (k_z imaginary) for $k < k_c$

Rectangular Waveguide TE: Case ($E_z=0$)



$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}$$

(**cutoff frequency** of TE_{mn} mode)

frequency at which k_z becomes imaginary (5.21)

exponential attenuation when $f < f_c$

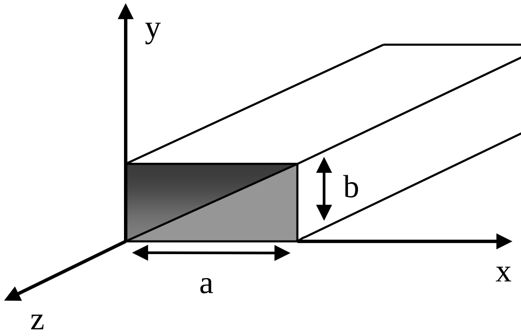
$$\lambda_g = \frac{2\pi}{k_z} = \frac{2\pi}{\sqrt{k^2 - k_c^2}}$$

(**guide wavelength** of TE_{mn} mode)

wavelength at which k_z becomes imaginary (5.20)

defines a doubly infinite set of modes TE_{mn}

Rectangular Waveguide TM: Case (H_z=0)



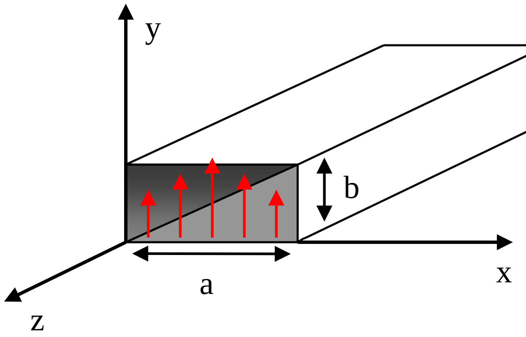
$$E_z = E_0 \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-jk_z z} \qquad (5.18)$$

Same k_z, k_c, f_c, λ_g as TE

For TM $m \neq 0$; and $n \neq 0$ lowest order (smallest f_c) is TM_{11}

The lowest order of all is TE_{10} called the **DOMINANT MODE** for rectangular waveguides (no TEM possible)
(there exists a frequency range where only it can propagate)

Rectangular Waveguide TE_{10} mode (m=1 , n=0)

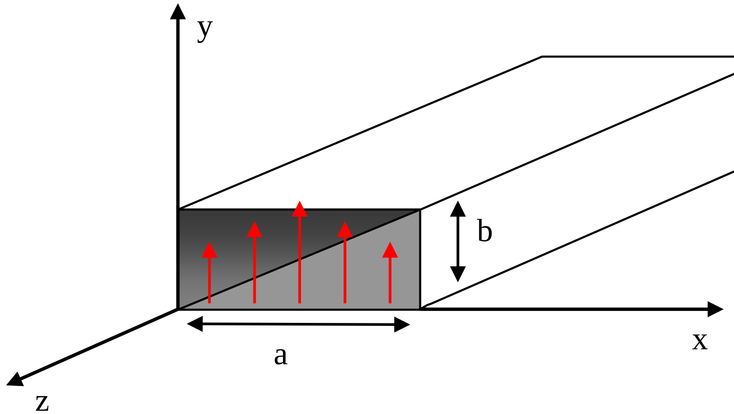


$$H_z = H_0 \cos\frac{\pi x}{a} e^{-jk_z z}$$

$$E_x = -\frac{j\omega\mu}{k_c^{\,2}}\frac{\partial H_z}{\partial y} = 0 \quad ; \quad E_y = \frac{j\omega\mu}{k_c^{\,2}}\frac{\partial H_z}{\partial x} = -\frac{j\omega\mu}{k_c} H_0 \sin\frac{\pi x}{a} e^{-jk_z z}$$

$$H_x = \frac{-jk_z}{k_c^{\,2}}\frac{\partial H_z}{\partial x} = \frac{jk_z}{k_c} H_0 \sin\frac{\pi x}{a} e^{-jk_z z} \quad ; \quad H_y = \frac{-jk_z}{k_c^{\,2}}\frac{\partial H_z}{\partial y} = 0$$

Rectangular Waveguide TE_{10} mode (m=1 , n=0)



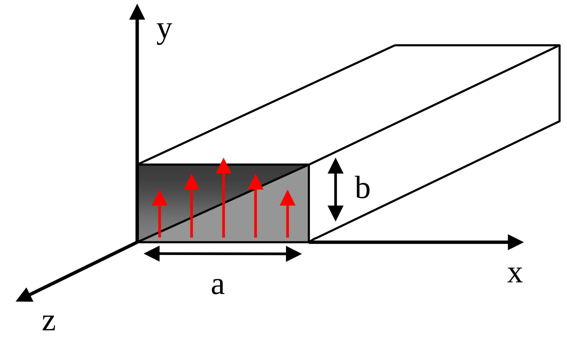
$$k_z = \sqrt{k_2{}^2 - \left(\frac{\pi}{a}\right)^2}$$

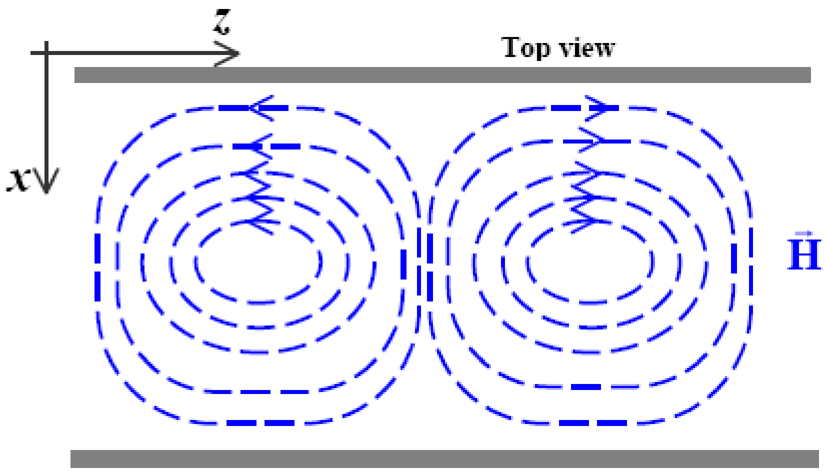
$$\lambda_c = 2a \quad ; \quad f_c = \frac{1}{2a\sqrt{\mu\varepsilon}} \quad ; \quad k_c = \frac{\pi}{a}$$

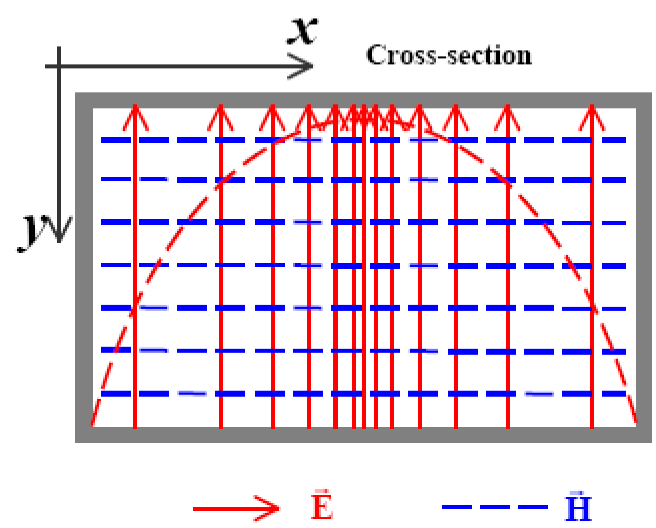
$$\lambda_g = \frac{2\pi}{k_z} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$$

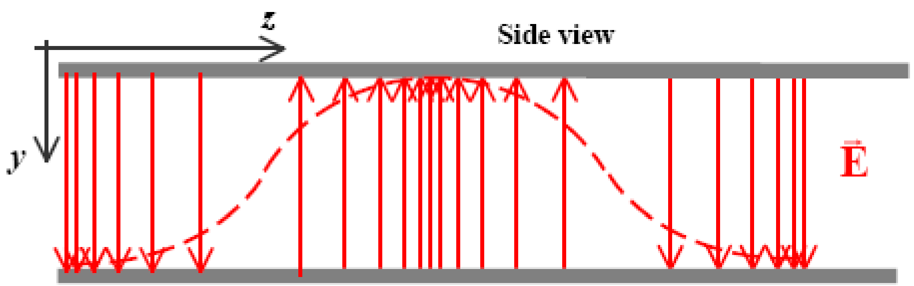
Note: these are the eqns. found in slide 5-32 and 5-33 with $m=1$ and $n=0$

Physical Interpretation TE_{10} mode (m=1 , n=0)

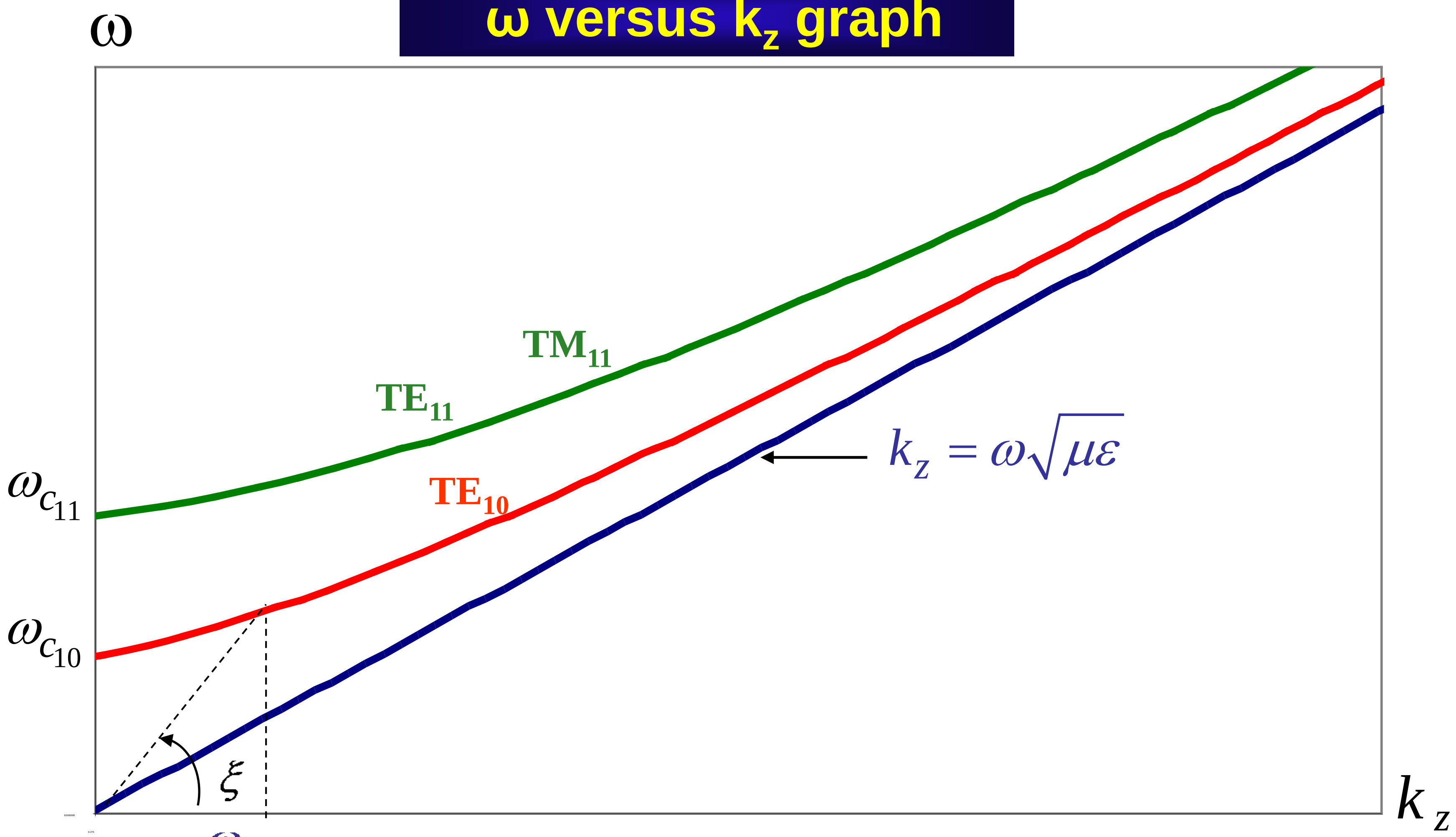








ω versus k_z graph



$$v_p = \frac{\omega}{k_z} = \tan\xi$$ (function of frequency)

where $k_z = \sqrt{k^2 - k_c^{\ 2}}$ and $v_g = \dfrac{\partial\omega}{\partial k_z}$

Design of Practical Rectangular Waveguide

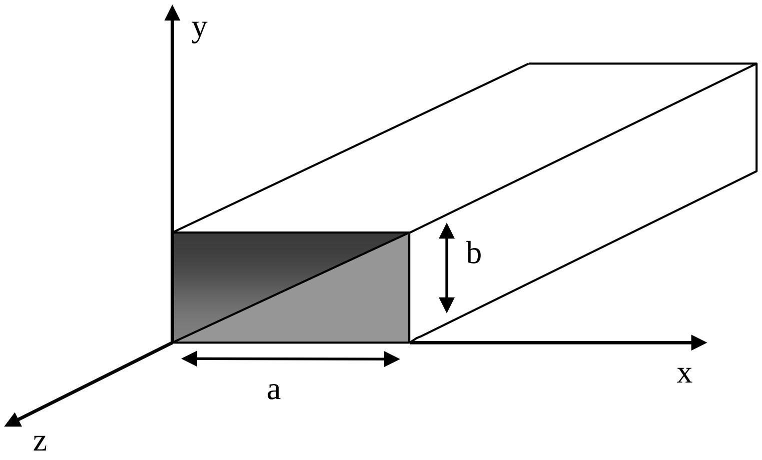
Example 5.4

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$f_{c\,\mathrm{TE}_{10}} = \frac{1}{2\sqrt{\mu\varepsilon}}\left(\frac{1}{a}\right)$$

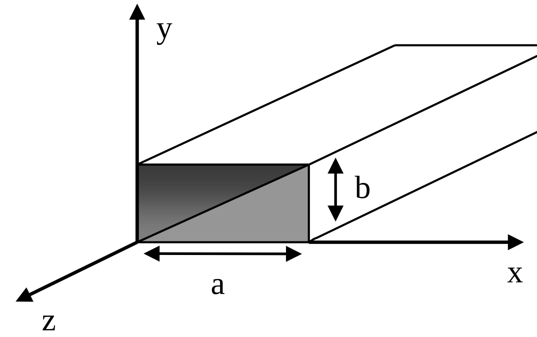
$$f_{c\,\mathrm{TE}_{01}} = \frac{1}{2\sqrt{\mu\varepsilon}}\left(\frac{1}{b}\right)$$

$$f_{c\,\mathrm{TE}_{20}} = \frac{1}{2\sqrt{\mu\varepsilon}}\left(\frac{2}{a}\right)$$

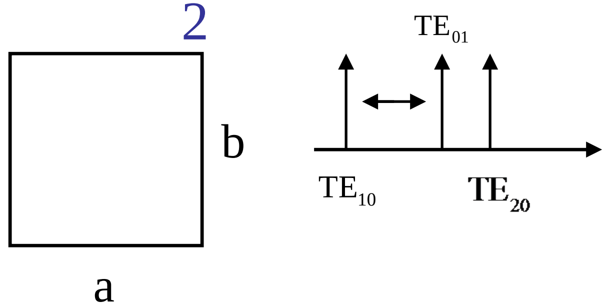


Design of Practical Rectangular Waveguide

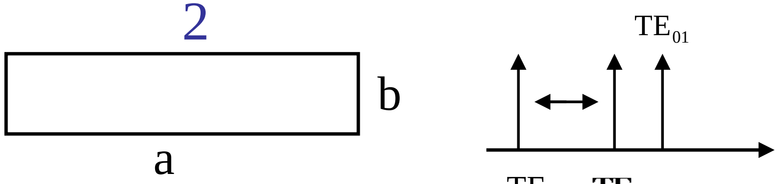
(Cont. e.g 5.4)



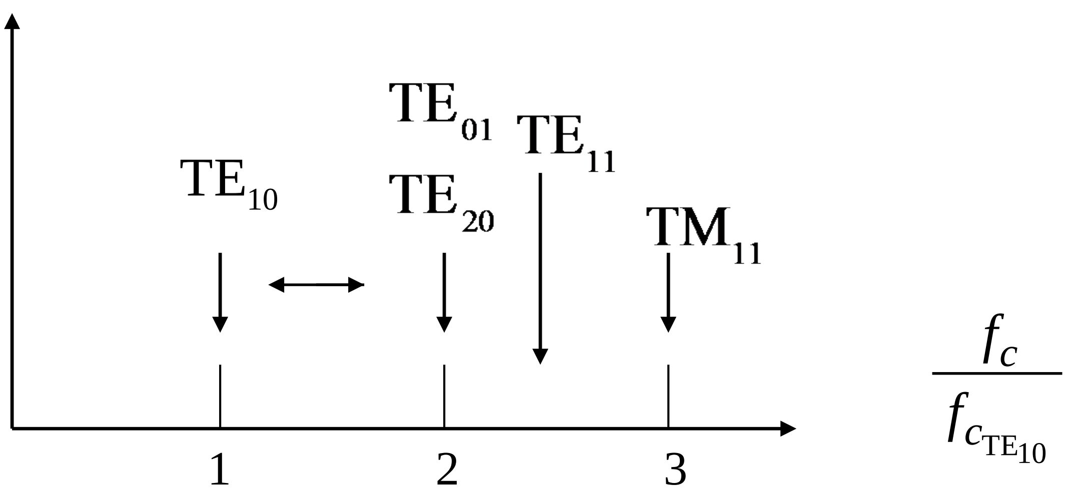
for $b > \frac{a}{2}$



for $b < \frac{a}{2}$

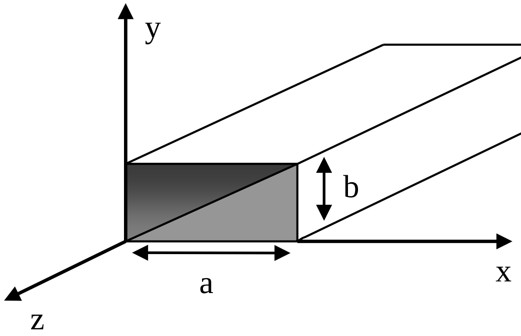


for $b = \frac{a}{2}$



Design of Practical Rectangular Waveguide

(Example 5.3)



Transmitting Power in waveguide $\left[\mathrm{TE}_{10}\ \text{mode}\right]$

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$$

$$\langle \mathbf{S} \rangle = \hat{z}\frac{E_0^2 k_z}{2\omega\mu}\sin^2\left(\frac{\pi x}{a}\right) \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Time-average Poynting vector

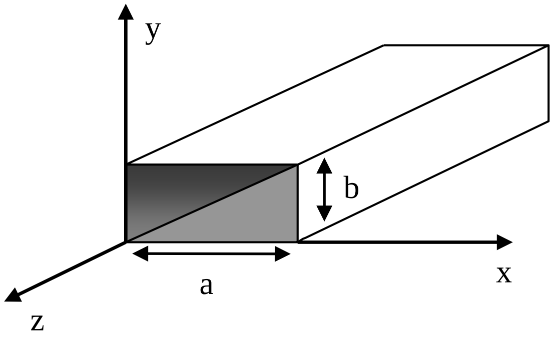
$$P = \int_0^a \int_0^b \langle \mathbf{S} \rangle \bullet \hat{z}\, dx dy$$

$$P = \frac{E_0^2 abk_z}{4\omega\mu} \quad [\mathrm{W}]$$

Total transmitted power

Design of Practical Rectangular Waveguide

(Cont. e.g 5.3)



Note:

- Do not want $b > \frac{a}{2}$ for bandwith
- Do not want b small for power handling $P = \frac{E_0{}^2 abk_z}{4\omega\mu}$

Usual case

$$b = \frac{a}{2}$$

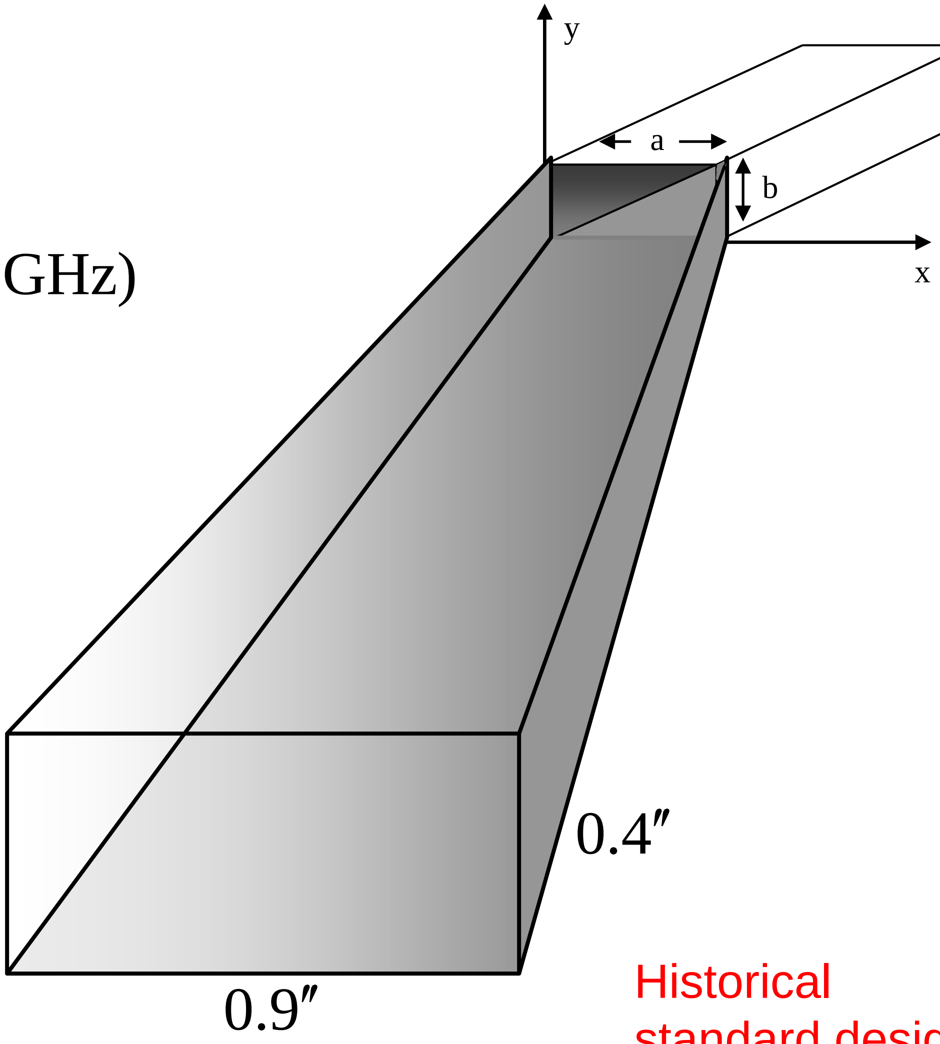
Design of Practical Rectangular Waveguide

X-Band Waveguide (8-12 GHz)

$f_{c_{TE_{10}}} = 6.55$ GHz

$f_{c_{TE_{20}}} = 13.1$ GHz

$f_{c_{TE_{01}}} = 14.7$ GHz



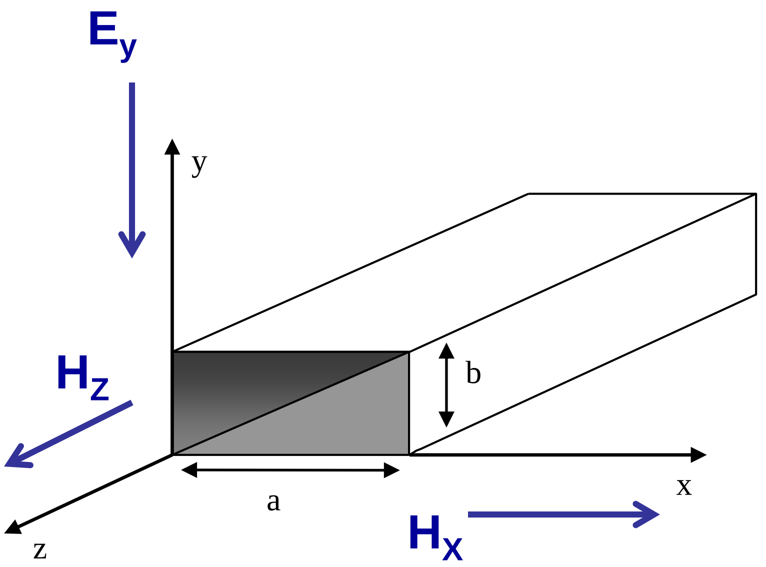
Historical standard design

Rectangular Waveguide TE_{10} mode

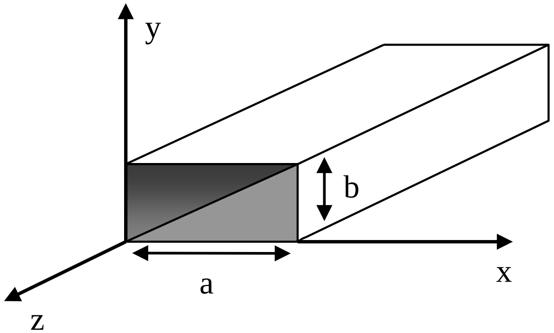
$$H_z = H_0 \cos\frac{\pi x}{a} e^{-jk_z z}$$

$$E_y = -\frac{j\omega\mu}{\pi/a} H_0 \sin\frac{\pi x}{a} e^{-jk_z z}$$

$$H_x = \frac{jk_z}{\pi/a} H_0 \sin\frac{\pi x}{a} e^{-jk_z z}$$



Rectangular Waveguide TE$_{10}$ mode



or

$$\mathrm{E_y} = E_0 \sin\frac{\pi x}{a} e^{-jk_z z}$$

where

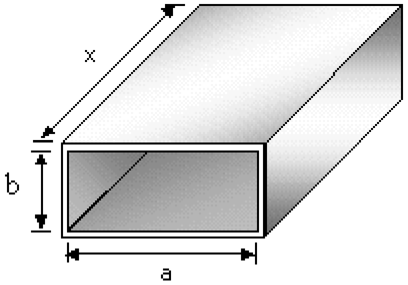
$$E_0 = -\frac{j\omega\mu}{\pi/a} H_0$$

$$\mathrm{H_x} = \frac{-k_z}{\omega\mu} E_0 \sin\frac{\pi x}{a} e^{-jk_z z} \qquad (5.23)$$

$$\mathrm{H_z} = \frac{\frac{\pi}{a} E_0}{-j\omega\mu} \cos\frac{\pi x}{a} e^{-jk_z z}$$

Example

Example 5.5 & 5.6



Design a Rectangular waveguide with 10 GHz (λ=3cm) at Mid-Band and $b = \frac{a}{2}$

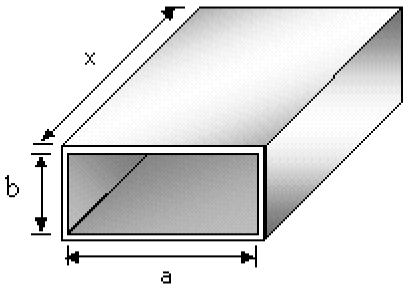
$$\text{BW} \Rightarrow \frac{3\times10^8}{2a} < f < \frac{3\times10^8}{a}$$

$$\text{let } 10\times10^9 = \frac{1}{2}\left[\frac{3\times10^8}{2a} + \frac{3\times10^8}{a}\right] \Rightarrow \boxed{a = 2.25 \text{ cm}}$$

$$b = \frac{a}{2} \Rightarrow \boxed{b = 1.125 \text{ cm}}$$

Example

(Cont. e.g 5.5 & 5.6)



Max power handling $\Rightarrow P = \dfrac{E_0^{\ 2}ab}{4Z_{\mathrm{TE}}}$

where $Z_{\mathrm{TE}} = \dfrac{\omega\mu}{k_z}\ [\Omega]$

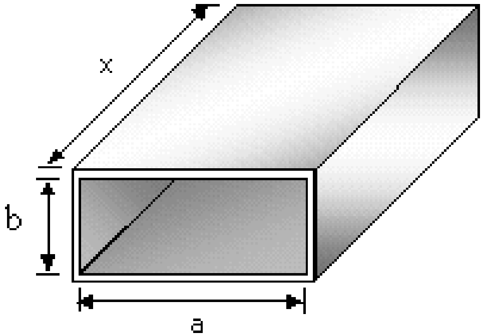
$$E_{BDair} = 2\times10^{6}\left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

take $E_{\max} = 2\times10^{5}\left[\dfrac{\mathrm{V}}{\mathrm{m}}\right]$ (safety factor of 10)

$$Z_{\mathrm{TE}} = \frac{(2\pi\times10^{10})(4\pi\times10^{-7})}{\sqrt{k^2-(\pi/a)^2}} = 505.8\ [\Omega]$$

Example

(Cont. e.g 5.5 & 5.6)



note:

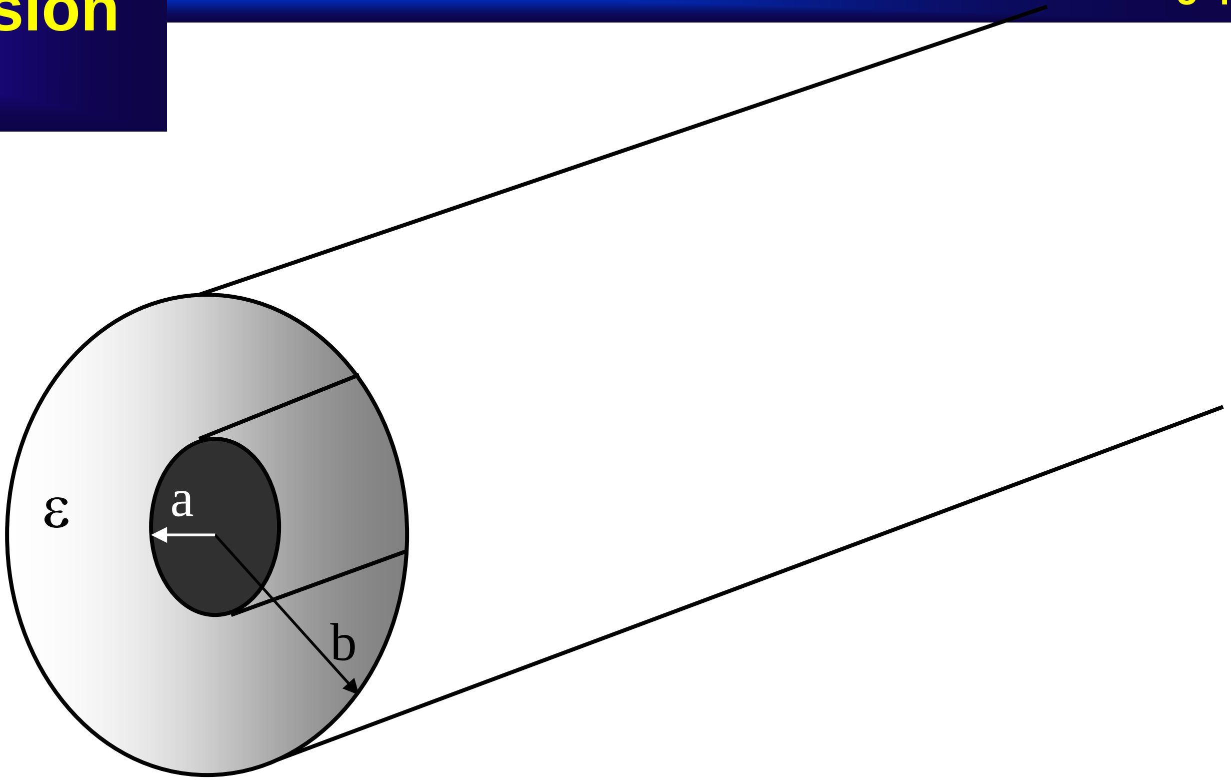
$$k_z = \frac{2\pi\sqrt{1-(\lambda/2a)^2}}{\lambda} = \frac{2\pi\sqrt{1-(3/4.5)^2}}{3}$$

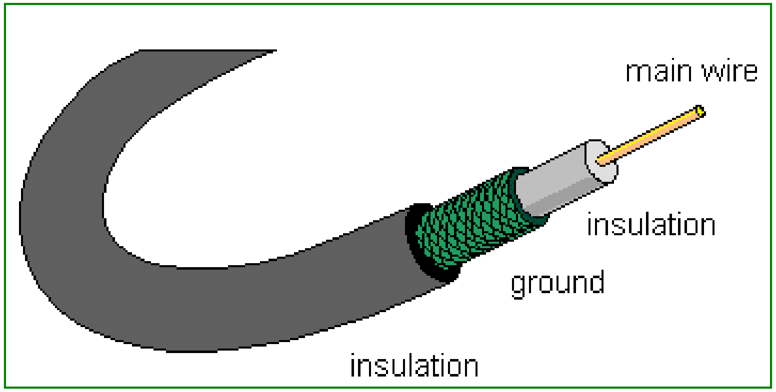
$$k_z = 1.561\ [\text{cm}^{-1}] = \boxed{156.1\ [\text{m}^{-1}]}$$

$$P_{\max} = \frac{(2\times10^5)^2(0.0225)(0.01125)}{4(505.8)} = \boxed{5.004\ [\text{KW}]}$$

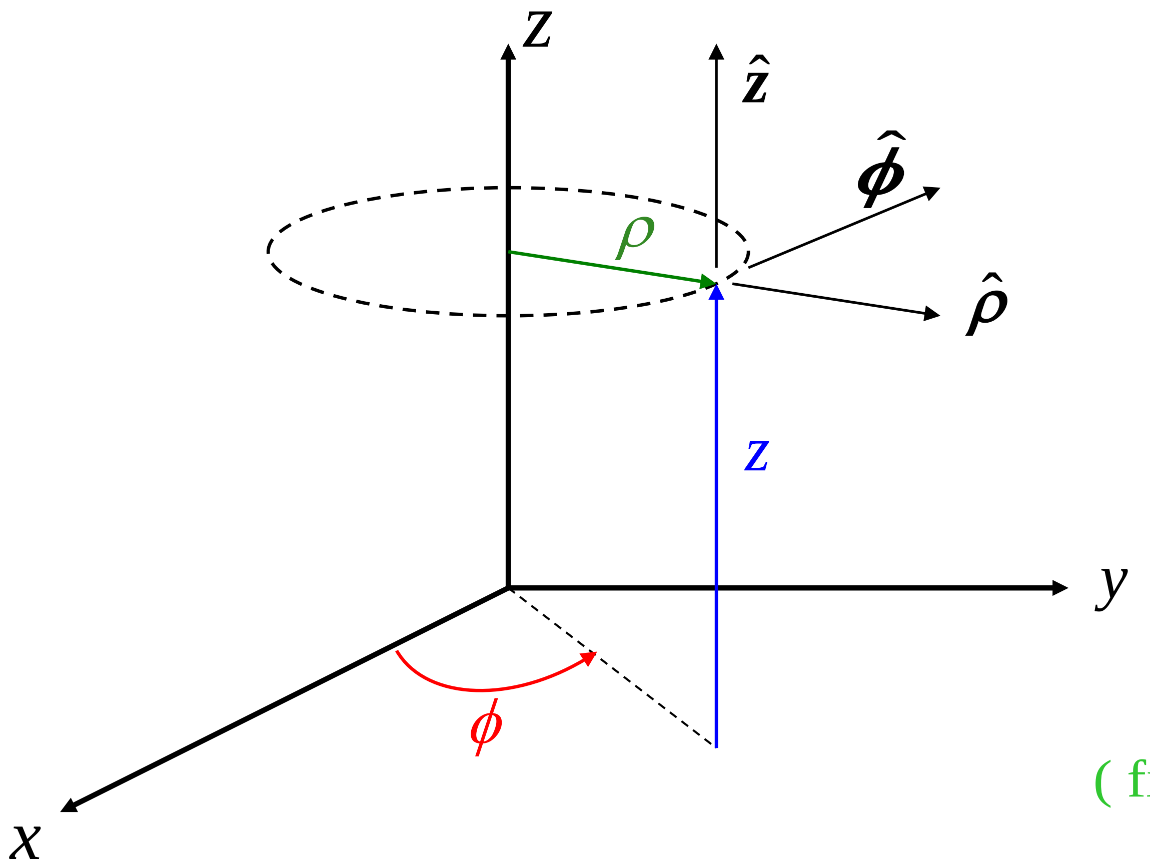
Rectangular Waveguide Simulation

Coaxial Transmission Lines





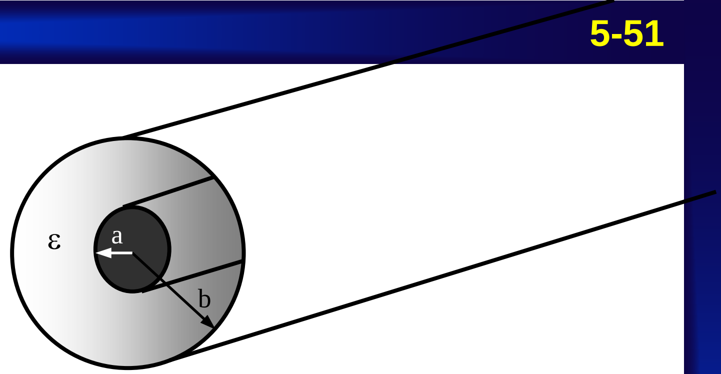
Cylindrical Coordinates

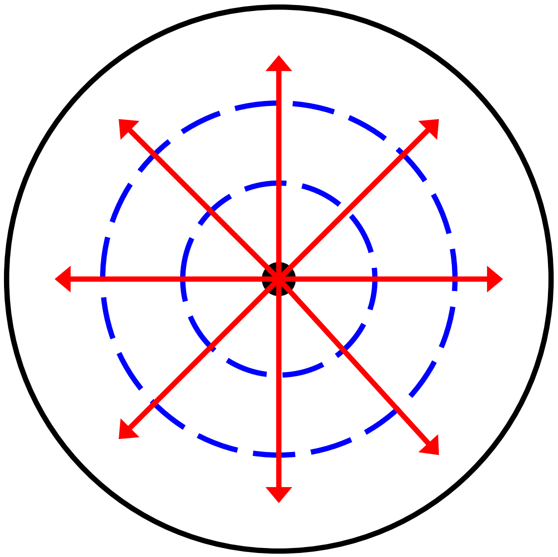


(fig. 5.16)

$$\hat{\boldsymbol{\rho}} \times \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{z}}$$

Coaxial Transmission Lines





Maxwell Eqns. $\Rightarrow$

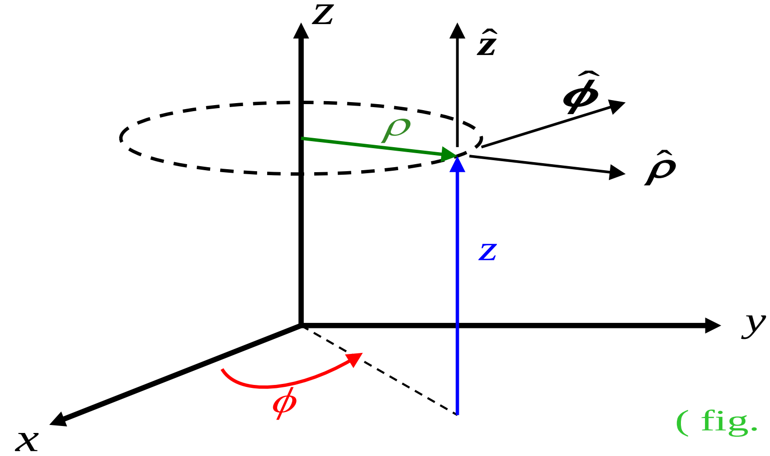
$$\mathbf{E} = \frac{V_0}{\rho} e^{-jkz} \hat{\boldsymbol{\rho}} \quad (5.48a)$$

$$\mathbf{H} = \frac{V_0}{\eta\rho} e^{-jkz} \hat{\boldsymbol{\phi}} \quad (5.48b)$$

$$k = \omega\sqrt{\mu\varepsilon} \quad (5.49a)$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad (5.49b)$$

The "Del" Operator in Cylindrical Coordinates



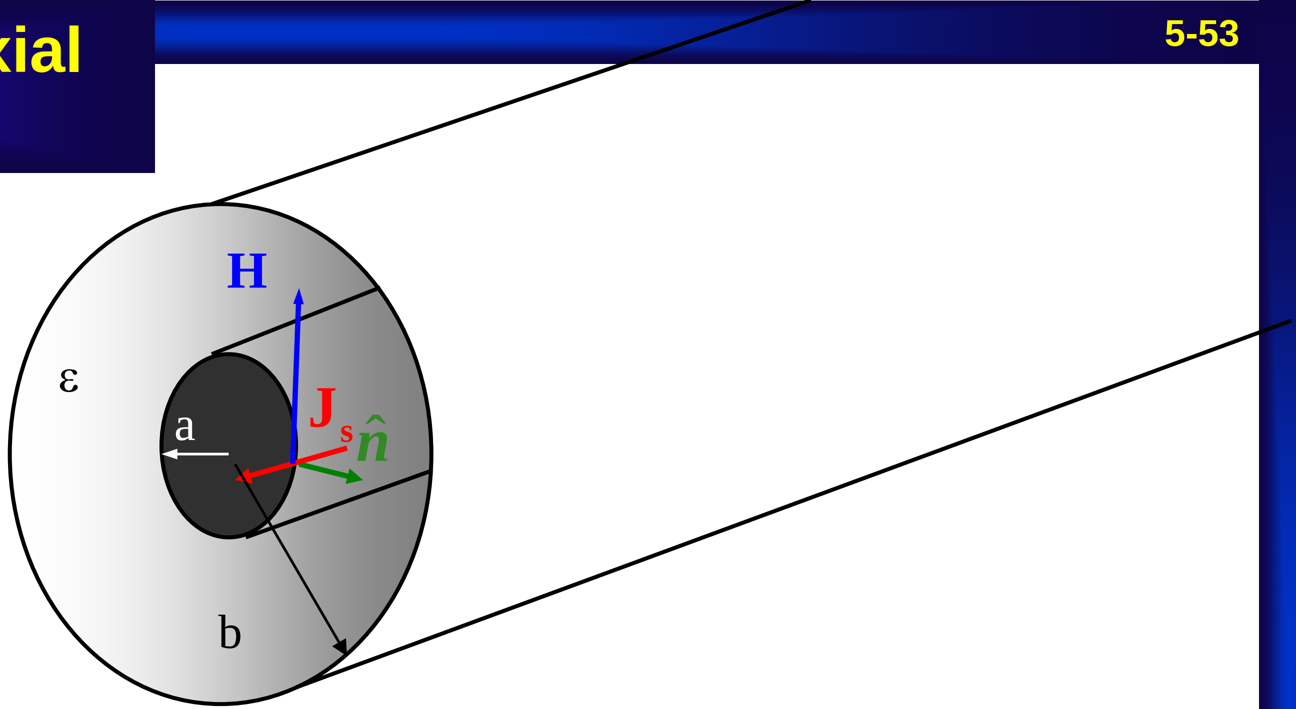
(fig. 5.16)

$$\hat{\rho} \times \hat{\phi} = \hat{z}$$

$$\nabla \bullet \mathbf{A} = \frac{1}{\rho}\frac{\partial\left(\rho A_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial\left(A_\phi\right)}{\partial \phi} + \frac{\partial A_z}{\partial z} \qquad (5.44)$$

$$\nabla \times \mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial\left(\rho A_\phi\right)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{z} \qquad (5.45)$$

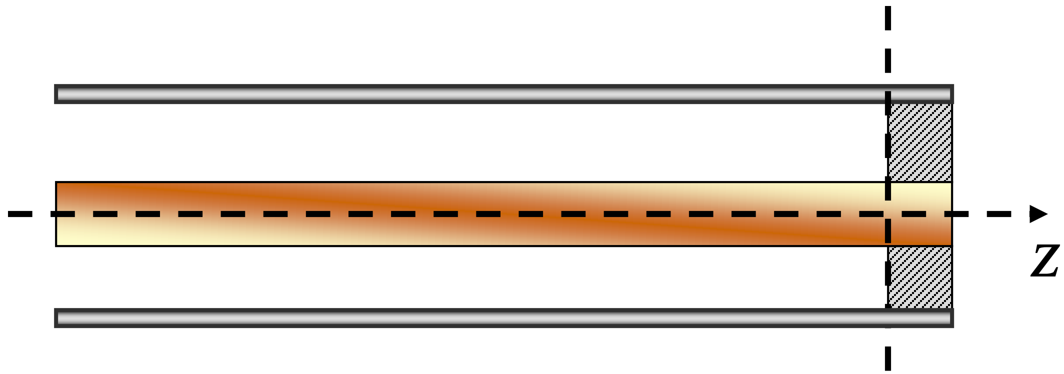
Currents on Coaxial Lines



$$\mathbf{J_s} = \hat{n} \times \mathbf{H}\Big|_{\rho=a} = \hat{\rho} \times \hat{\phi}\, \frac{V_0}{\eta\rho}\, e^{-jkz}\Big|_{\rho=a} = \hat{z}\, \frac{V_0}{\eta a}\, e^{-jkz} \qquad (5.50a)$$

$$I = \int_0^{2\pi} \mathbf{J_s}\Big|_{\rho=a} a\,\partial\phi = \frac{2\pi V_0}{\eta} e^{-jkz} \qquad (5.50b)$$

Short-Circuited Coax



(Fig. 5.18)

for $z < 0$

both Incident Fields and Reflected Fields exist

$$\mathbf{E} = \hat{\rho}\left(\frac{V_0}{\rho}e^{-jkz} + \frac{V_1}{\rho}e^{+jkz}\right)$$

$$\mathbf{H} = \hat{\phi}\left(\frac{V_0}{\rho\eta}e^{-jkz} - \frac{V_1}{\rho\eta}e^{+jkz}\right)$$

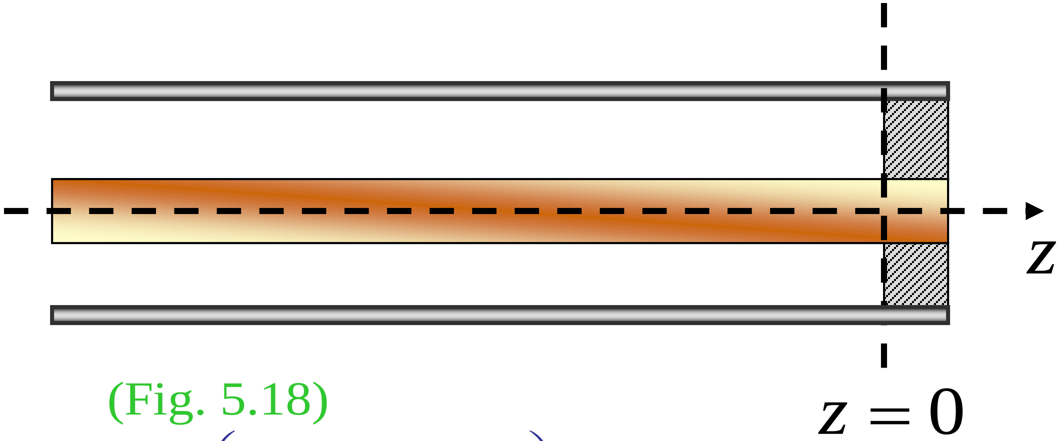
(5.52a)

Negative sign comes from same place as in reflected H-field in Chapter 4

(5.52b)

B.C at $z = 0 \Rightarrow$ tangential $\mathbf{E} = 0 \Rightarrow E_\rho = 0 \Rightarrow V_1 = -V_0$

Short-Circuited Coax



(Fig. 5.18)

$$\mathbf{E} = \hat{\rho}\frac{V_0}{\rho}\left(e^{-jkz} - e^{+jkz}\right) = \hat{\rho}\left(-\frac{2jV_0}{\rho}\sin kz\right) \qquad (5.53a)$$

Similarly,

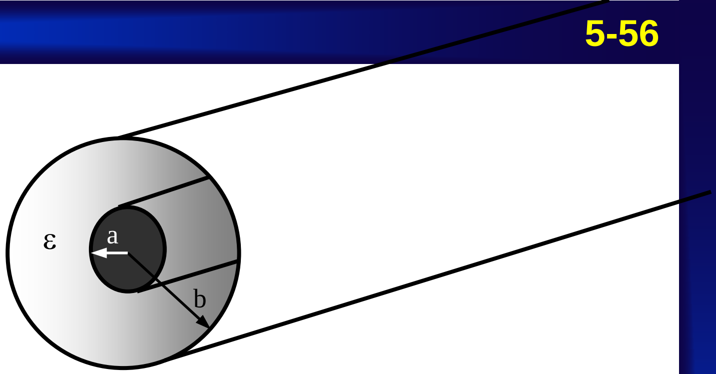
$$\mathbf{H} = \hat{\phi}\frac{2V_0}{\eta\rho}\cos kz \qquad (5.53b)$$

To measure standing wave pattern cut longitudinal slot in outer conductor for moveable probe.

Note: $\mathbf{H}$ totally in $\hat{\phi}$ direction so $\mathbf{J}_s$ totally in $\hat{z}$ direction

Example

(Example 5.8)



Calculate the transmitted power along a coaxial line.

$$\mathbf{E} = \frac{V_0}{\rho} e^{-jkz} \hat{\boldsymbol{\rho}}$$

$$\mathbf{H} = \frac{V_0}{\eta\rho} e^{-jkz} \hat{\boldsymbol{\phi}}$$

TEM mode in a coaxial line

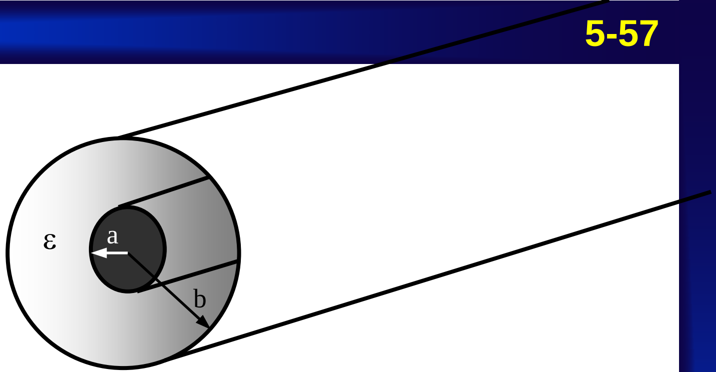
$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right] = \frac{1}{2}\mathrm{Re}\left\{\frac{V_0}{\rho}e^{-jkz}\hat{\boldsymbol{\rho}} \times \frac{V_0}{\eta\rho}e^{jkz}\hat{\boldsymbol{\phi}}\right\} = \hat{\mathbf{z}}\frac{V_0^{\,2}}{\eta\rho^2} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

$$P = \frac{\pi V_0^{\,2}}{\eta}\ln\left(\frac{b}{a}\right) \quad [\mathrm{W}]$$

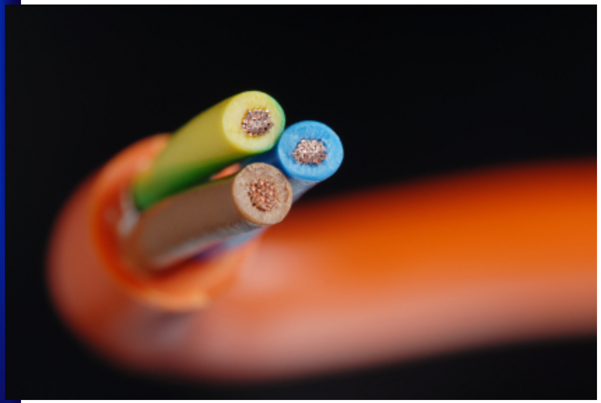
Transmission Lines

2 or more conductors

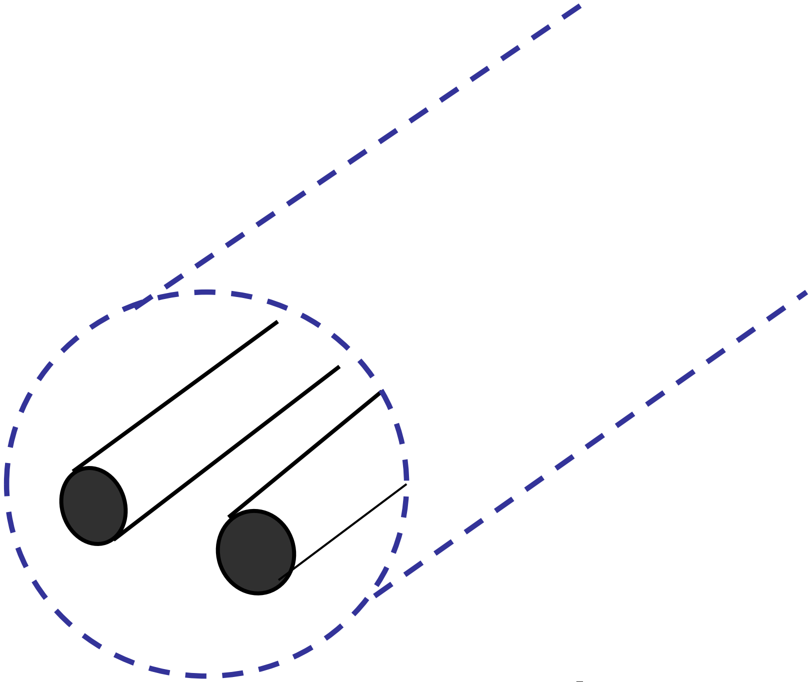
- ✓ TEM possible
- ✓ along with TE and TM



Coaxial



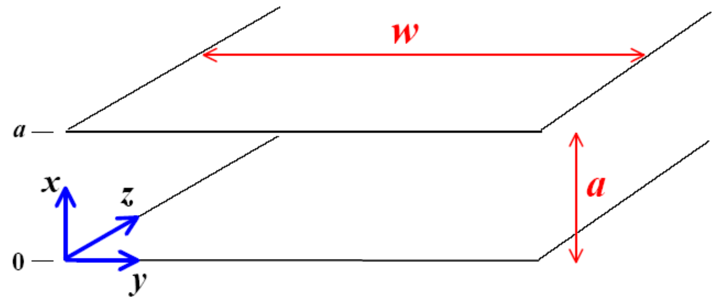
three-wire (shielded)



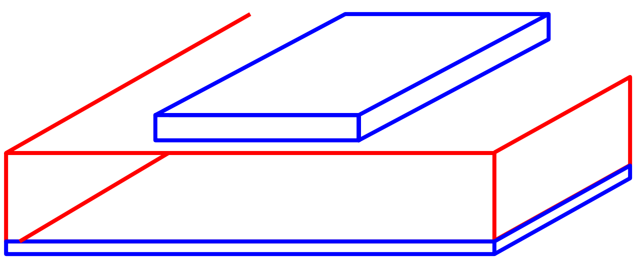
two-wire (shielded)

2 or more conductors

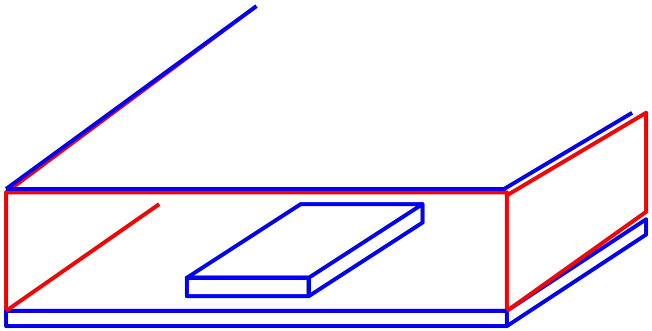
- ✓ TEM possible
- ✓ along with TE and TM



Parallel Plate



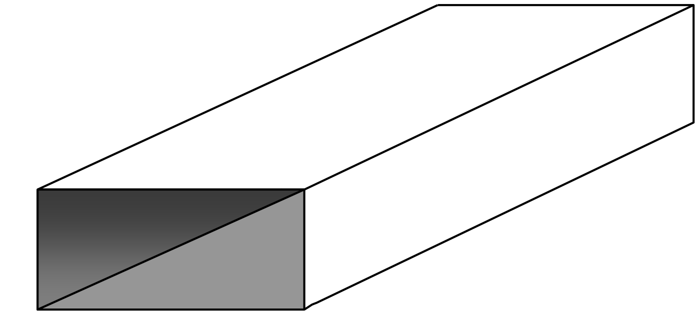
Microstrip



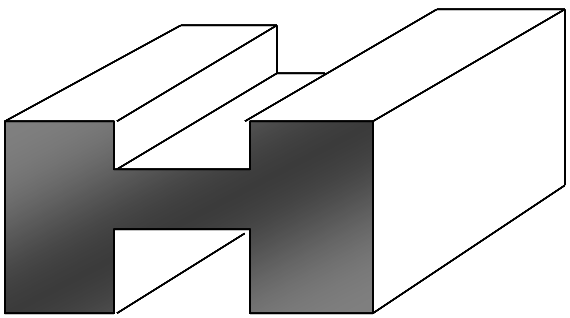
Stripline

Conductor   Dielectric

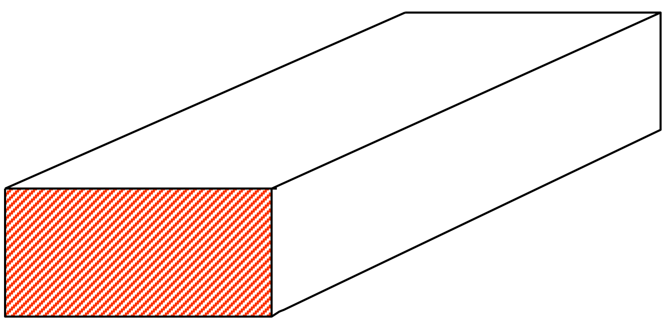
Waveguides



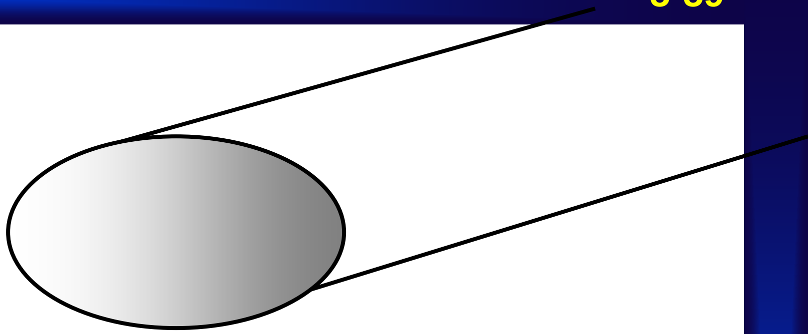
Rectangular



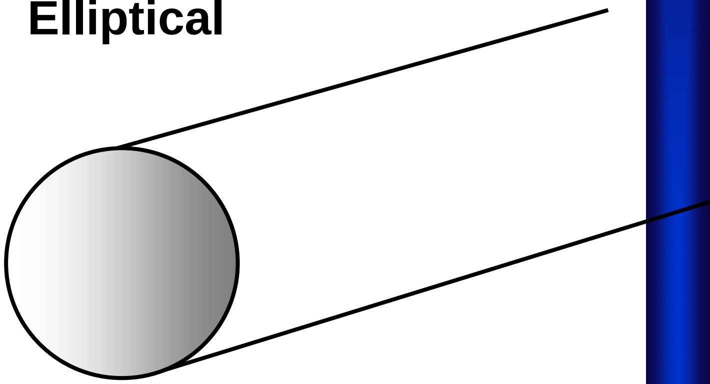
Ridged



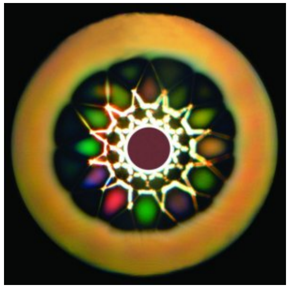
Dielectric Slab



Elliptical

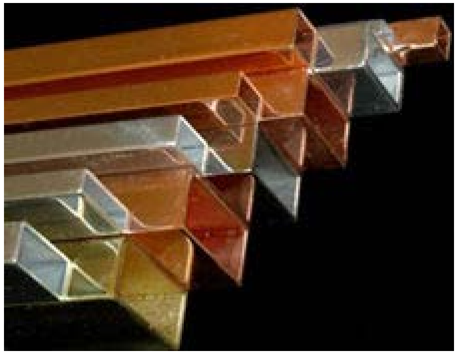


Circular



http://optics.org

Optical Fiber



Rectangular

Bends

Circular

Elliptical