Chapter 6
Transmission Lines

ECE 3317
Dr. Stuart A. Long

General Definitions

p.133

Voltage $\quad \mathrm{V}(z) = \alpha_1 \int_{C_t} \mathbf{E} \bullet d\boldsymbol{s} \quad (C_t \perp z) \quad (a)$

Current $\quad \mathrm{I}(z) = \alpha_2 \oint_{C_0} \mathbf{H} \bullet d\boldsymbol{s} \quad (C_0 \text{ closed}) \quad (b)$

General Definitions

p.133

Power

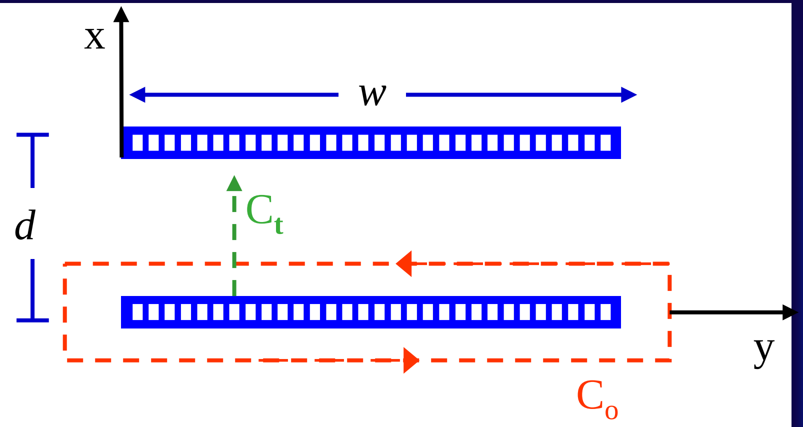
$$\frac{1}{2}\mathrm{Re}\left[V(z)\,I^*(z)\right] = \int_A \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]\bullet\hat{\boldsymbol{z}}\; dA \qquad (c)$$

Characteristic Impedance

$$Z_0 = \frac{V(z)}{I(z)}$$

infinite line with with no reflection

Parallel Plate Waveguide (TEM mode)



The Transverse Electromagnetic Fields in a Parallel Waveguide are approximately as follows:

$$\mathbf{E} = \hat{\boldsymbol{x}}\, E_0 e^{-jkz} \qquad 6.1a$$

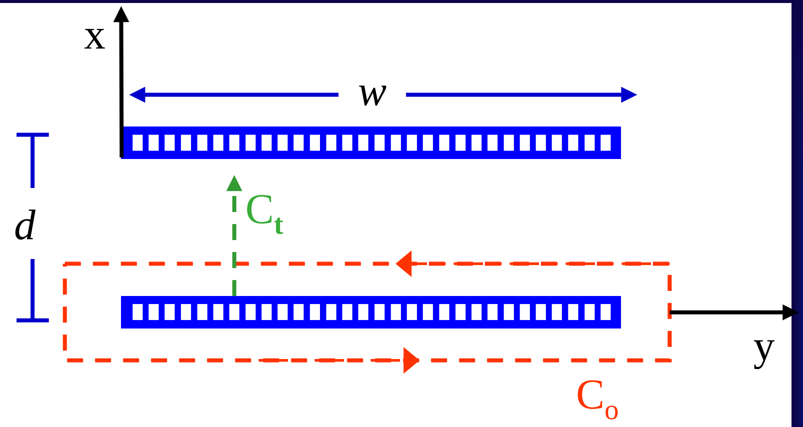
$$\mathbf{H} = \hat{\boldsymbol{y}}\, \frac{E_0}{\eta} e^{-jkz} \qquad 6.1b$$

Using the General Definitions

$$\mathrm{V}(z) = \alpha_1 \int_0^d \mathbf{E} \bullet \hat{\boldsymbol{x}}\, dx = \alpha_1 E_0\, d\, e^{-jkz} \qquad 6.2a$$

$$\mathrm{I}(z) = \alpha_2 \int_0^w \mathbf{H} \bullet \hat{\boldsymbol{y}}\, dy = \alpha_2 \frac{E_0 w}{\eta} e^{-jkz} \qquad 6.2b$$

Parallel Plate Waveguide (TEM mode)



The time-average power transmitted is given by

$$P_t = \int_A \frac{E_0^2}{2\eta} \hat{z} \circ \hat{z} \, dA = \frac{E_0^2}{2\eta} wd$$

equate

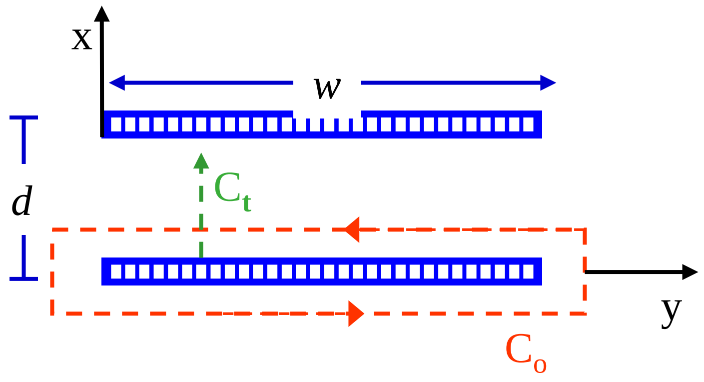
$$\frac{1}{2}\mathrm{Re}\left[\mathrm{V}(z)\mathrm{I}(z)^*\right] = \frac{E_0^2}{2\eta} wd$$

$$\frac{1}{2}(\alpha_1 E_0 d)\left(\alpha_2 \frac{E_0}{\eta} w\right) = \frac{E_0^2}{2\eta} wd$$

$$\alpha_1\alpha_2 \frac{E_0^2}{2\eta} wd = \frac{E_0^2}{2\eta} wd \Rightarrow \alpha_1\alpha_2 = 1$$

choose $\alpha_1 = 1$ and $\alpha_2 = 1$

Parallel Plate Waveguide (TEM mode)

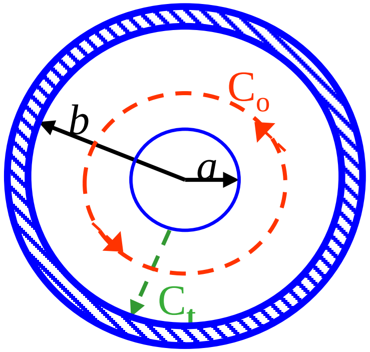


$$V(z) = E_0 d\, e^{-jkz} \qquad 6.3a$$

$$I(z) = \frac{E_0 w}{\eta} e^{-jkz} \qquad 6.3b$$

$$Z_0 = \eta \frac{d}{w} \ [\Omega] \qquad 6.4$$

Coaxial Line



The fields inside a coaxial line for the TEM mode are given by

$$\mathbf{E} = \hat{\rho}\frac{V_0}{\rho}e^{-jkz} \qquad 6.5a$$

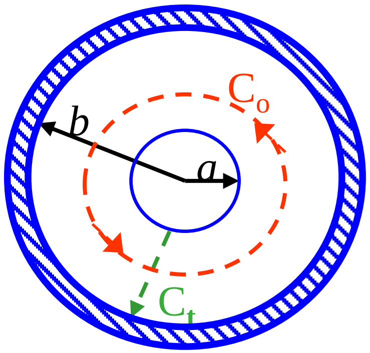
$$\mathbf{H} = \hat{\phi}\frac{V_0}{\eta\rho}e^{-jkz} \qquad 6.5b$$

Using the General Definitions

$$\mathrm{V}(z) = \alpha_1\int_a^b \mathbf{E}\bullet\hat{\rho}\,d\rho = \alpha_1 V_0 \ln\frac{b}{a}e^{-jkz} \qquad 6.6a$$

$$\mathrm{I}(z) = \alpha_2\oint \mathbf{H}\bullet\hat{\phi}\,\rho\,d\phi = \alpha_2\frac{2\pi V_0}{\eta}e^{-jkz} \qquad 6.6b$$

Coaxial Line



The time-average power transmitted is given by

$$P_t = \frac{\pi V_0^2}{\eta} \ln \frac{b}{a}$$

equate

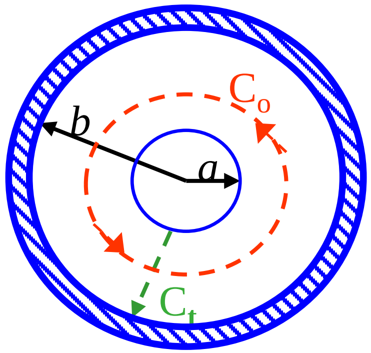
$$\frac{1}{2} \mathrm{Re}\left[\mathrm{V}(z) \mathrm{I}(z)^* \right] = \frac{\pi V_0^2}{\eta} \ln \frac{b}{a}$$

$$\frac{1}{2}\left(\alpha_1 V_0 \ln \frac{b}{a} \right)\left(\alpha_2 \frac{2\pi V_0}{\eta} \right) = \frac{\pi V_0^2}{\eta} \ln \frac{b}{a}$$

$$\alpha_1 \alpha_2 \frac{\pi V_0^2}{\eta} \ln \frac{b}{a} = \frac{\pi V_0^2}{\eta} \ln \frac{b}{a} \Rightarrow \alpha_1 \alpha_2 = 1$$

choose $\alpha_1 = 1$ *and* $\alpha_2 = 1$

Coaxial Line



$$V(z) = V_0 \ln\frac{b}{a} e^{-jkz}$$

$$I(z) = \frac{2\pi V_0}{\eta} e^{-jkz}$$

$$Z_0 = \frac{\eta}{2\pi}\ln\frac{b}{a} \quad [\Omega]$$

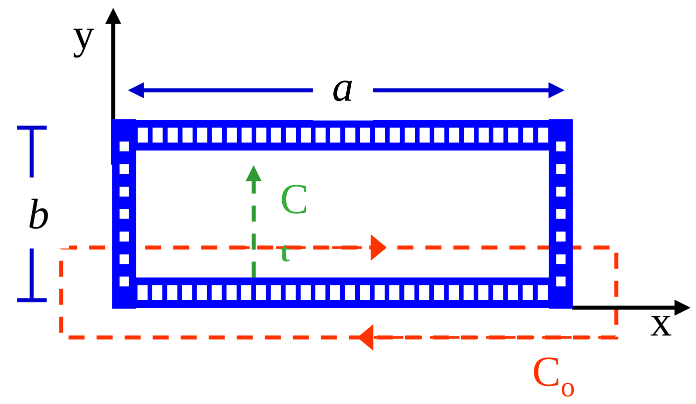
6.7

Example: What is the ratio b/a for an air-filled coax and 50[Ω] line?

$$50 = \frac{377}{2\pi}\ln(b/a)$$

$$\Rightarrow \frac{b}{a} = 2.3$$

Rectangular Waveguide (Dominant TE_{10} mode)



The Electromagnetic fields in a Rectangular Waveguide for the are TE_{10} mode are

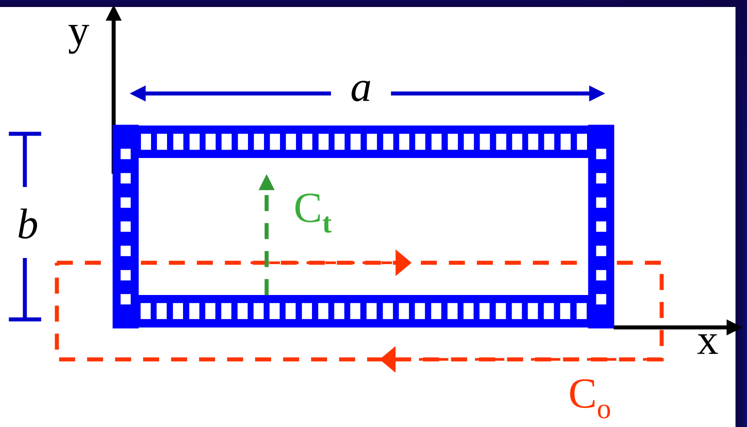
$$E_y = E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8a$$

$$H_x = -\frac{k_z}{\omega\mu} E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8b$$

$$H_z = j\frac{\pi/a}{\omega\mu} E_0 \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8c$$

Rectangular Waveguide (Dominant TE_{10} mode)

Using the General Definitions



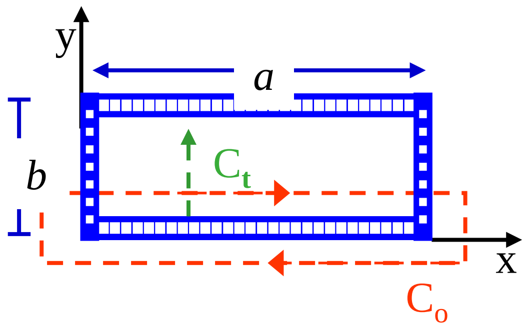
$$V(z) = \alpha_1 \int_0^b E_y \Big|_{x=a/2} dy$$

$$= \alpha_1 E_0\, b\, e^{-jk_z z} \qquad 6.9a$$

$$I(z) = \alpha_2 \oint_{C_0} \hat{\mathbf{x}} \bullet \mathbf{H}\, dx$$

$$= \alpha_2 \frac{2E_0 a k_z}{\pi\omega\mu} e^{-jk_z z} \qquad 6.9b$$

Rectangular Waveguide (Dominant TE_{10} mode)



The time-average power transmitted is given by

$$P_t = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

equate

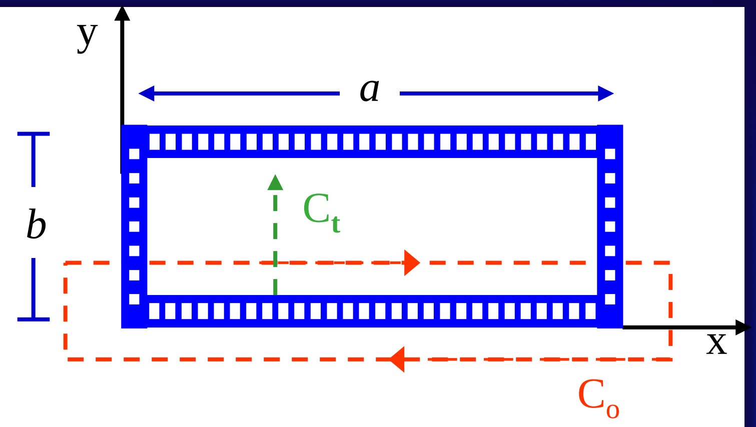
$$\frac{1}{2}\mathrm{Re}\left[\mathrm{V}(z)\mathrm{I}(z)^*\right] = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

$$\frac{1}{2}(\alpha_1 E_0 b)\left(\alpha_2 \frac{2E_0 a k_z}{\pi\omega\mu}\right) = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

$$\alpha_1\alpha_2 \frac{ab}{\pi}\frac{k_z}{\omega\mu}E_0^2 = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2 \Rightarrow \alpha_1\alpha_2 = \frac{\pi}{4}$$

choose $\alpha_1 = \sqrt{\dfrac{ak_z}{2b\omega\mu}}$ **and** $\alpha_2 = \sqrt{\dfrac{\pi^2 b\omega\mu}{8ak_z}} \Rightarrow Z_0 = 1$

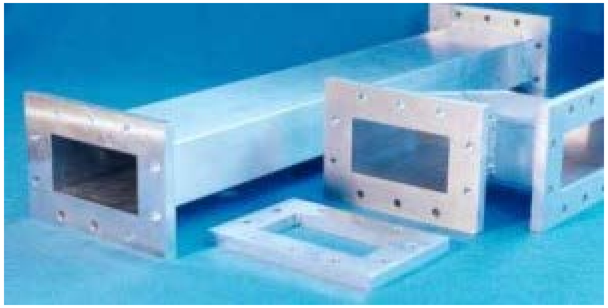
Rectangular Waveguide (Dominant TE_{10} mode)



$$V(z) = \sqrt{\frac{a k_z}{2 b \omega \mu}} E_0 \, b \, e^{-j k_z z}$$

$$I(z) = \sqrt{\frac{\pi^2 b \omega \mu}{8 a k_z}} \frac{2 E_0 a k_z}{\pi \omega \mu} e^{-j k_z z}$$

$$Z_0 = 1 \ [\Omega]$$



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Transmission Line Equations

(Parallel plate waveguide)

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\frac{\partial}{\partial z} E_x = -j\omega\mu H_y$$

$$\mathrm{V}(z) = E_0 d e^{-jkz} = d\, E_x$$

$$\frac{\partial \mathrm{V}}{\partial z} = -j\omega \mathrm{L} \mathrm{I}$$

$$\mathrm{L} = \frac{\mu d}{w} \left[\frac{\mathrm{H}}{\mathrm{m}} \right]$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

$$-\frac{\partial}{\partial z} H_y = j\omega\varepsilon E_x$$

$$\mathrm{I}(z) = \frac{E_0 w}{\eta} e^{-jkz} = w H_y$$

$$\frac{\partial \mathrm{I}}{\partial z} = -j\omega \mathrm{C} \mathrm{V}$$

$$\mathrm{C} = \frac{\varepsilon w}{d} \left[\frac{\mathrm{F}}{\mathrm{m}} \right]$$

Transmission Line Equations (Parallel plate waveguide)

$$\frac{\partial^2 V}{\partial z^2} + \omega^2 LCV = 0 \quad \text{wave equation} \qquad 6.15$$

$$V = V_+ e^{-jkz} + V_- e^{+jkz} \qquad 6.16$$

$$I = \frac{1}{Z_0}\left[V_+ e^{-jkz} - V_- e^{+jkz} \right] \qquad 6.17$$

$$k = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad 6.18$$

Standing Waves on Terminated Transmission Lines

$$V(z) = V^{+} e^{-jkz} + V^{-} e^{+jkz} \quad 6.22$$

$$I(z) = \frac{V^{+}}{Z_0} e^{-jkz} - \frac{V^{-}}{Z_0} e^{+jkz} \quad 6.23$$

Impedance $Z(z) = \dfrac{V(z)}{I(z)}$

$$Z_n(z) = \frac{Z(z)}{Z_0} = \frac{V^{+} e^{-jkz} + V^{-} e^{+jkz}}{V^{+} e^{-jkz} - V^{-} e^{+jkz}} \quad 6.24$$

(note $Z(0) = Z_L$)

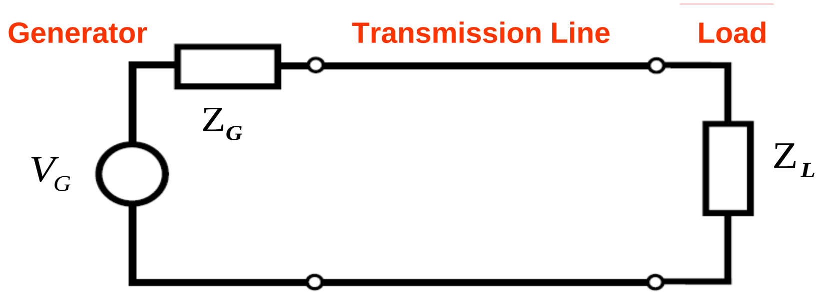
Reflection coefficient

$$\Gamma_L = \frac{V^-}{V^+}$$

With the reflection coefficient now defined, we can rewrite 6.24 as

$$Z_n(z) = \frac{e^{-jkz} + \Gamma_L e^{+jkz}}{e^{-jkz} - \Gamma_L e^{+jkz}}$$

Standing Waves on Terminated Transmission Lines



Evaluating Z_n at the Load ($z = 0$)

$$Z_{nL}(z=0) = \frac{Z(z=0)}{Z_0} = \frac{1+\Gamma_L}{1-\Gamma_L} \qquad \left(\text{or } \Gamma_L = \frac{Z_{nL}-1}{Z_{nL}+1} \right) \qquad 6.25$$

Evaluating Z_n at a point along line ($z = -l$)

$$Z_n(z=-l) = \frac{Z(z=-l)}{Z_0} = \frac{Z_L + jZ_0 \tan kl}{Z_0 + jZ_L \tan kl} \qquad 6.27$$

Standing Waves on Terminated Transmission Lines

$$\left|V_{(z)}\right| = \left|V^{+}\right|\left|1+\Gamma_L e^{j2kz}\right|$$

with $\Gamma_L = \left|\Gamma_L\right| e^{j\varphi}$

$$\left|V_{(z)}\right| = \left|V^{+}\right|\left|1+\left|\Gamma_L\right| e^{j(\varphi+2kz)}\right|$$

V_{max} when $\varphi + 2kz = 0\ , -2\pi, \ldots$

V_{min} when $\varphi + 2kz = -\pi\ , -3\pi, \ldots$

Standing Wave Pattern

$$\left|V_{(z)}\right| = \left|V^{+}\right| \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

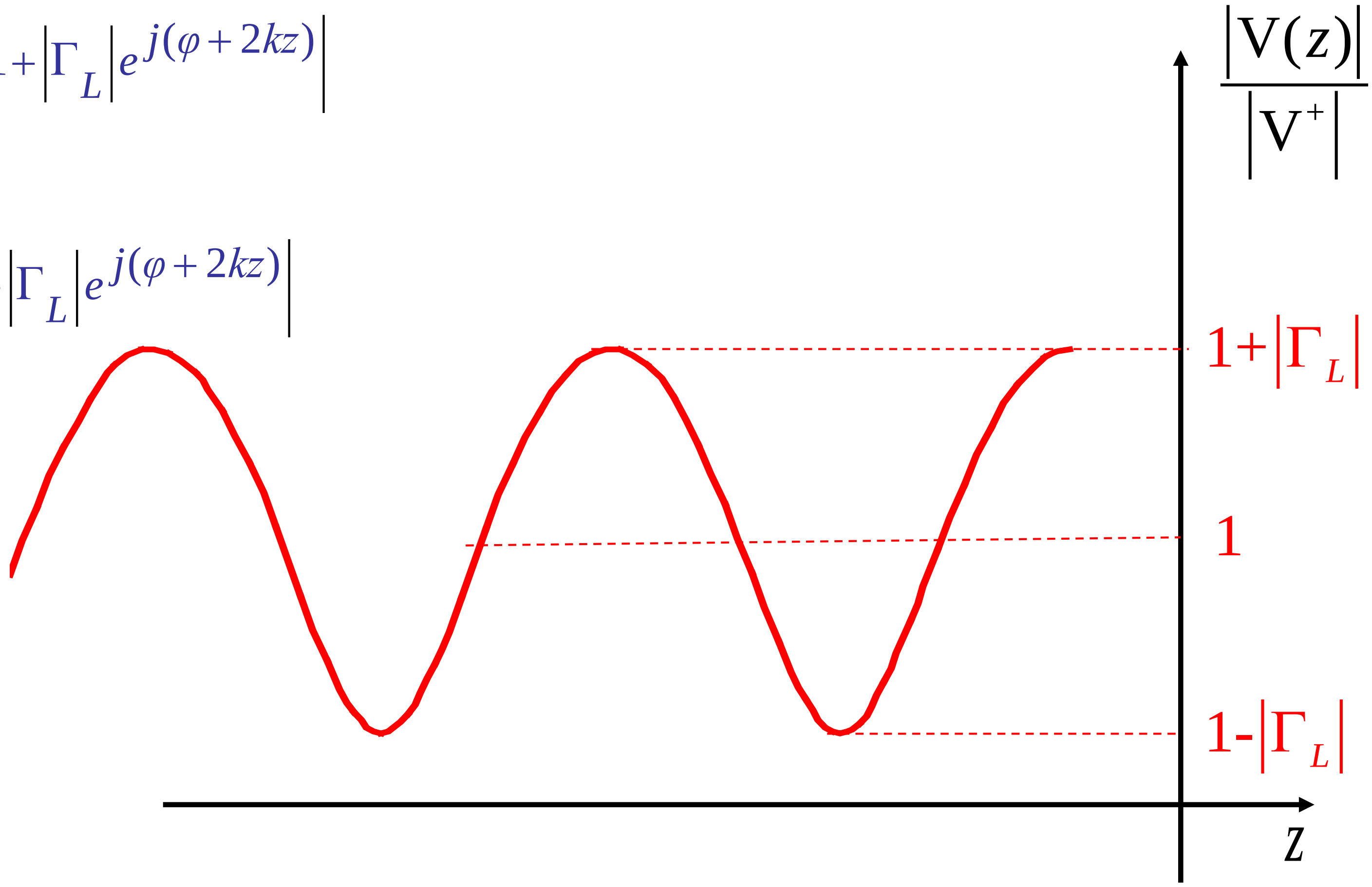


Fig. 6.6

Crank Diagram

$$\left|V_{(z)}\right| = \left|V^+\right| \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

$$\frac{\left|V_{(z)}\right|}{\left|V^+\right|} = \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

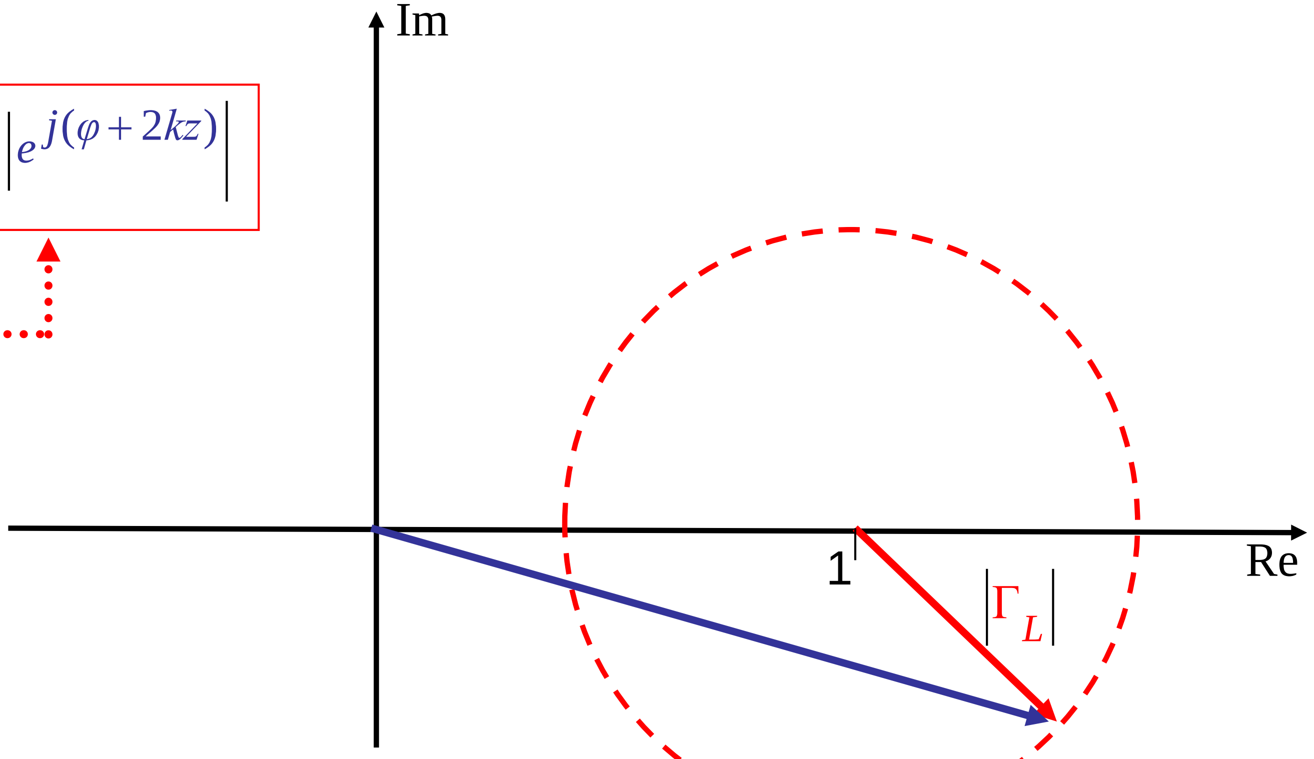
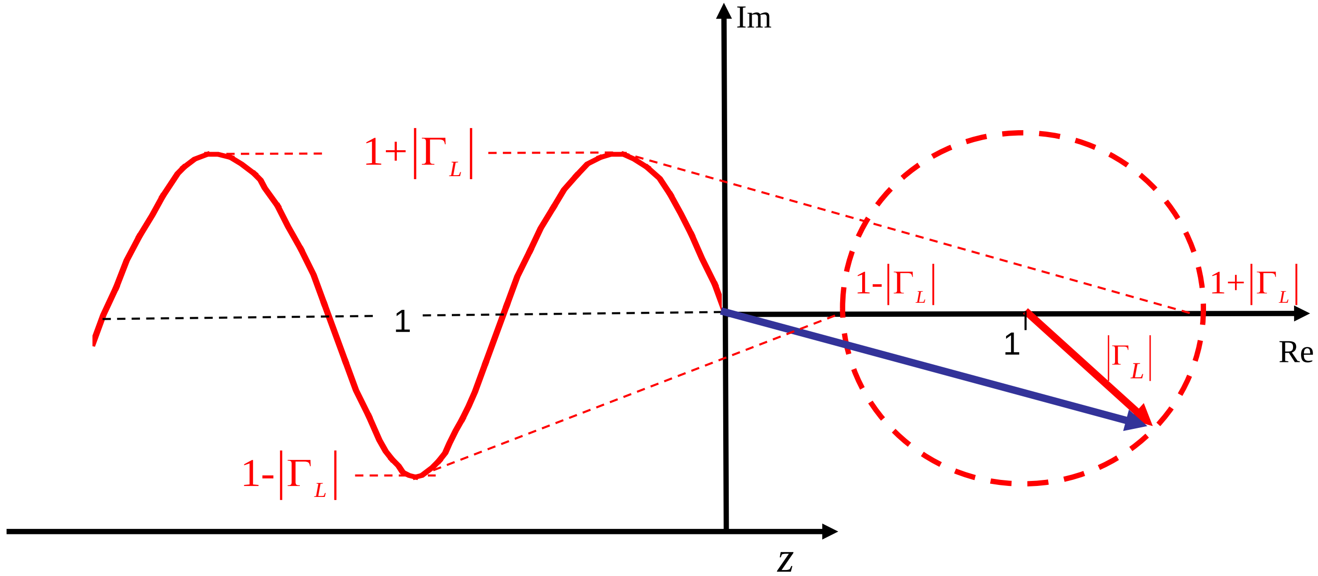


Fig. 6.6

$$\left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

Crank Diagram



$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1+\left|\Gamma_L\right|e^{j(\varphi+2kz)}\right|$$

Standing Wave Ratio (SWR or VSWR)

$$[\text{VSWR}] = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$[\text{VSWR}] = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \quad 6.29$$

$$|\Gamma_L| = \frac{[\text{VSWR}]-1}{[\text{VSWR}]+1} \quad 6.30$$

To determine $(\angle\Gamma_L)$ or Z_L also need position of V_{min}

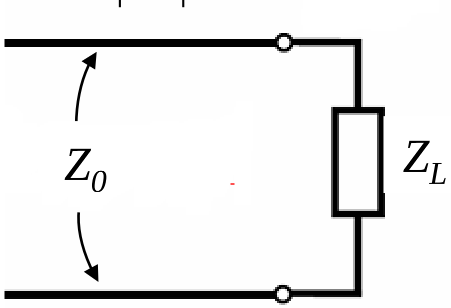
Note:

Power $\sim V^2 \Rightarrow \frac{P^-}{P^+} = |\Gamma_L|^2$

Power to load $\sim 1-|\Gamma_L|^2$

Ex. 6.6

$Z_L = 17.4 - j30[\Omega]$ and $Z_0 = 50[\Omega]$



$$\Gamma_L = \frac{Z_{nL} - 1}{Z_{nL} + 1} = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.24 - j.55$$

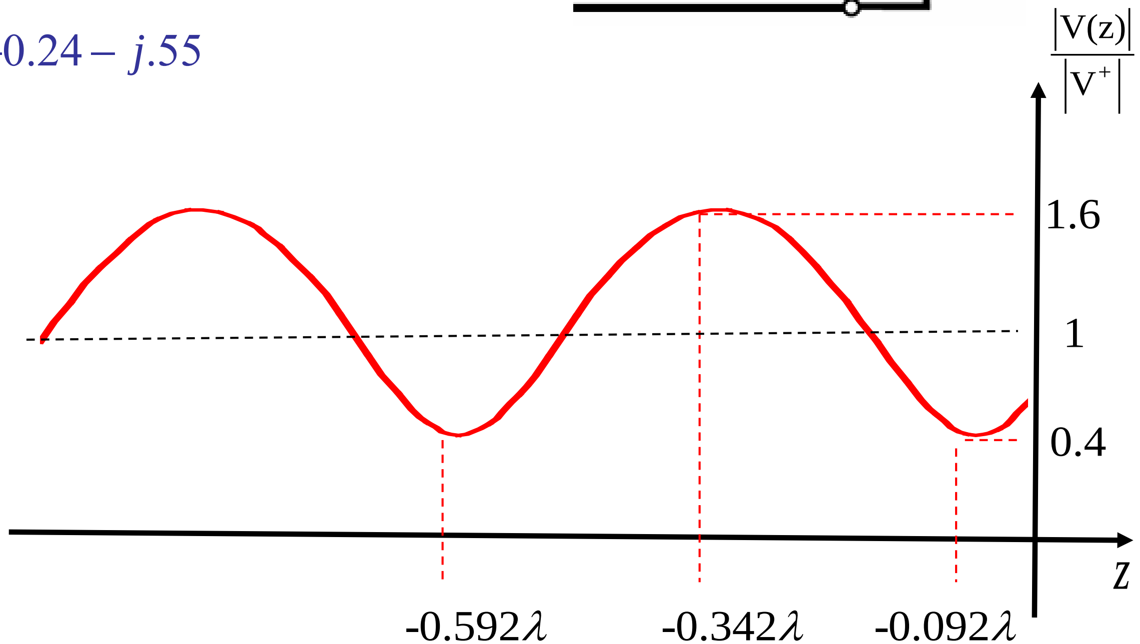
or

$$\Gamma_L = 0.6e^{-j1.99}$$

where

$$|\Gamma_L| = 0.6$$

$$\phi = \angle\Gamma_L = 1.99$$

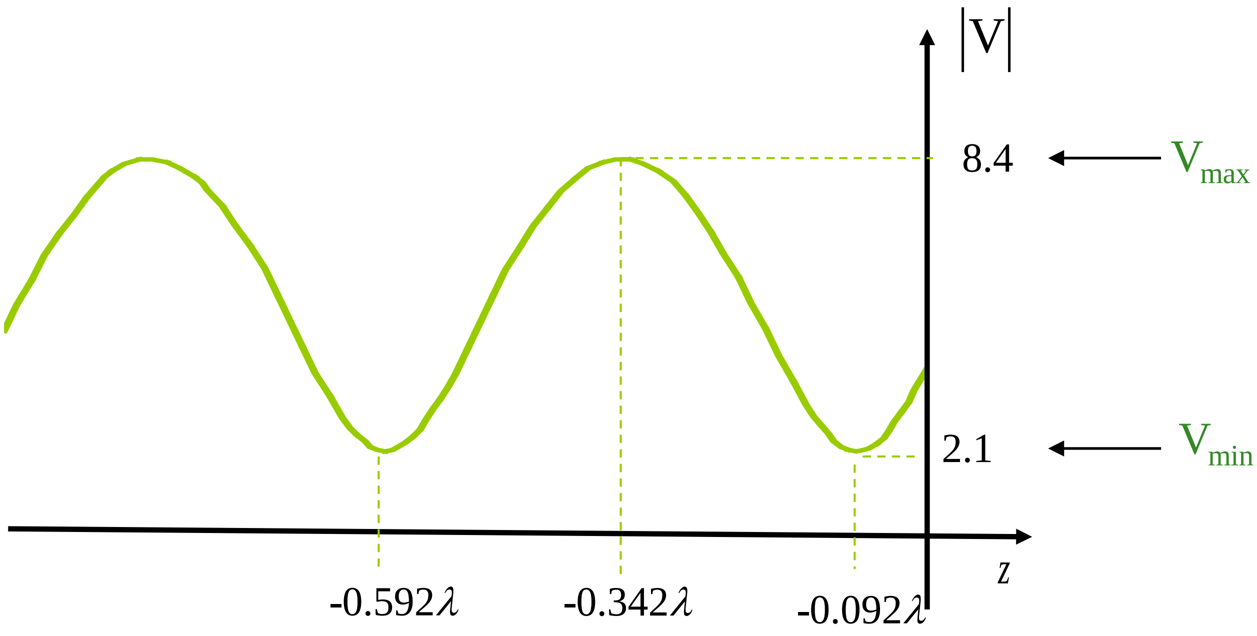


V_{max} when $\varphi + 2kz = 0, -2\pi, \ldots$

V_{min} when $\varphi + 2kz = -\pi, -3\pi, \ldots$

Ex. 6.6 V_{min} where $\phi + 2k\, z_m = -\pi$

$$z_m = \frac{-\pi - \phi}{2k} = \frac{(-\pi + 1.99)\lambda}{2(2\pi)} = -.092\lambda$$



Ex. 6.6

Reverse problem

$$VSWR = \frac{\text{Vmax}}{\text{Vmin}} = \frac{8.4}{2.1} = 4.0$$

$$\left|\Gamma_L\right| = \frac{VSWR\text{-}1}{VSWR+1} = \frac{4-1}{4+1} = 0.6$$

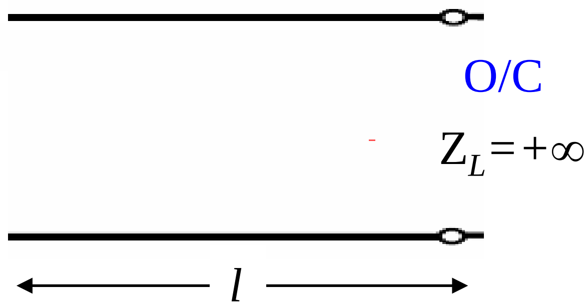
To determine ($\angle\Gamma_L$) or Z_L also need position of V_{min}

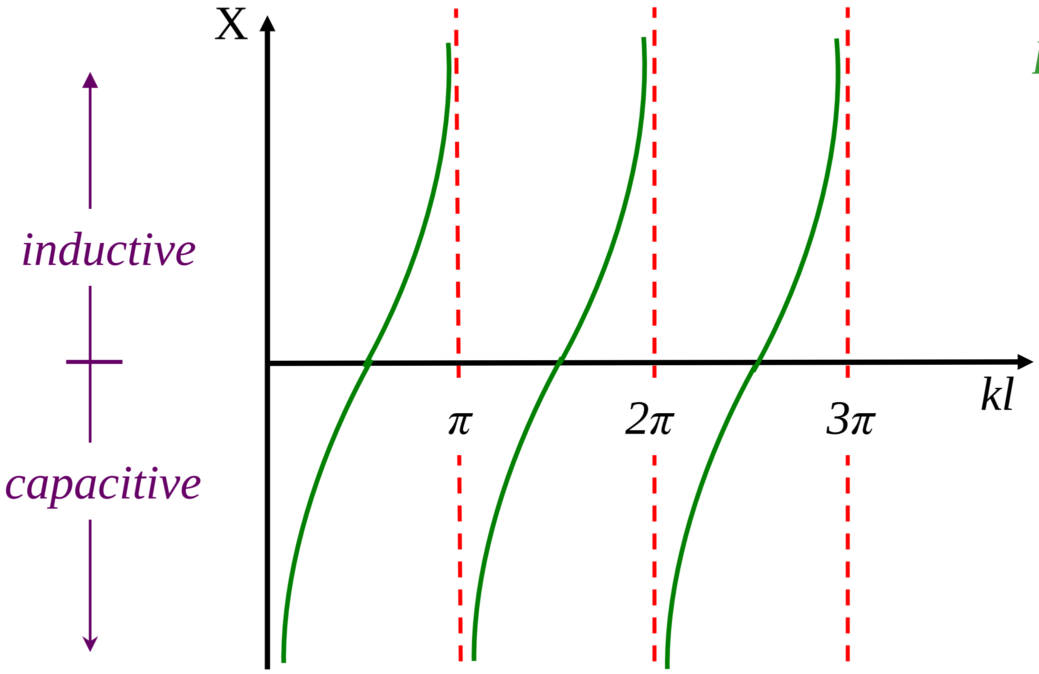
$$\angle\Gamma_L = \phi = -\pi - 2k\,z_m = -\pi - 2\left(\frac{2\pi}{\lambda}\right)\left(-.092\lambda\right) = -1.99$$

$$Z_L = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] = \boxed{17.4 - j30 \quad \Omega}$$

Open Circuit

$$Z(-l) = Z_0 \frac{Z_L + jZ_0 \tan kl}{Z_0 + jZ_L \tan kl} = Z_0 \frac{1}{j \tan kl}$$



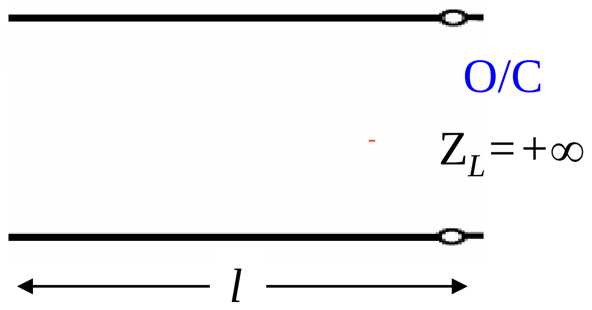


pure imaginary

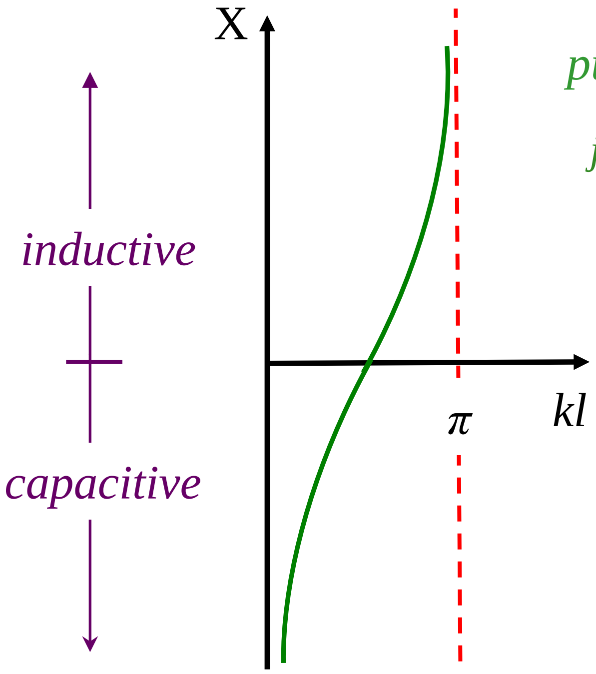
$$jX = -jZ_0 \cot kl$$

If the impedance is purely imaginary, Z = R+jX becomes Z = jX, where X is called the reactance.

Open Circuit



$\lim kl \ll 1$ (small l or low freq)



pure imaginary

$jX = -jZ_0 \cot kl$

Note: at $kl = \frac{\pi}{2} \Rightarrow \left(l = \frac{\lambda}{4} \right)$

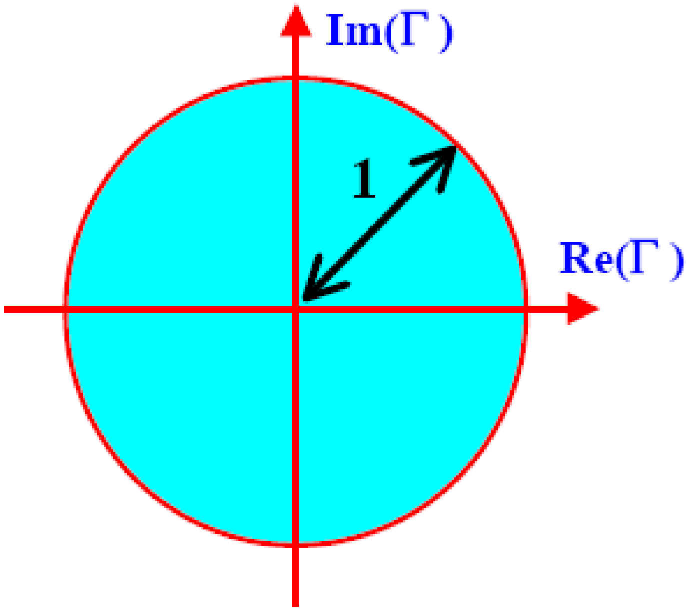
$Z\left(-\lambda/4\right) = 0$ (short circuit)

near $kl \le \pi$ Z is inductive

near $kl \ge 0$ Z is capacitive

Smith Chart

Philip Smith



• Shows the entire universe of complex impedances in one convenient circle.

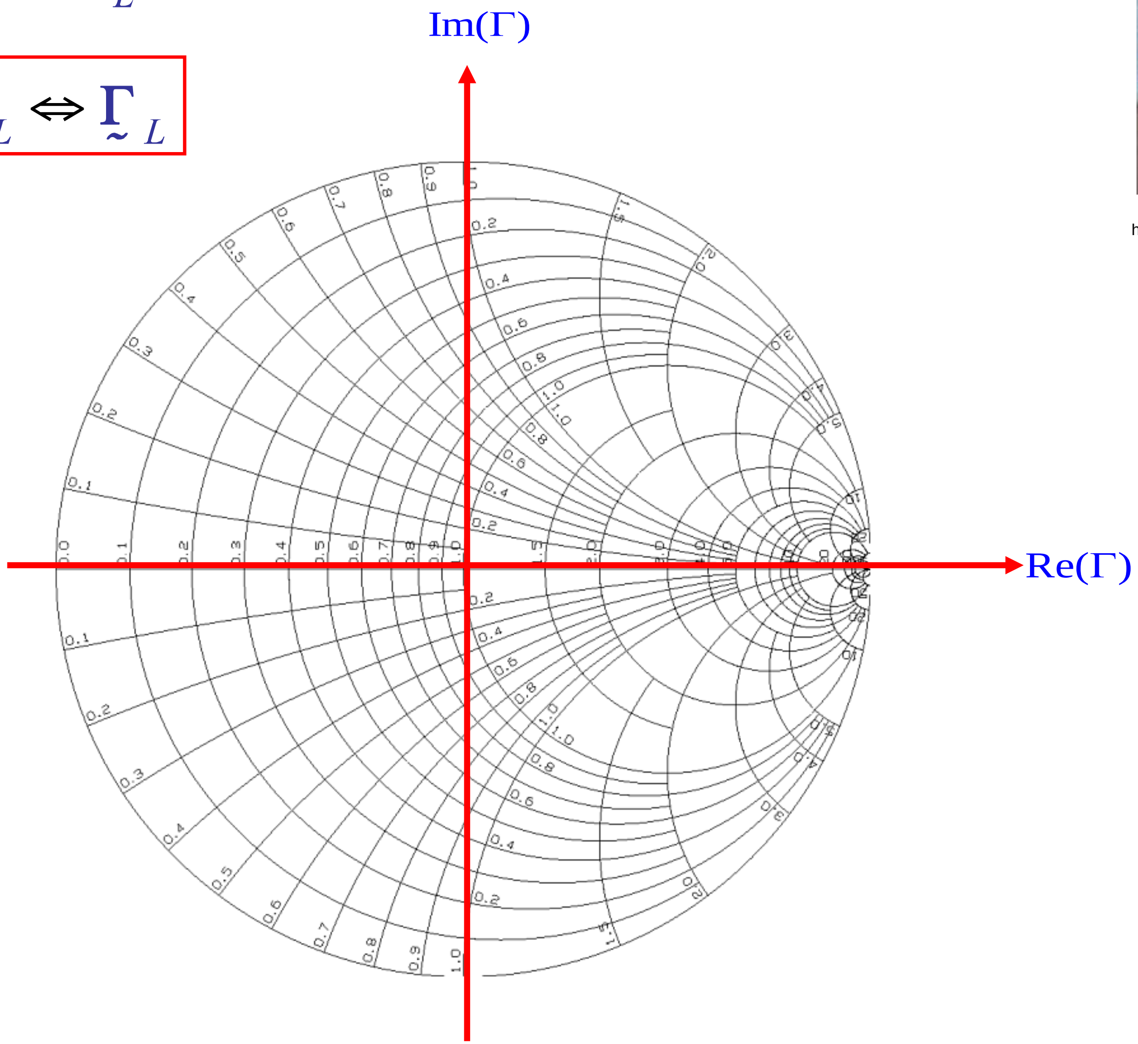
• Invented at Bell Labs by Philip Smith in 1937.

• By 1975 about 9 million copies of his chart sold to microwave engineers all over the world.

• Its usefulness continues to this day as a method of displaying measured and calculated data produced by computer software and modern measurement instruments.

$$Z_{nL} = \frac{1+\Gamma_L}{1-\Gamma_L} = R + jX$$

$$\underset{\sim}{Z}_{nL} \Leftrightarrow \underset{\sim}{\Gamma}_L$$



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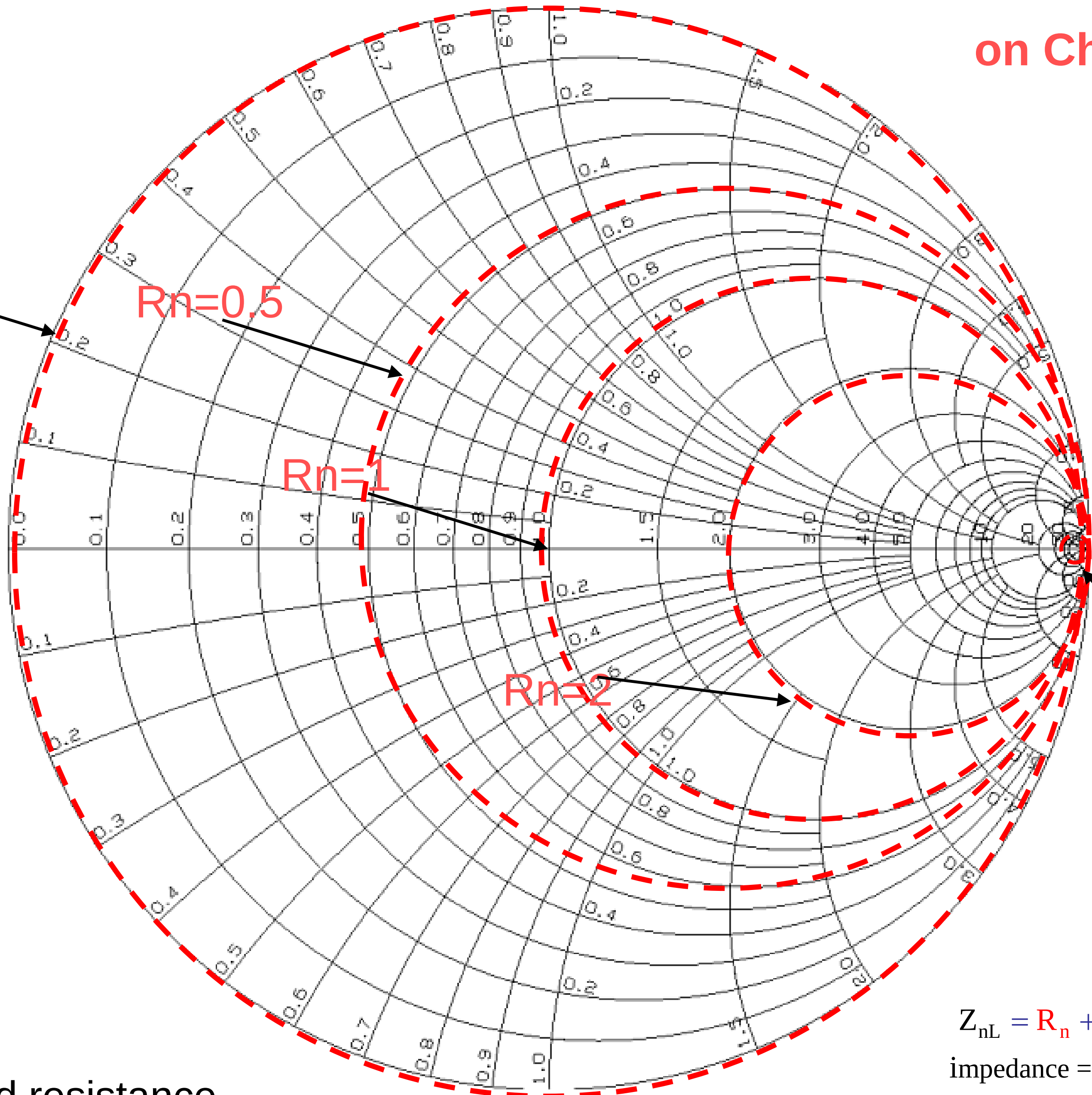
Z Chart

Constant R Circles on Chart



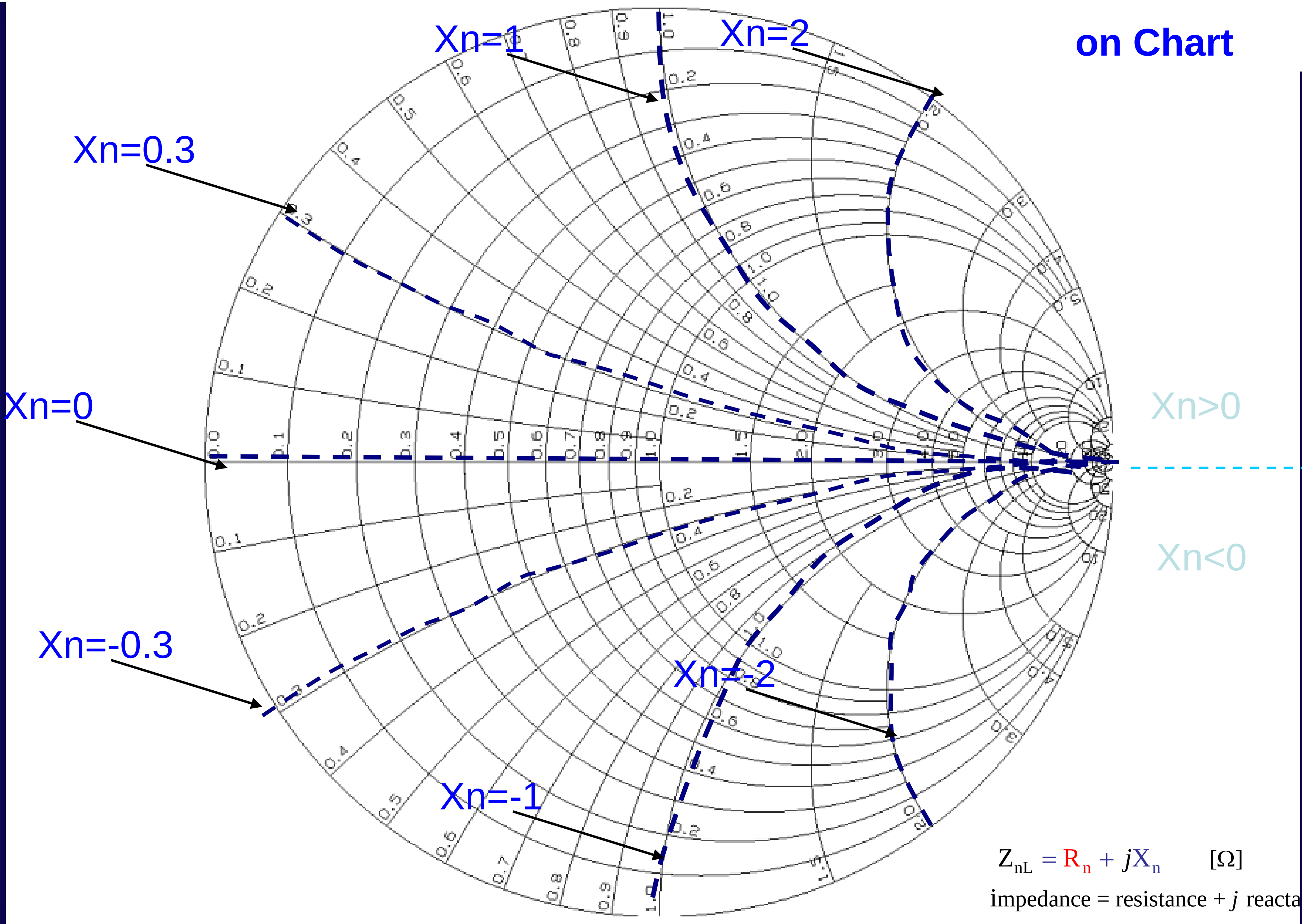
$Z_{nL} = R_n + jX_n \quad [\Omega]$

impedance = resistance + *j* reactance

Rn: normalized resistance

Z Chart

Constant X Arcs on Chart

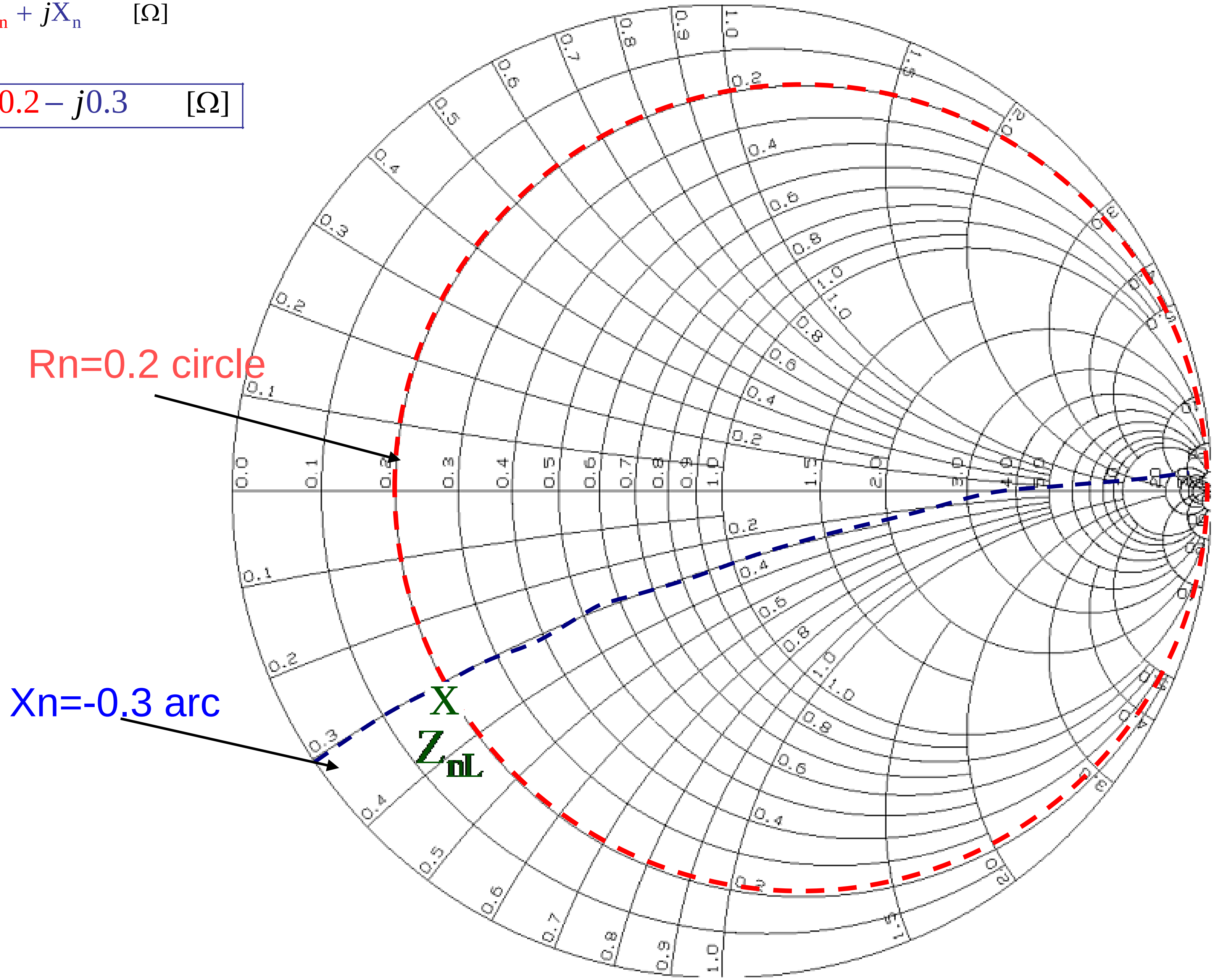


$Z_{nL} = R_n + jX_n \quad [\Omega]$

impedance = resistance + j reactance

$Z_{nL} = R_n + jX_n \quad [\Omega]$

$Z_{nL} = 0.2 - j0.3 \quad [\Omega]$

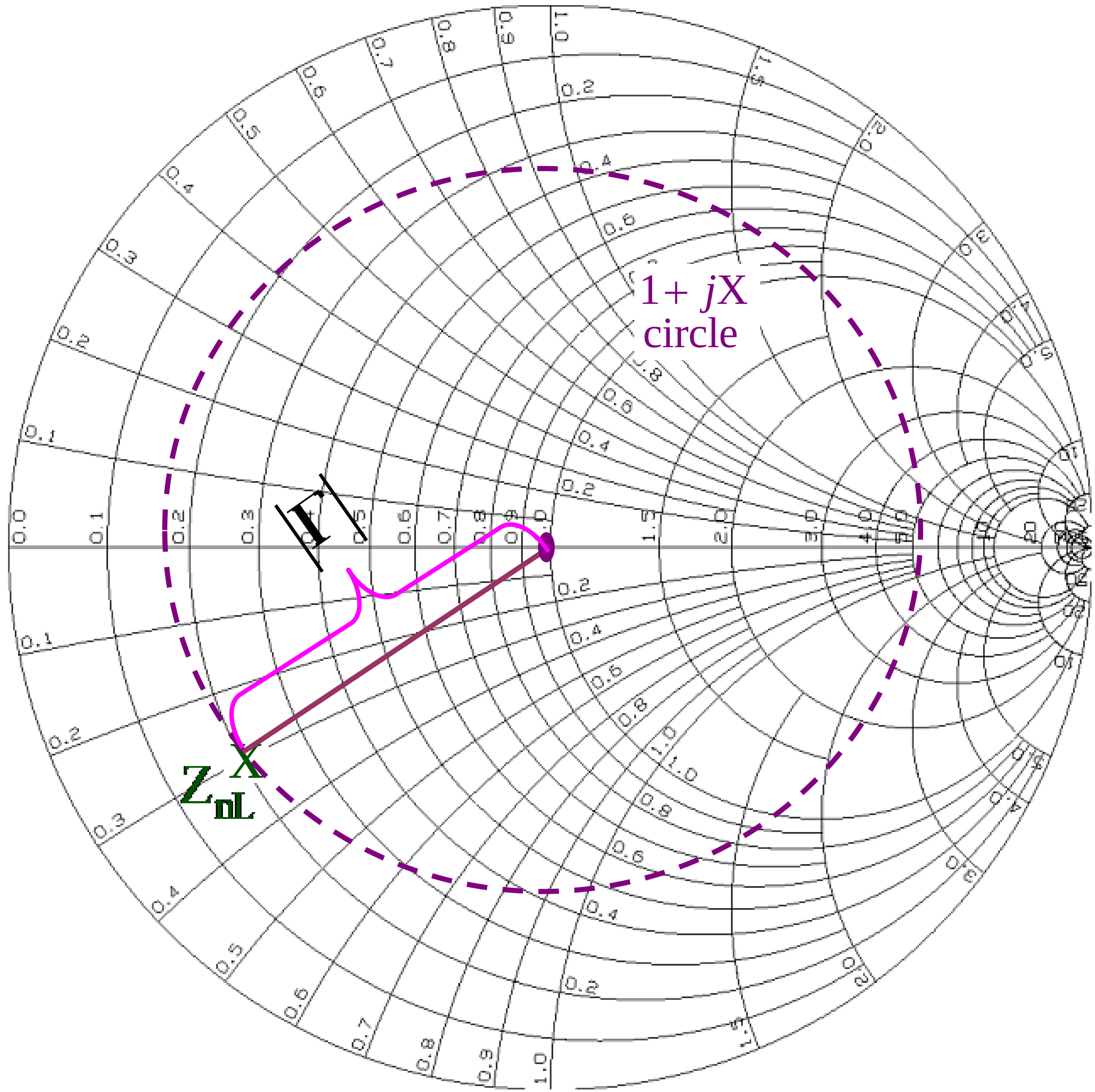


$Z_{nL} = R_n + jX_n \quad [\Omega]$

$Z_{nL} = 0.2 - j0.3 \quad [\Omega]$

Using your compass draw the 0.2-j0.3 circle with center at z_n=1.

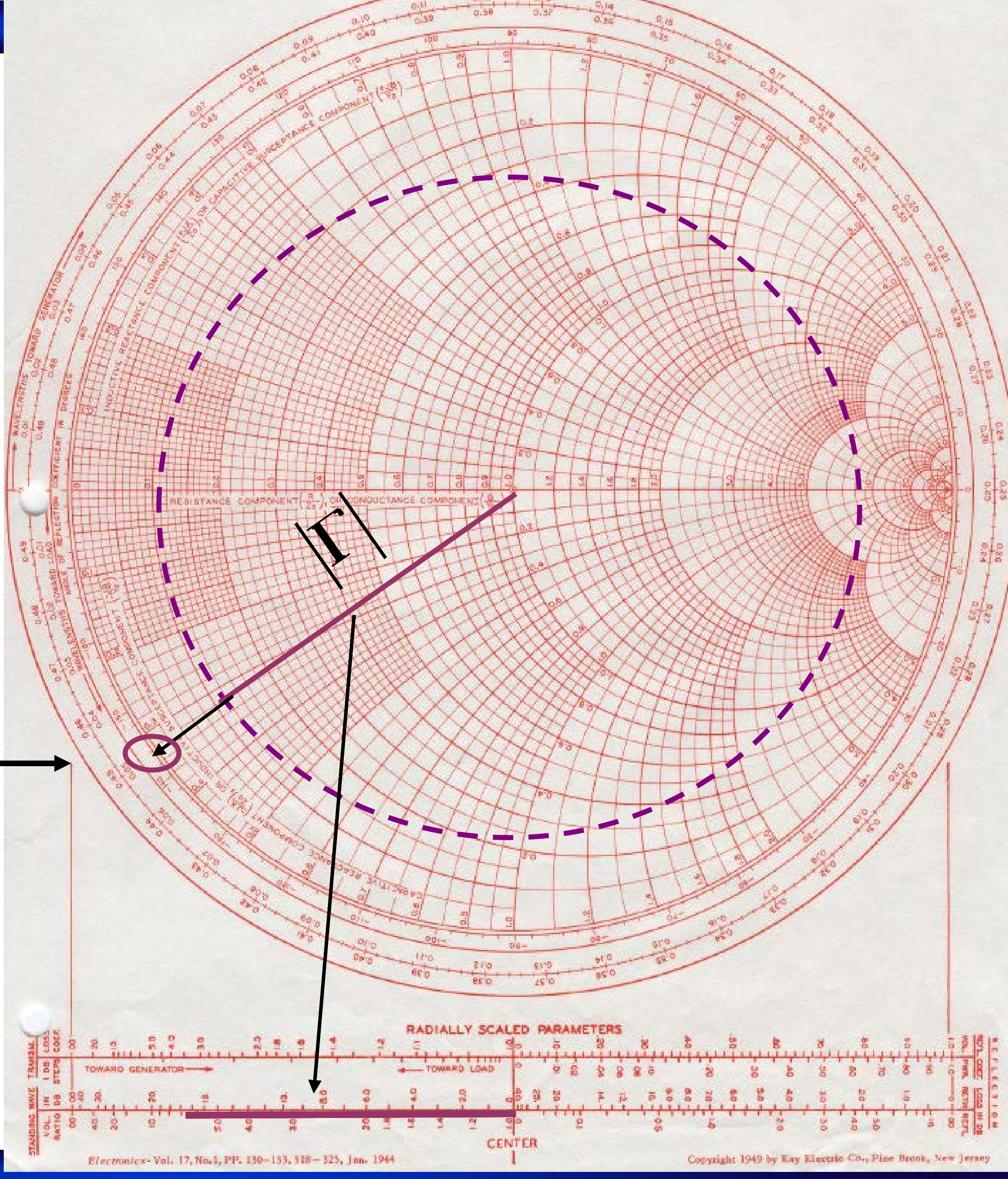
(NOTE: this circle is not on the chart until you draw it.)



Find the value of reflection coefficient

Reflection Coeff PHASE

Reflection Coeff MAGNITUDE



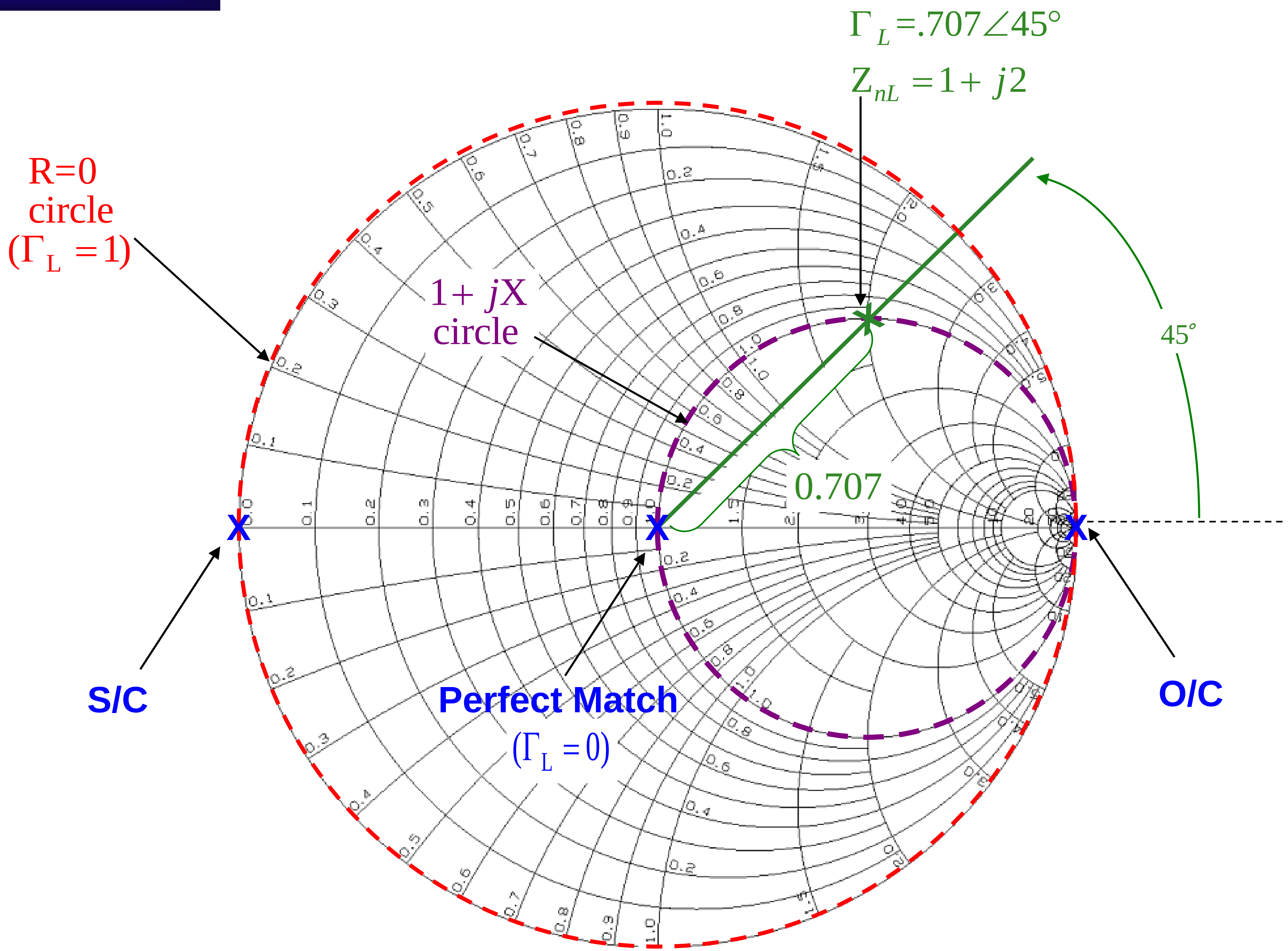
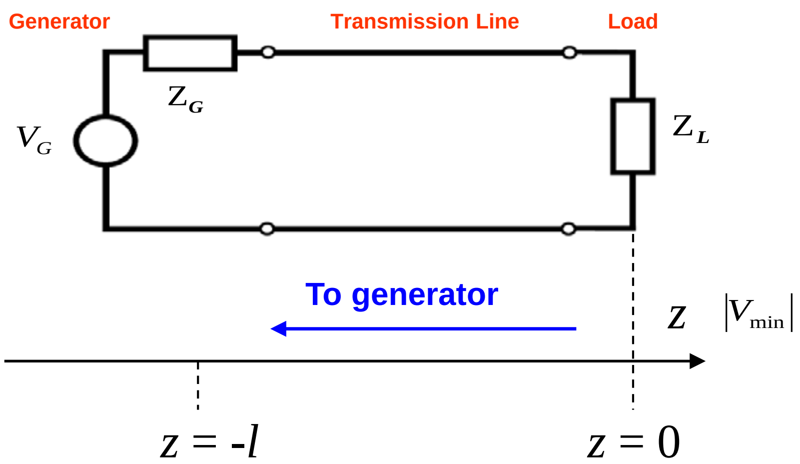

$\Gamma_L = .707\angle 45°$
$Z_{nL} = 1 + j2$
R=0 circle ($\Gamma_L = 1$)
1+ jX circle
45°
0.707
S/C
Perfect Match ($\Gamma_L = 0$)
O/C

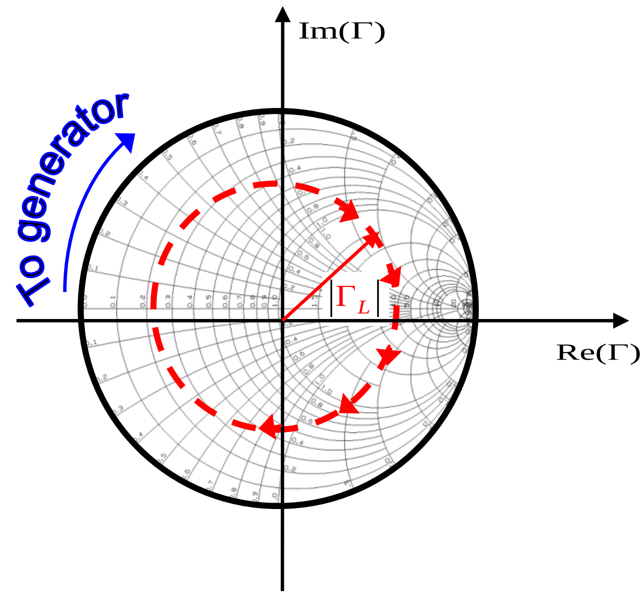
fig 6.13

$$Z(z) = Z_0 \frac{e^{-jkz} + \Gamma_L e^{+jkz}}{e^{-jkz} - \Gamma_L e^{+jkz}} = Z_0 \frac{1+\Gamma_L e^{+2jkz}}{1-\Gamma_L e^{+2jkz}}$$

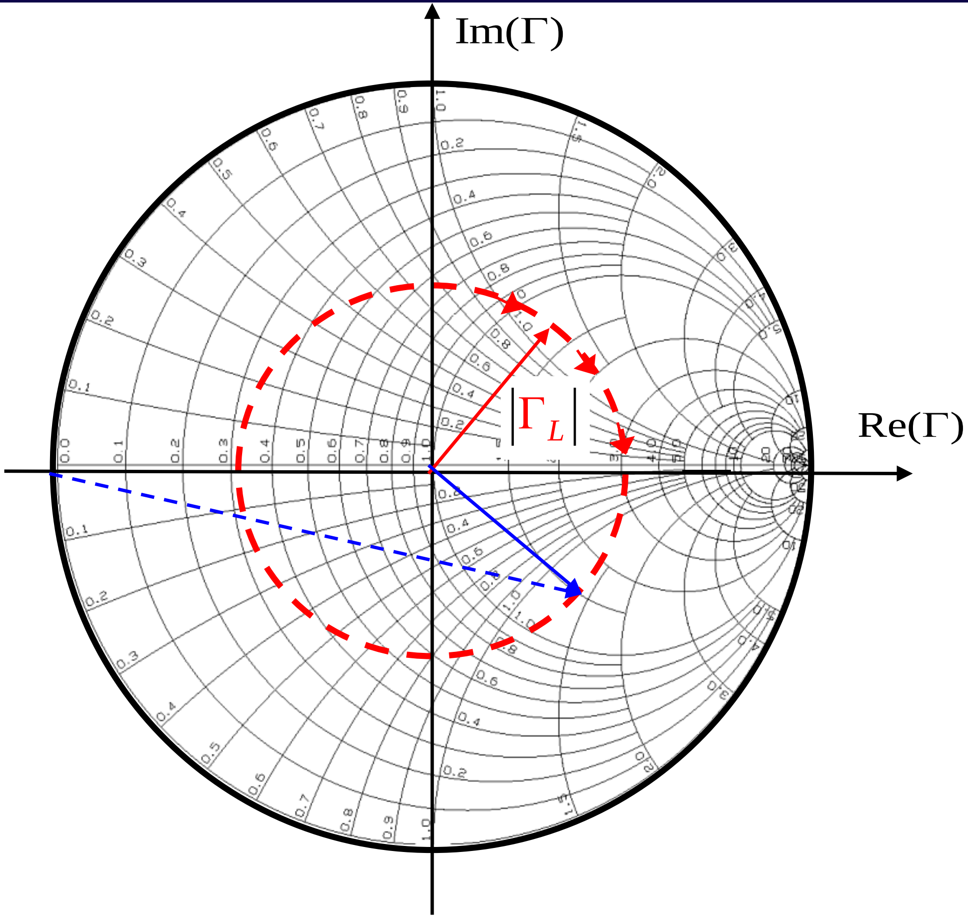
movement in negative $\hat{z}$ direction $\Leftrightarrow$ clockwise motion on circle of constant $|\Gamma_L|$

(toward generator)





$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|}=\left|1+\Gamma_L e^{j2kz}\right|$$



Note:

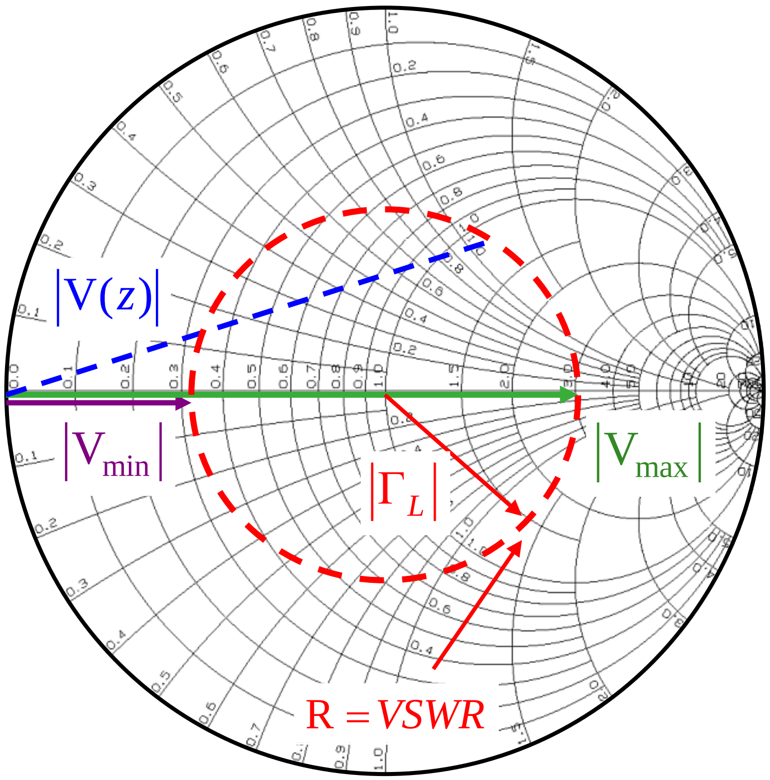
complete circle ($360° = 2\pi$ rad) $= \frac{\lambda}{2}$

$Y_n(z) = \frac{1}{Z_n(z)} \Rightarrow$ can just replace Γ_L by $(-\Gamma_L)$

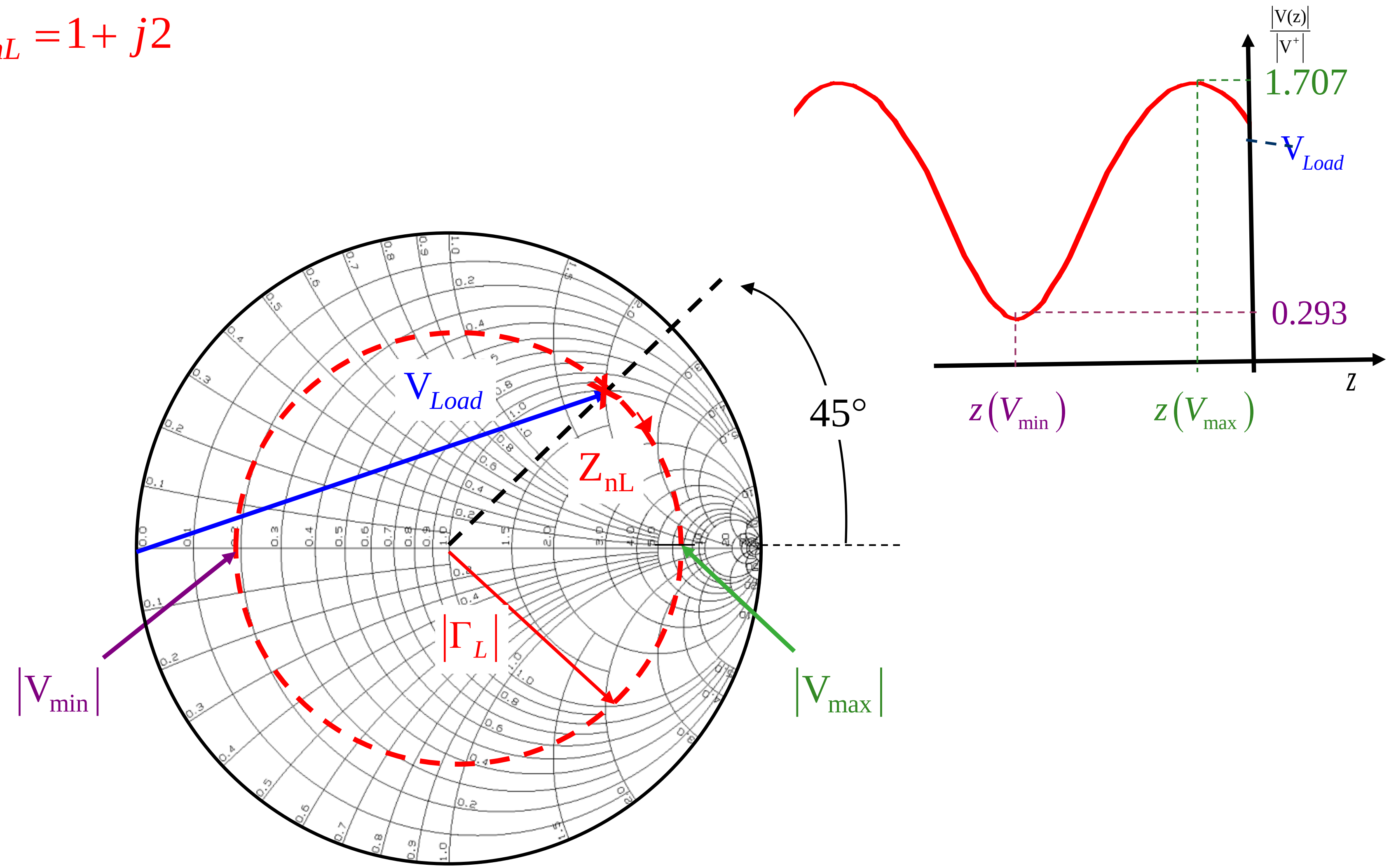
$$\left|V_{max}\right| = 1 + \left|\Gamma_L\right| \quad \text{(RH Real axis)}$$

$$\left|V_{min}\right| = 1 - \left|\Gamma_L\right| \quad \text{(LH Real axis)}$$

$$\frac{\left|V(z)\right|}{\left|V^{+}\right|} = \left|1 + \Gamma_L e^{2jkz}\right|$$



$Z_{nL} = 1 + j2$



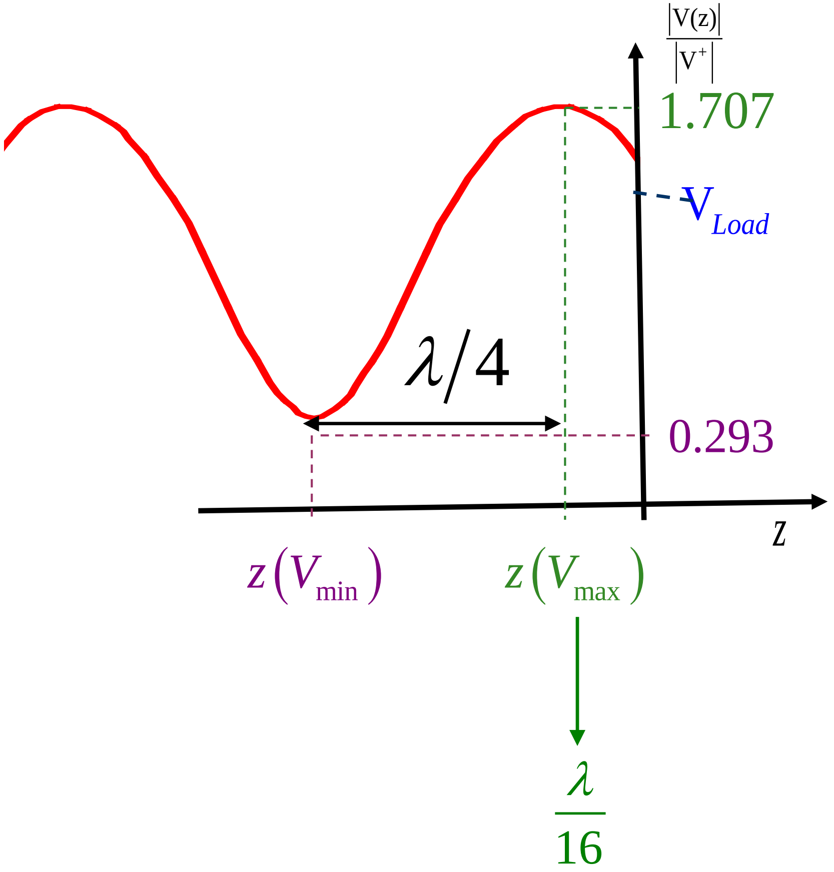
Smith Chart

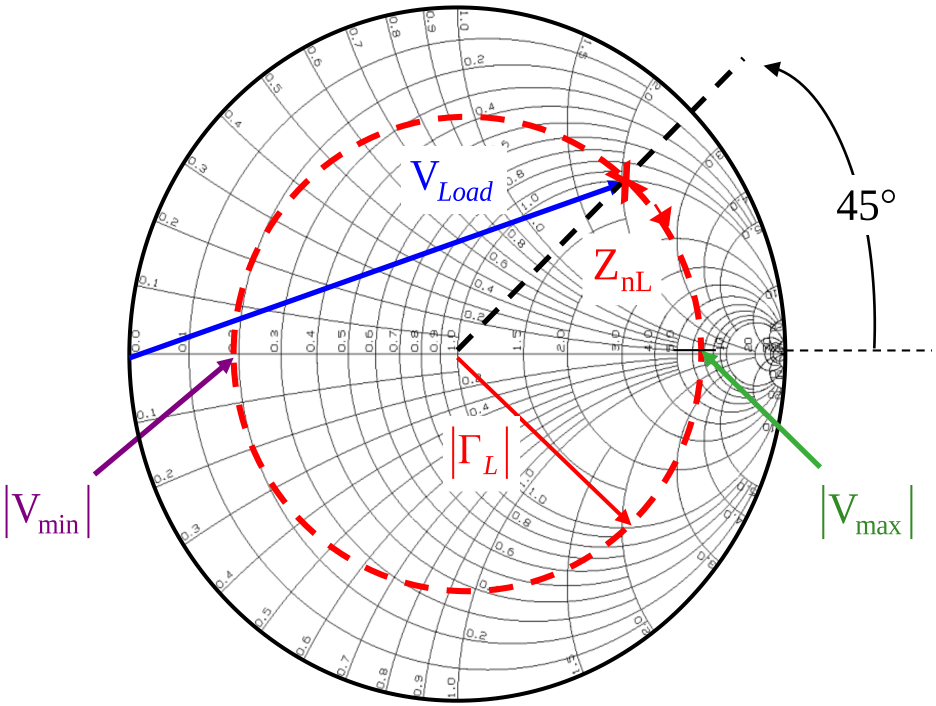
$Z_{nL} = 1 + j2$

$\Gamma = .707\angle 45°$

$$360° \text{ on Smith Chart} = \frac{\lambda}{2}$$

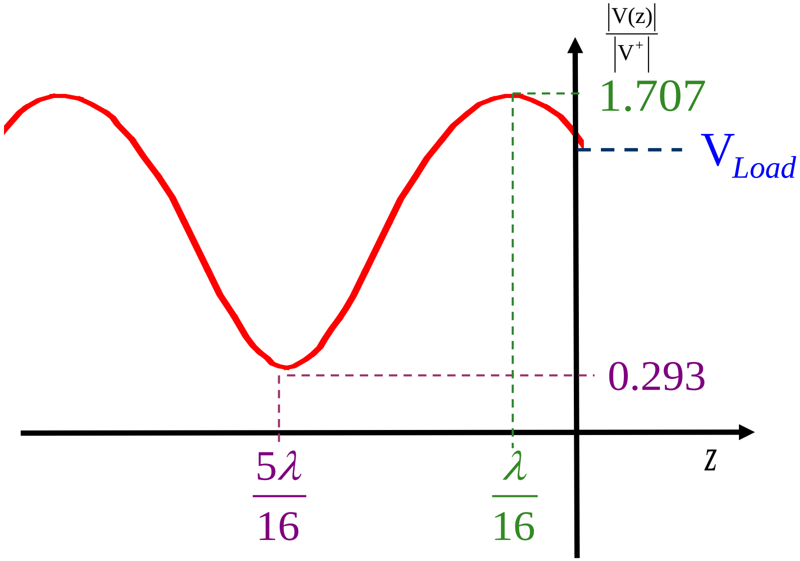
$$180° \text{ on Smith Chart} = \frac{\lambda}{4}$$

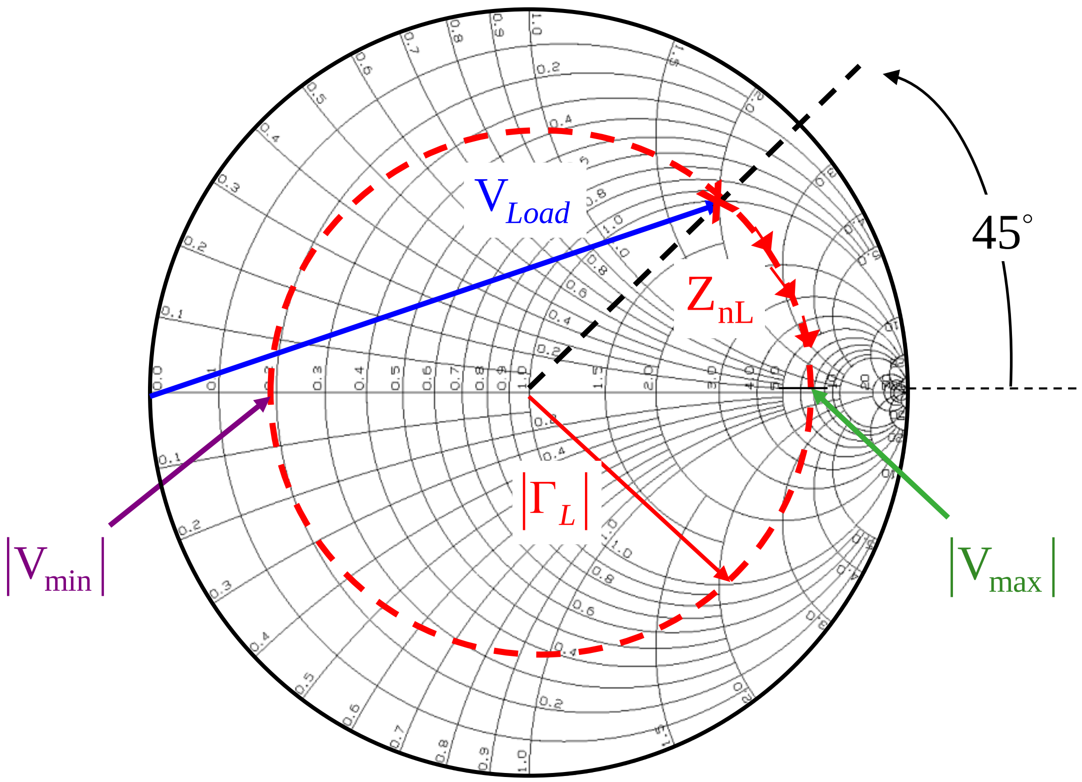


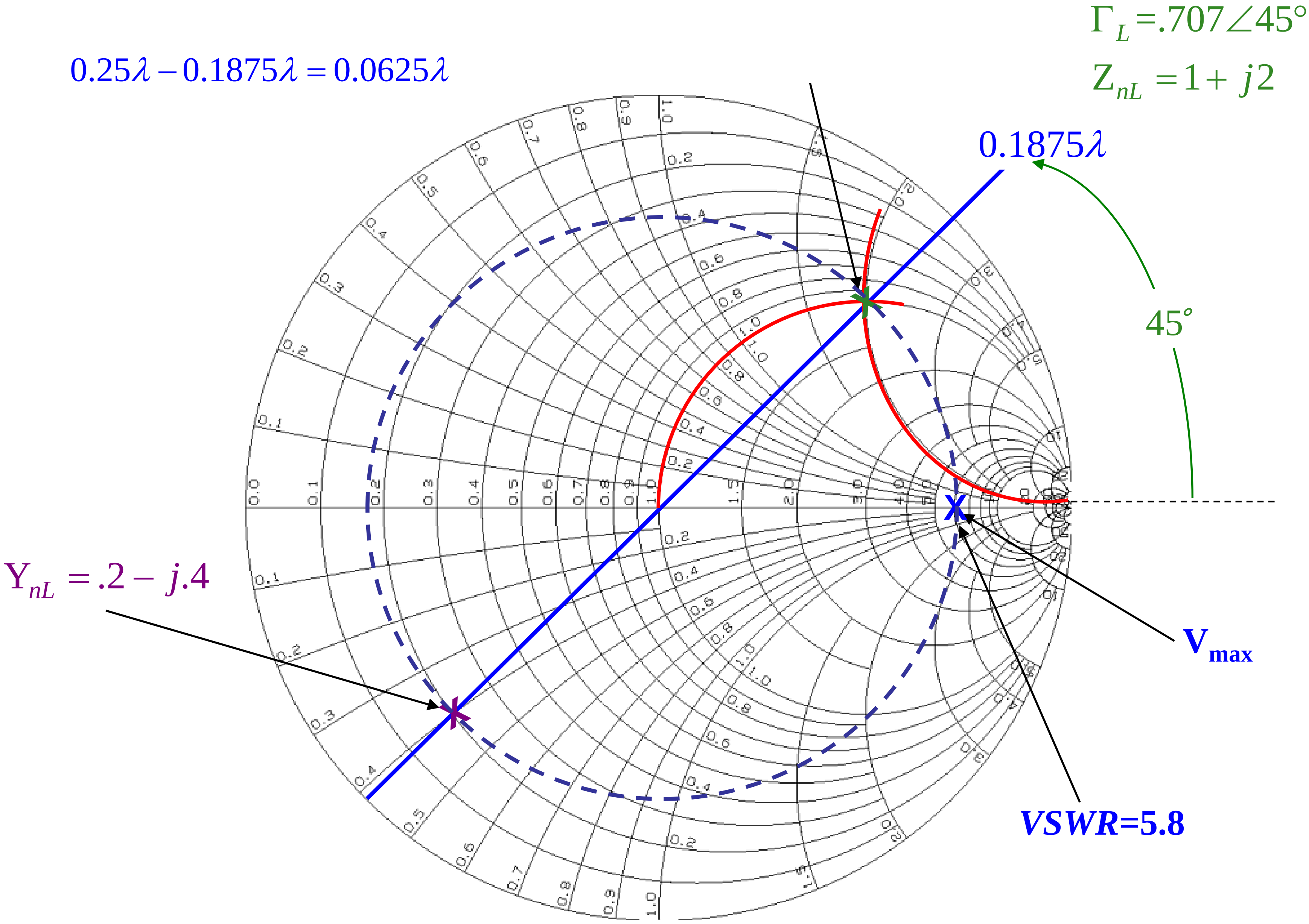


$Z_{nL} = 1 + j2$

$\Gamma = .707\angle 45°$







$\Gamma_L = .707\angle 45°$
$Z_{nL} = 1 + j2$
$0.25\lambda - 0.1875\lambda = 0.0625\lambda$
0.1875λ
$45°$
$Y_{nL} = .2 - j.4$
V_{max}
VSWR=5.8

Usually want power to be absorbed by load (minimize $|\Gamma_L|^2$).
To do so one adds pure reactances (or susceptances) to tune or match the network.

$$Z = R + jX \qquad [\Omega]$$

impedance = resistance + j reactance

$$Y = G + jB \qquad [S]$$

admittance = conductance + j susceptance

Note: It is physically easier to add a shunt susceptance than series reactance.

Example: Given $Z_{nL} = 2 + j2 \Rightarrow \Gamma = 0.62\angle 30°$

$$|\Gamma|^2 = 0.62^2 = 38\% \text{ power reflected}$$

change from $Z_{nL} \Rightarrow Y_{nL} = G_{nL} + jB_{nL}$

$$Y_{nL} = \frac{1}{Z_{nL}} = \frac{1}{2 + j2} = 0.25 - j0.25$$

could add $+j.25$ at load

$$Y_{nL} = 0.25 \Rightarrow \Gamma = 0.6\angle 0°$$

$$|\Gamma|^2 = 0.6^2 = 36\% \text{ power reflected}$$

instead rotate toward generator to $1+jB$ circle and add a $-jB$ there.

Smith Chart

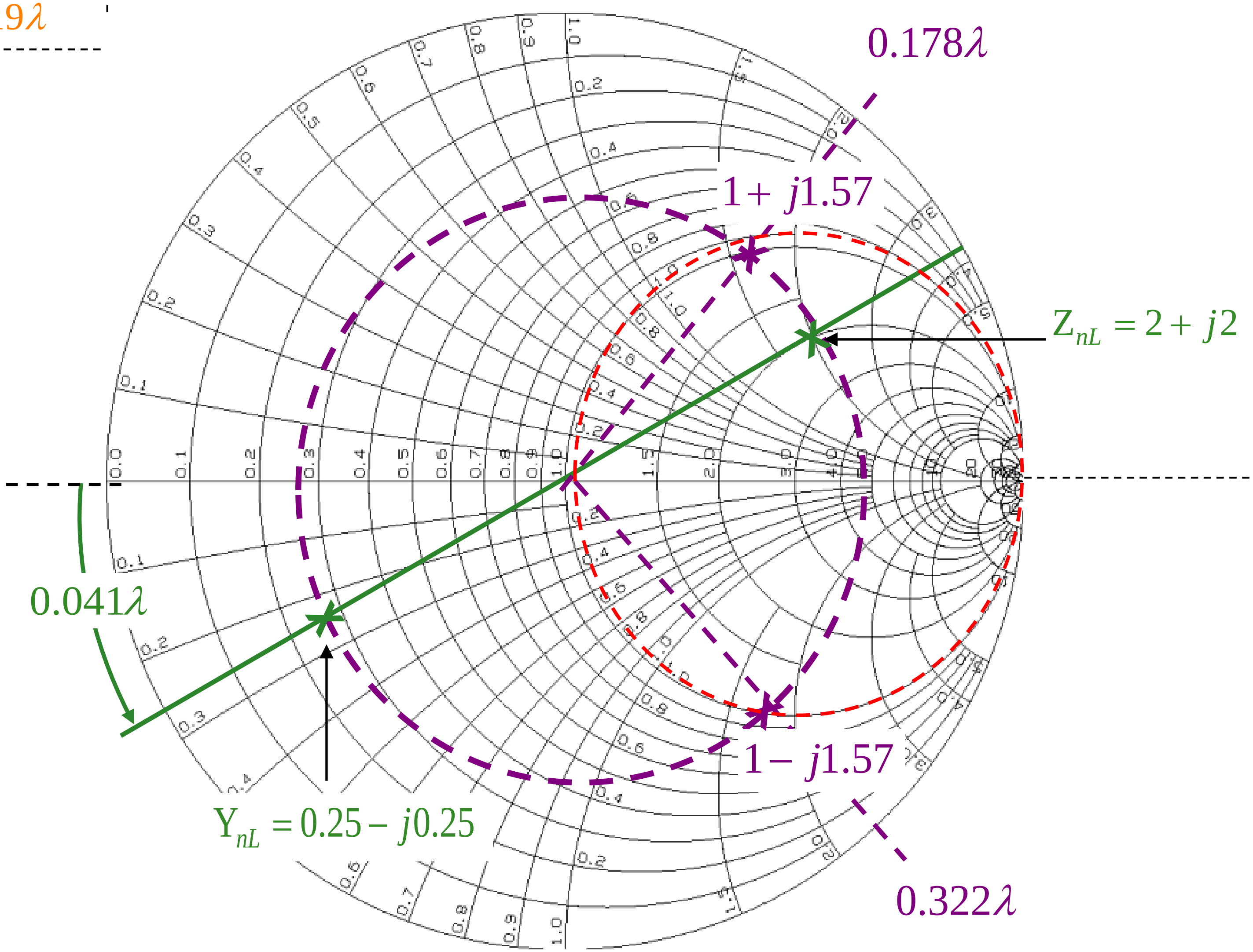
Solution:

Add $+j1.57$ at 0.362λ

Or $-j1.57$ at 0.219λ

$$0.041\lambda + 0.178\lambda = 0.219\lambda$$

$$0.041\lambda + 0.322\lambda = 0.363\lambda$$



Smith Chart

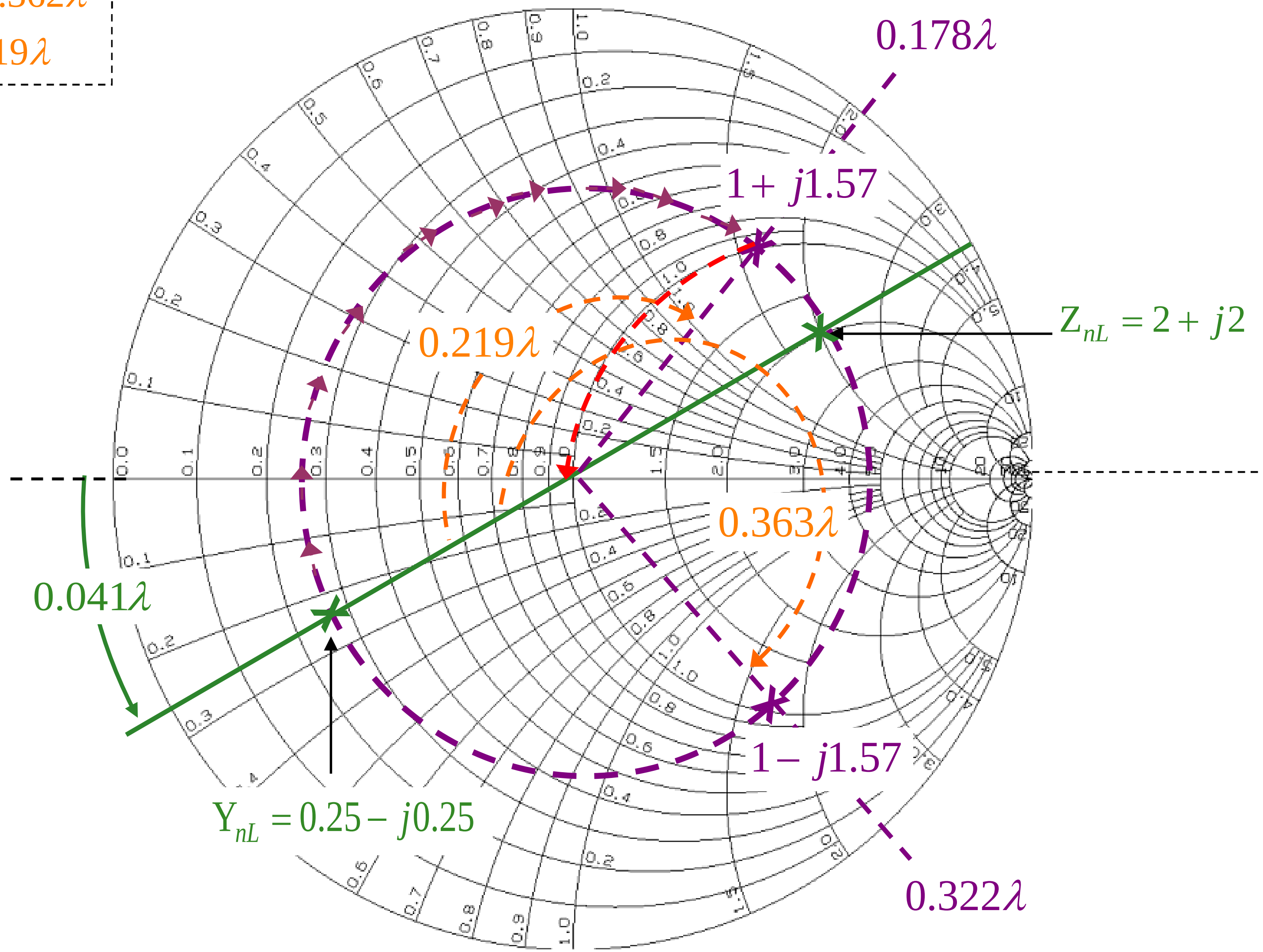
$$0.041\lambda + 0.178\lambda = 0.219\lambda$$

$$0.041\lambda + 0.322\lambda = 0.363\lambda$$

Solution:

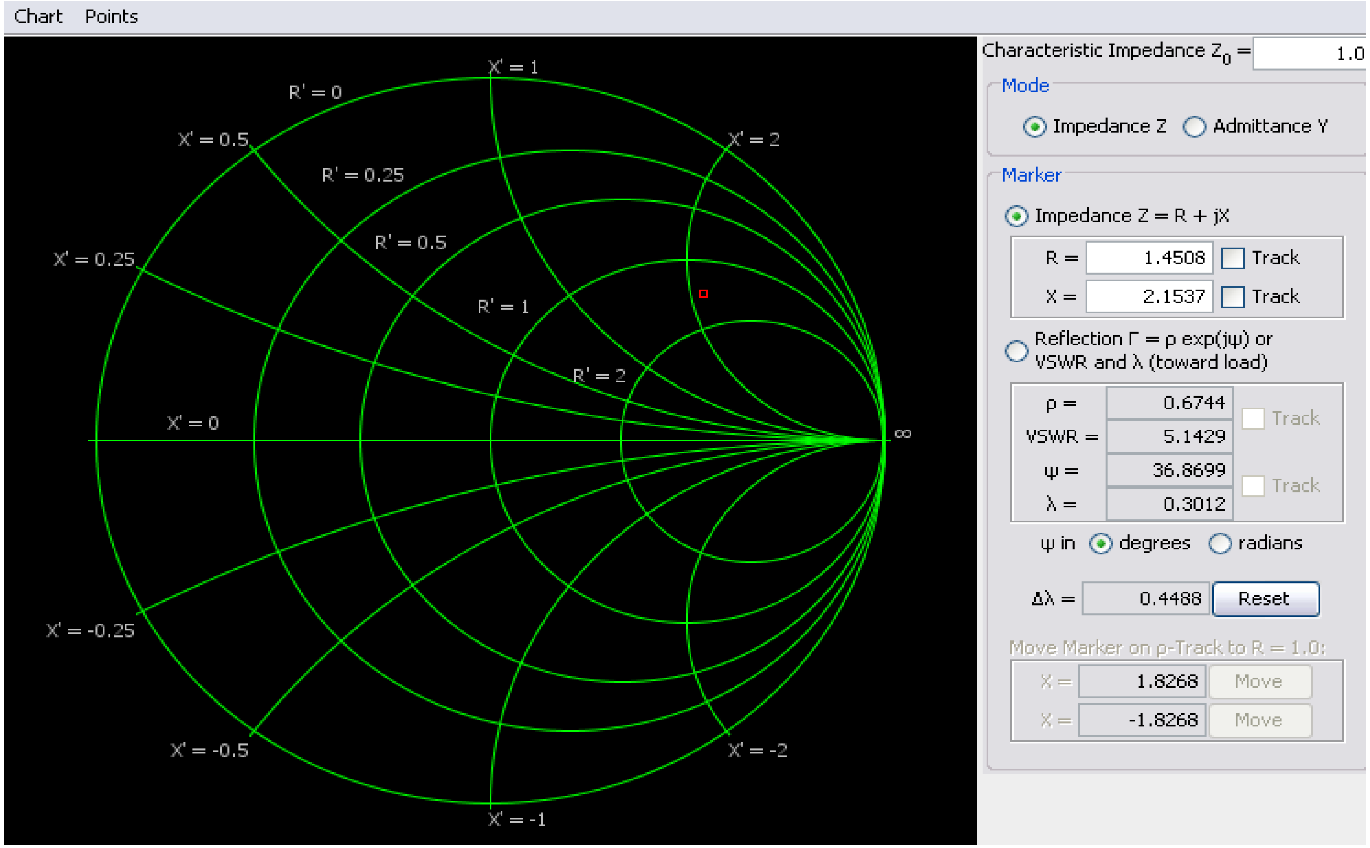
Add $+\,j1.57$ at 0.362λ

Or $-\,j1.57$ at 0.219λ

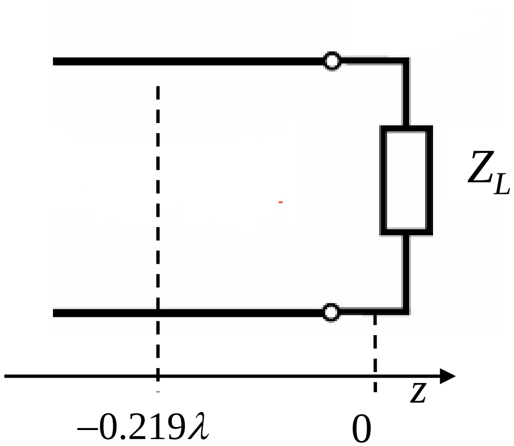


Smith Chart Applet

http://www.ocf.berkeley.edu/~joydip/smithchart/index.html

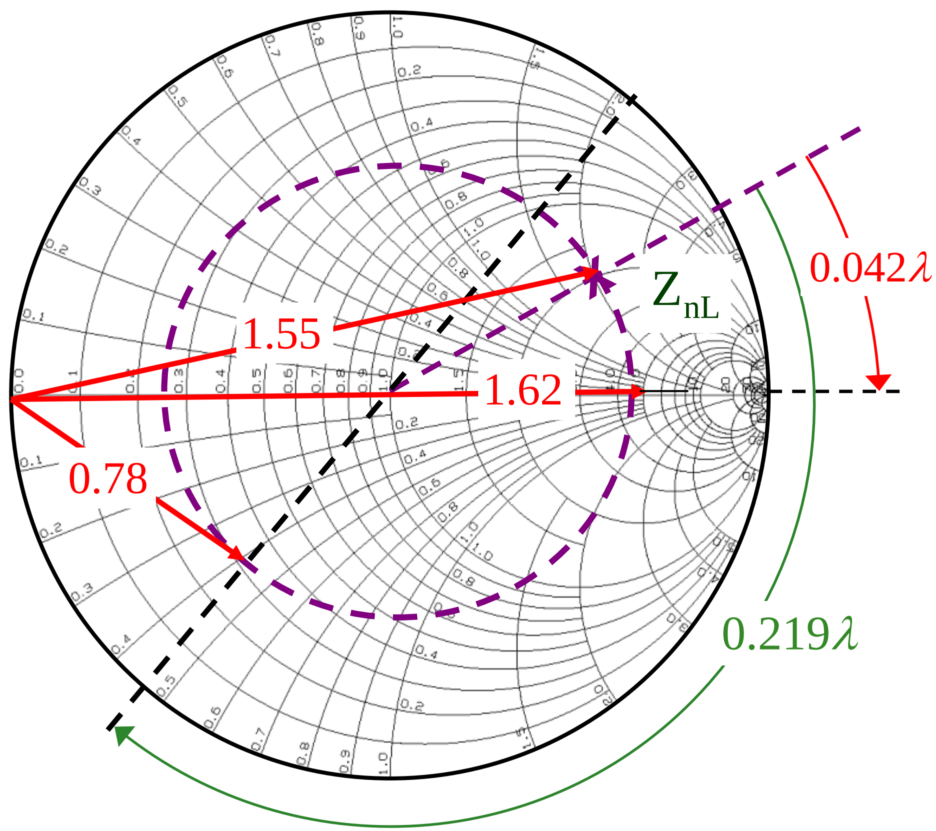


Standing Wave Pattern

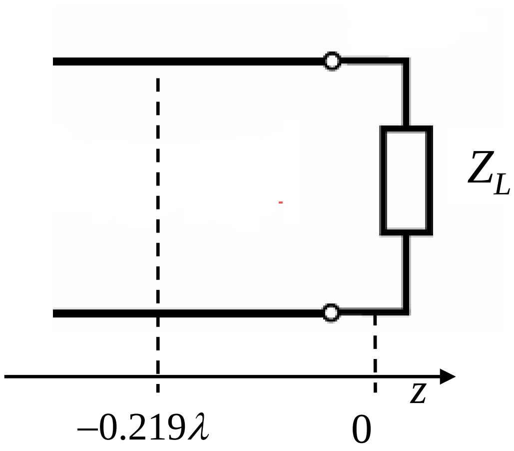


$$V_{load} = V(0) = 1.55$$
$$V_{max} = 1.62$$
$$V_{min} = 0.38$$
$$V(-0.219\lambda) = 0.78$$

Voltages: Unmatched Line



Standing Wave Pattern



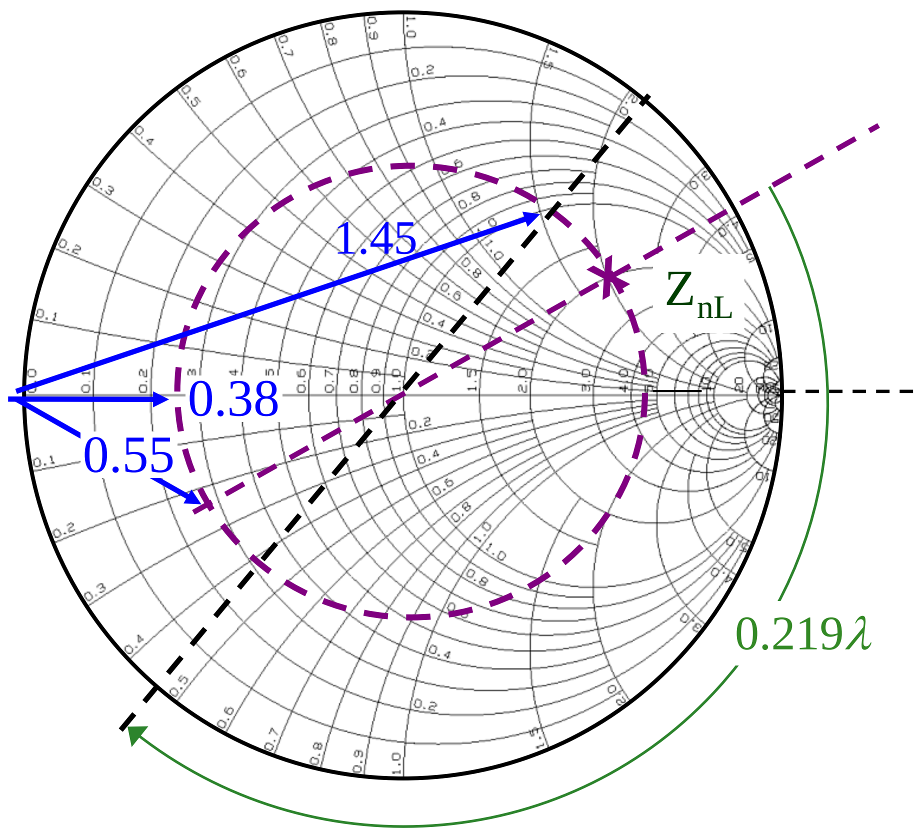
$$I_{load} = 0.55$$

$$I_{max} = 1.62$$

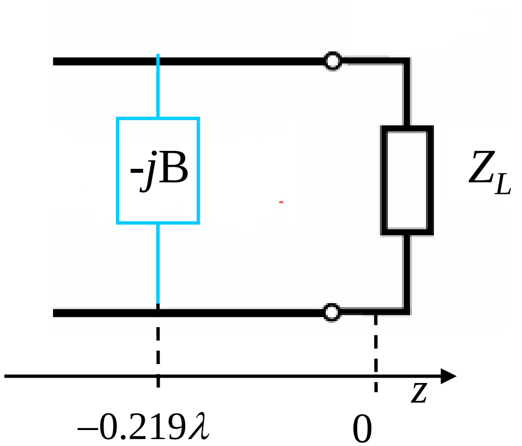
$$I_{min} = 0.38$$

$$I(-0.219\lambda) = 1.45$$

Currents: Unmatched Line

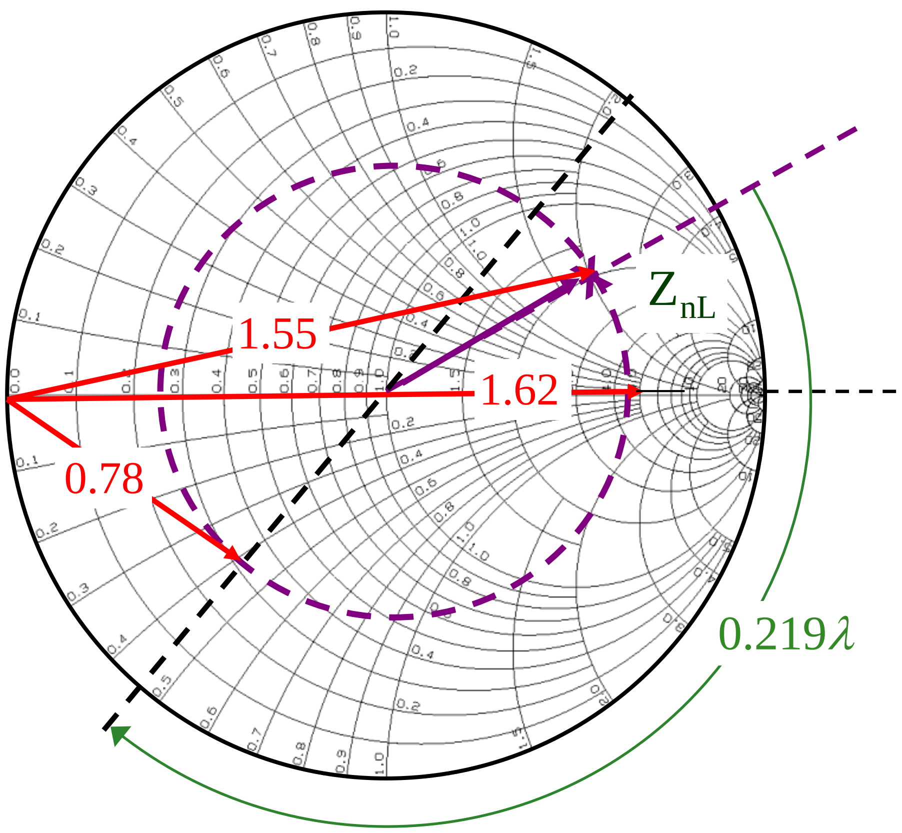


Standing Wave Pattern

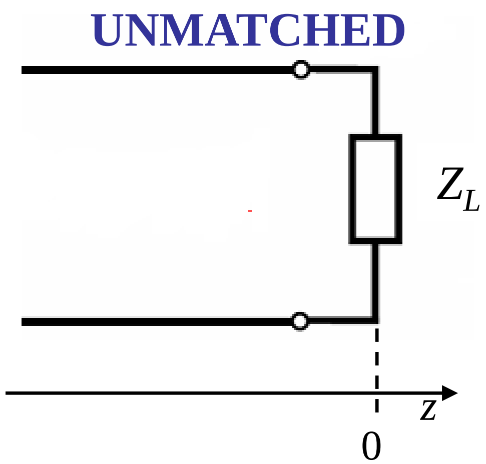


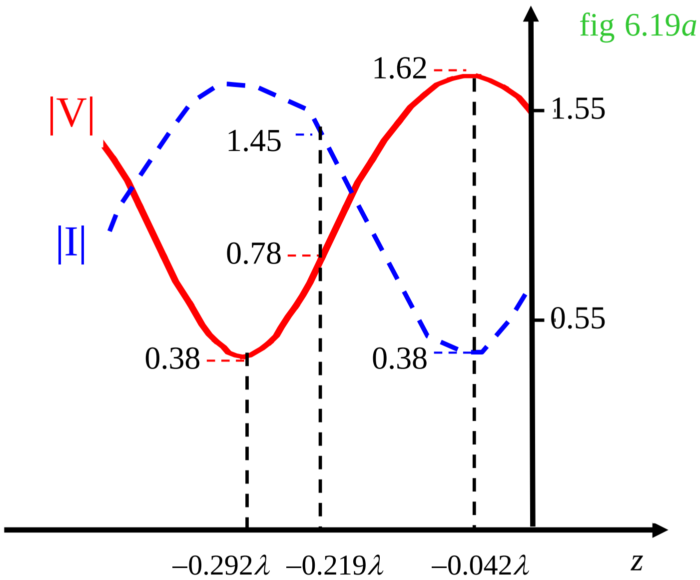
$$V_{load} = 1.55$$
$$V_{max} = 1.62$$
$$V_{min} = 0.38$$
$$V(-0.219\lambda) = 0.78$$

Voltages: Matched Line



Standing Wave Pattern





$V_{load} = 1.55$

$V_{max} = 1.62$

$V_{min} = 0.38$

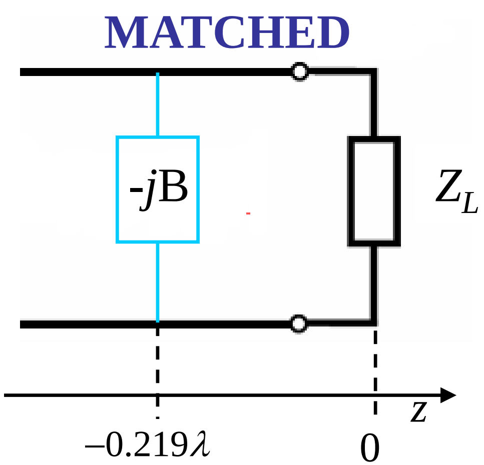
$V(-0.219\lambda) = 0.78$

$I_{load} = 0.55$

$I_{max} = 1.55$

$I_{min} = 0.38$

$I(-0.219\lambda) = 1.45$



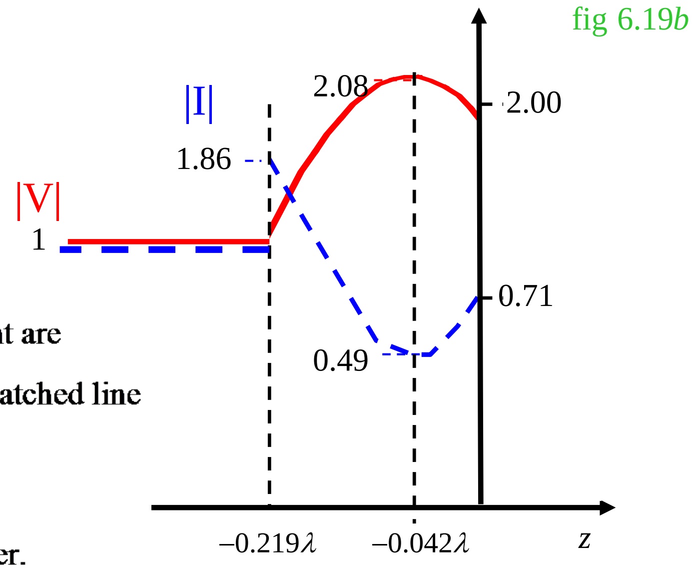
For tuned line

V,I between load and shunt are

greater than those for unmatched line

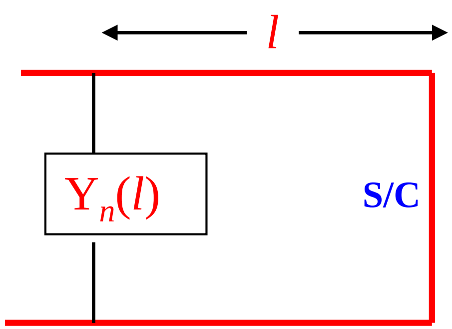
by a factor of $\frac{1}{0.78}$

for the same incident power.



How to Add Shunt Susceptance

For a transmission line ⇒ usually add a short circuit section of a line placed perpendicular to the main line.



$Y_n(l) = -j \cot kl$

6.36

for kl varying from 0 to π

⇓

$$0 \le l \le \frac{\lambda}{2}$$

all possible values
$-j\infty \le jB \le +j\infty$

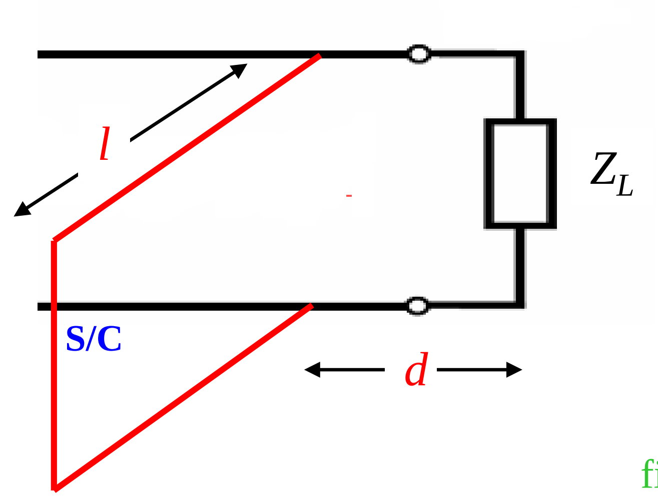
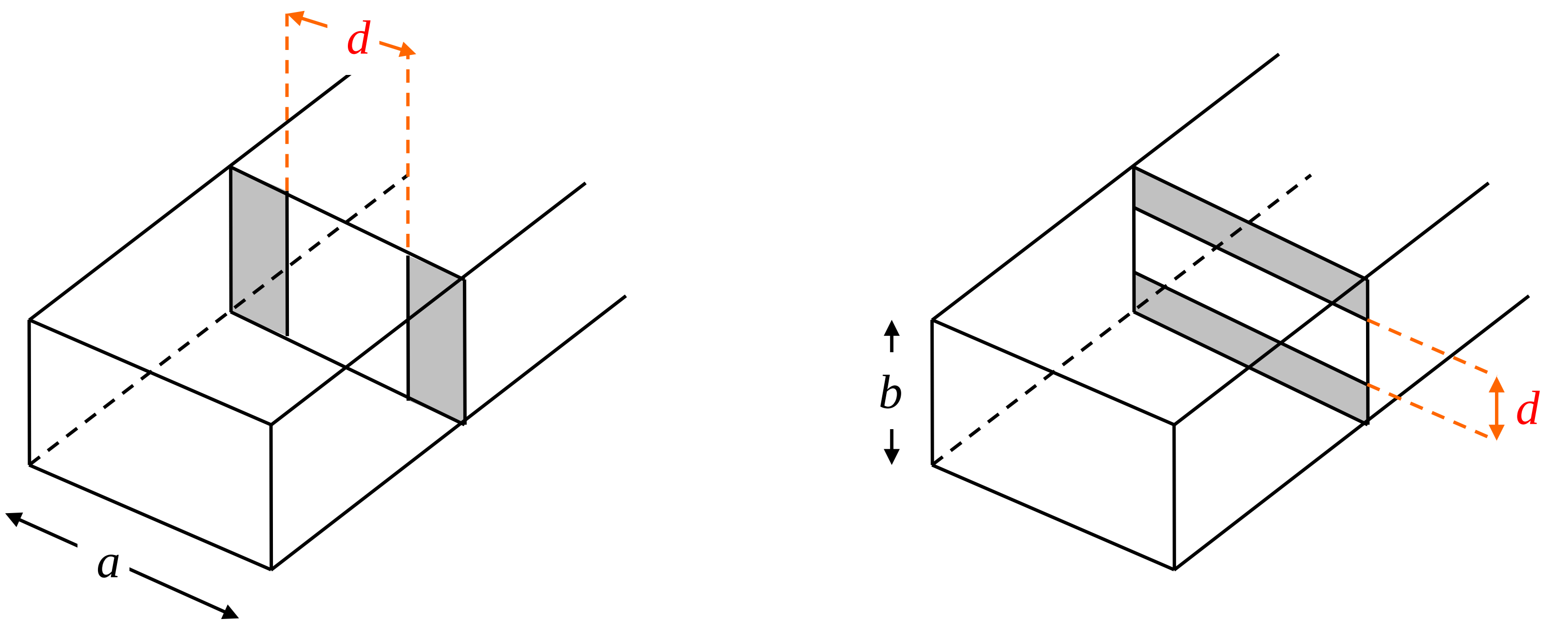


fig 6.20

How to Add Shunt Susceptance

For a rectangular waveguide ⇒ usually insert a metal iris inside the waveguide.



Inductive Iris

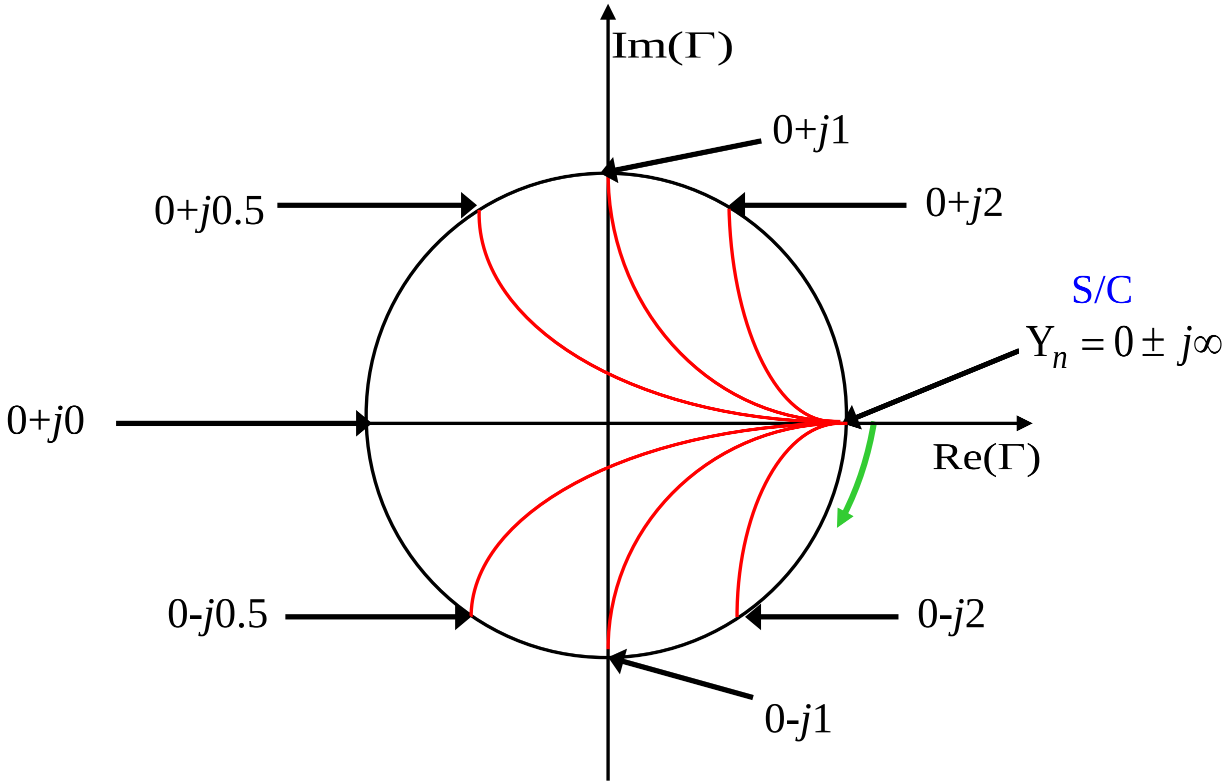
$$B_0 = \frac{-\lambda_g}{a}\cot^2\left(\frac{\pi d}{2a}\right) \quad 6.37$$

Capacitive Iris

$$B_0 = \frac{4b}{\lambda_g}\ln\left[\csc\left(\frac{\pi d}{2b}\right)\right] \quad 6.38$$

S/C Stub – Smith Chart Solution

Rotate clockwise from S/C to desired jB



Example: To add

$$jB = -j1.57$$

analytically

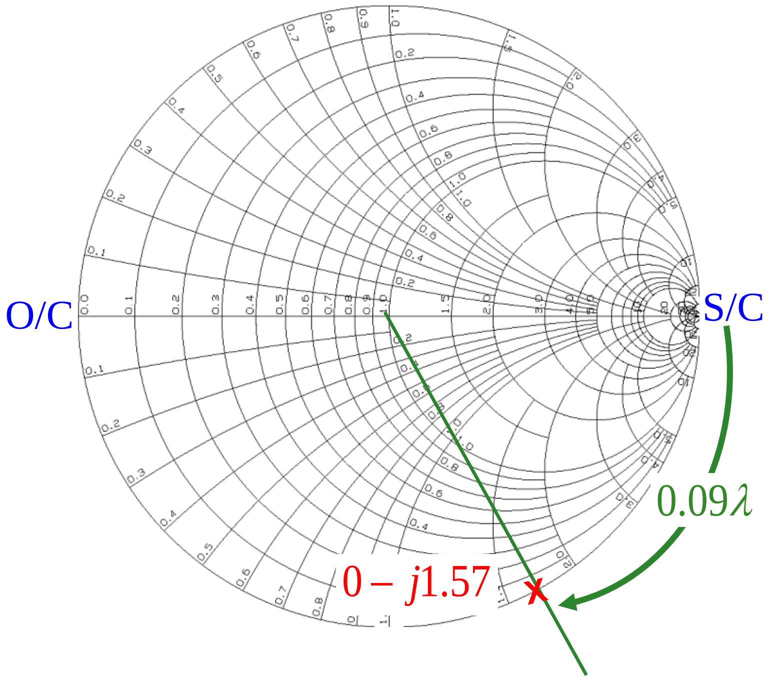
$$Y_n = -j\cot kl$$

$$-j1.57 = -j\cot kl$$

$$\cot kl = 1.57 \quad ; \quad \tan kl = \frac{1}{1.57}$$

$$kl = \frac{2\pi}{\lambda} l = 0.567 \text{ [radians]}$$

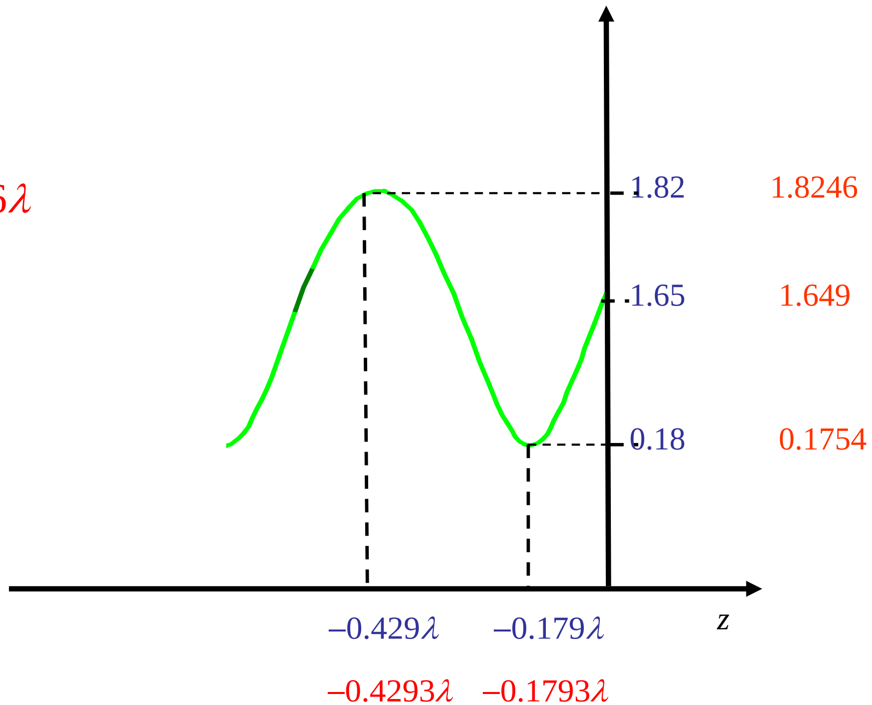
$$l = 0.0903\lambda$$

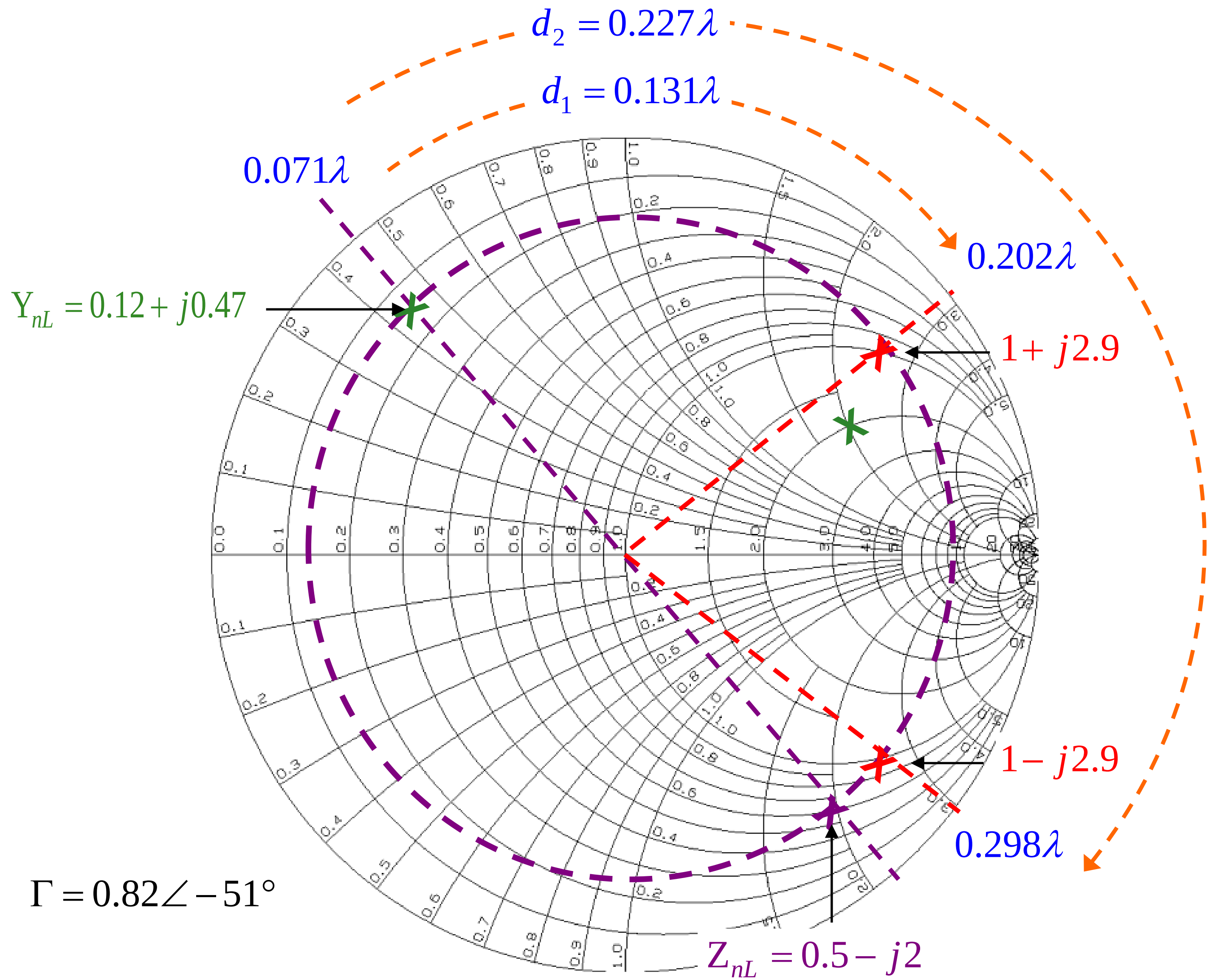


S/C Stub – Smith Chart Solution

Example: Given Z_{nL} =0.5-j2 find everything

Smith chart	Analytically
$\Gamma = 0.82\angle{-51^\circ}$	$\Gamma = 0.8246\angle{-50.9^\circ}$
$Y_n = 0.12 + j0.47$	$Y_n = 0.1176 + j0.4706$
$SWR = 10.5$	$SWR = 10.46$
$d_1 = 0.131\lambda$	$d_1 = 0.1315\lambda$
$B_1 = -2.9$	$B_1 = -2.915$
$l_1 = 0.053\lambda$	$l_1 = 0.0526\lambda$
$d_2 = 0.227\lambda$	
$B_2 = +2.9$	
$l_2 = 0.447\lambda$	





$d_2 = 0.227\lambda$
$d_1 = 0.131\lambda$
0.071λ
0.202λ
$Y_{nL} = 0.12 + j0.47$
$1 + j2.9$
$1 - j2.9$
0.298λ
$\Gamma = 0.82\angle -51°$
$Z_{nL} = 0.5 - j2$

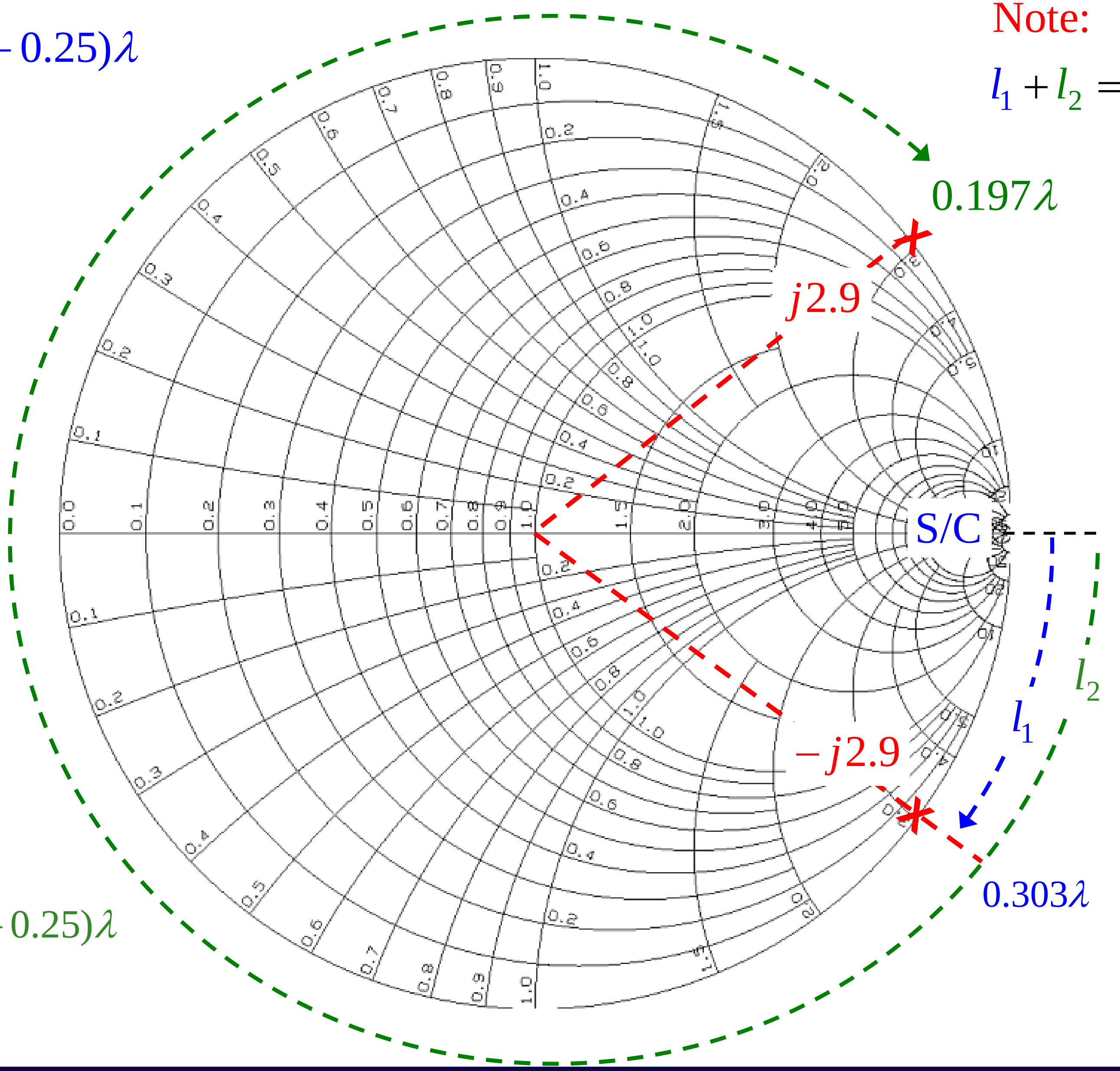
fig 6.18

$l_1 = (0.303 - 0.25)\lambda$

$l_1 = 0.053\lambda$

Note:

$l_1 + l_2 = 0.5\lambda$



$l_2 = (0.197 + 0.25)\lambda$

$l_2 = 0.447\lambda$