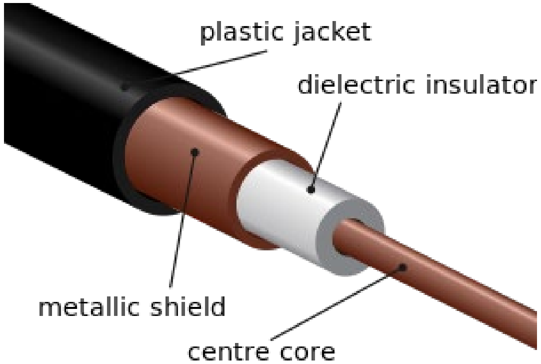


Chapter 6
Transmission Lines

ECE 3317
Dr. Stuart A. Long

Popular Mechanics, 1955

Bell scientist checks test equipment used to bounce microwaves back and forth for 40 miles in metal tubes

The flexibility of the coiled-wire tubing permits it to be laid over long distances and rough terrain

Long-Distance Microwave Pipe Carries Many Television Programs

Tens of thousands of cross-country telephone calls along with hundreds of television programs may someday be carried in a single two-inch metal tube. The long-distance wave guide, developed by Bell, could be buried underground and would funnel extremely short microwaves up hill, down dale and around corners. It is constructed of thin copper wire, tightly coiled like a spring under pressure and wrapped inside a flexible outer coating which holds the wire in place. In laboratory tests, microwaves have been carried for 40 miles in a metal tube with the same loss of strength encountered when the waves travel 12 miles in a coaxial cable. The system uses microwaves shorter than any previously used in communications.

General Definitions

p.133

Voltage $$V(z) = \alpha_1 \int_{C_t} \mathbf{E} \bullet d\boldsymbol{s} \qquad (C_t \perp z) \qquad (a)$$

Current $$I(z) = \alpha_2 \oint_{C_0} \mathbf{H} \bullet d\boldsymbol{s} \qquad (C_0 \text{ closed}) \qquad (b)$$

General Definitions

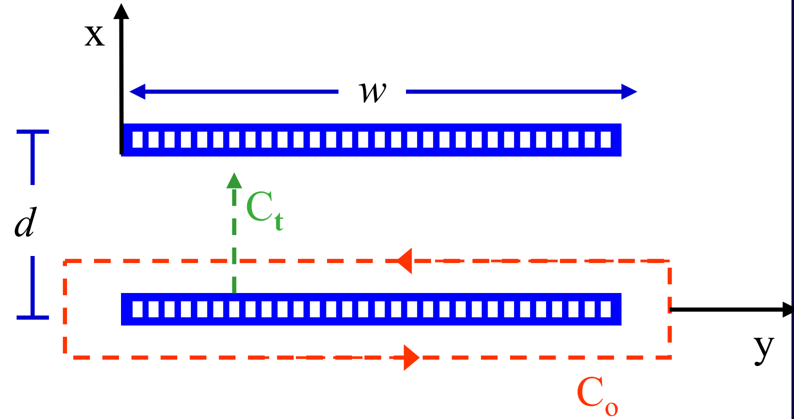
p.133

Power $$\frac{1}{2}\mathrm{Re}\left[\mathrm{V}(z)\,\mathrm{I}^{*}(z)\right] = \int_A \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^{*}\right]\bullet\hat{z}\;dA \qquad (c)$$

Characteristic Impedance $$\mathrm{Z}_0 = \frac{\mathrm{V}(z)}{\mathrm{I}(z)}$$

infinite line with
with no reflection

Parallel Plate Waveguide (TEM mode)



The Transverse Electromagnetic Fields in a Parallel Plate Waveguide are approximately as follows:

$$\mathbf{E} = \hat{\boldsymbol{x}}\, E_0 e^{-jkz} \qquad 6.1a$$

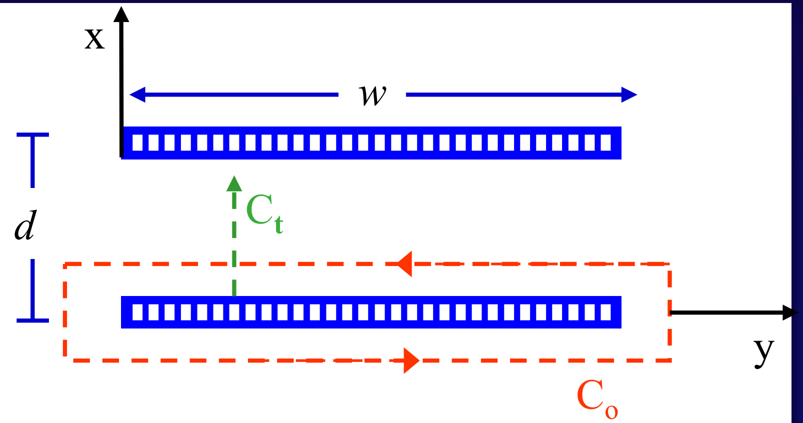
$$\mathbf{H} = \hat{\boldsymbol{y}}\, \frac{E_0}{\eta} e^{-jkz} \qquad 6.1b$$

Using the General Definitions

$$\mathrm{V}(z) = \alpha_1 \int_0^d \mathbf{E} \bullet \hat{\boldsymbol{x}}\, dx = \alpha_1 E_0\, d\, e^{-jkz} \qquad 6.2a$$

$$\mathrm{I}(z) = \alpha_2 \int_0^w \mathbf{H} \bullet \hat{\boldsymbol{y}}\, dy = \alpha_2 \frac{E_0 w}{\eta} e^{-jkz} \qquad 6.2b$$

Parallel Plate Waveguide (TEM mode)



The time-average power transmitted is given by

$$P_t = \int_A \frac{E_0^2}{2\eta} \hat{z} \cdot \hat{z}\, dA = \frac{E_0^2}{2\eta} wd$$

equate

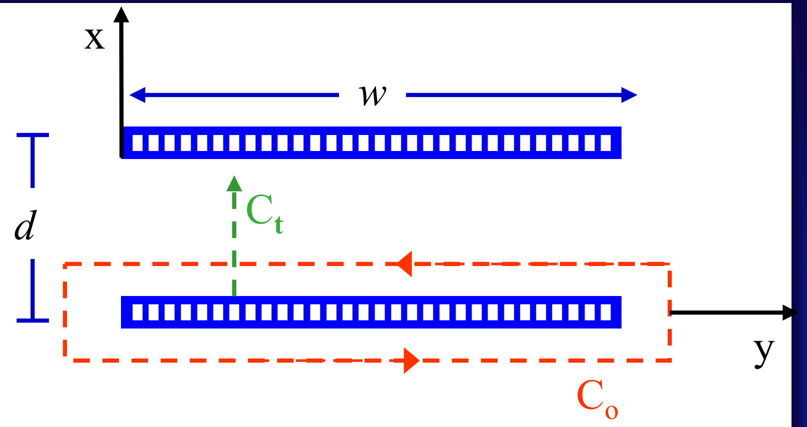
$$\frac{1}{2}\mathrm{Re}\left[\mathrm{V}(z)\mathrm{I}(z)^*\right] = \frac{E_0^2}{2\eta} wd$$

$$\frac{1}{2}\left(\alpha_1 E_0 d\right)\left(\alpha_2 \frac{E_0}{\eta} w\right) = \frac{E_0^2}{2\eta} wd$$

$$\alpha_1\alpha_2 \frac{E_0^2}{2\eta} wd = \frac{E_0^2}{2\eta} wd \Rightarrow \alpha_1\alpha_2 = 1$$

choose $\alpha_1 = 1$ and $\alpha_2 = 1$

Parallel Plate Waveguide (TEM mode)

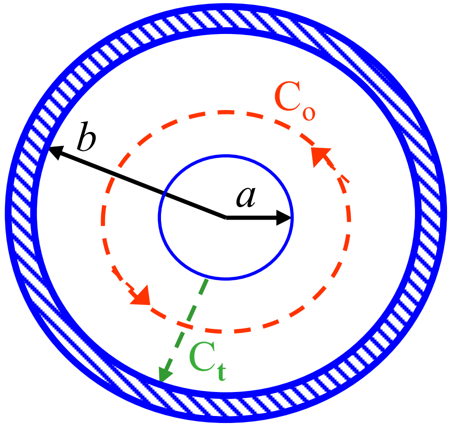


$$V(z) = E_0 d\, e^{-jkz} \quad 6.3a$$

$$I(z) = \frac{E_0 w}{\eta} e^{-jkz} \quad 6.3b$$

$$Z_0 = \eta \frac{d}{w} \ [\Omega] \quad 6.4$$

Coaxial Line



The fields inside a coaxial line for the TEM mode are given by

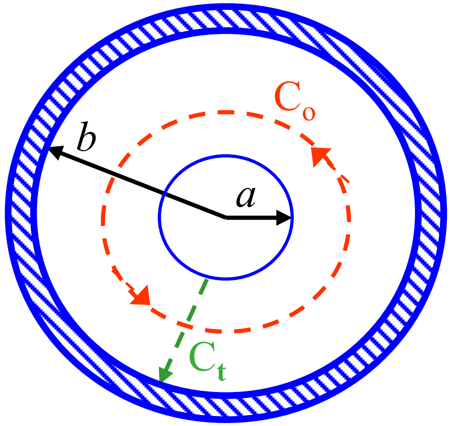
$$\mathbf{E} = \hat{\rho}\frac{V_0}{\rho}e^{-jkz} \qquad 6.5a$$

$$\mathbf{H} = \hat{\phi}\frac{V_0}{\eta\rho}e^{-jkz} \qquad 6.5b$$

Using the General Definitions

$$\mathrm{V}(z) = \alpha_1\int_a^b \mathbf{E}\bullet\hat{\rho}\,d\rho = \alpha_1 V_0 \ln\frac{b}{a}e^{-jkz} \qquad 6.6a$$

$$\mathrm{I}(z) = \alpha_2\oint \mathbf{H}\bullet\hat{\phi}\,\rho\,d\phi = \alpha_2\frac{2\pi V_0}{\eta}e^{-jkz} \qquad 6.6b$$



The time-average power transmitted is given by

$$P_t = \frac{\pi V_0^2}{\eta}\ln\frac{b}{a}$$

equate

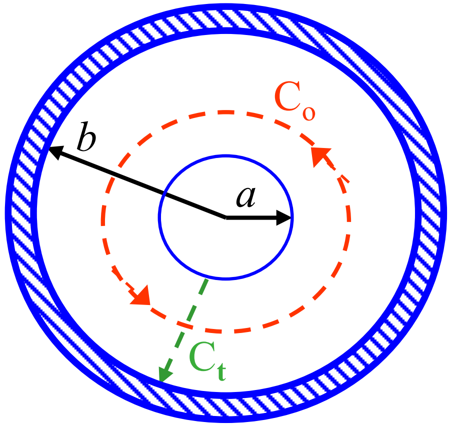
$$\frac{1}{2}\mathrm{Re}\left[\mathrm{V}(z)\mathrm{I}(z)^*\right] = \frac{\pi V_0^2}{\eta}\ln\frac{b}{a}$$

$$\frac{1}{2}\left(\alpha_1 V_0 \ln\frac{b}{a}\right)\left(\alpha_2\frac{2\pi V_0}{\eta}\right) = \frac{\pi V_0^2}{\eta}\ln\frac{b}{a}$$

$$\alpha_1\alpha_2\frac{\pi V_0^2}{\eta}\ln\frac{b}{a} = \frac{\pi V_0^2}{\eta}\ln\frac{b}{a} \Rightarrow \alpha_1\alpha_2 = 1$$

choose $\alpha_1 = 1$ *and* $\alpha_2 = 1$

Coaxial Line



$$V(z) = V_0 \ln\frac{b}{a} e^{-jkz}$$

$$I(z) = \frac{2\pi V_0}{\eta} e^{-jkz}$$

$$Z_0 = \frac{\eta}{2\pi} \ln\frac{b}{a} \quad [\Omega]$$

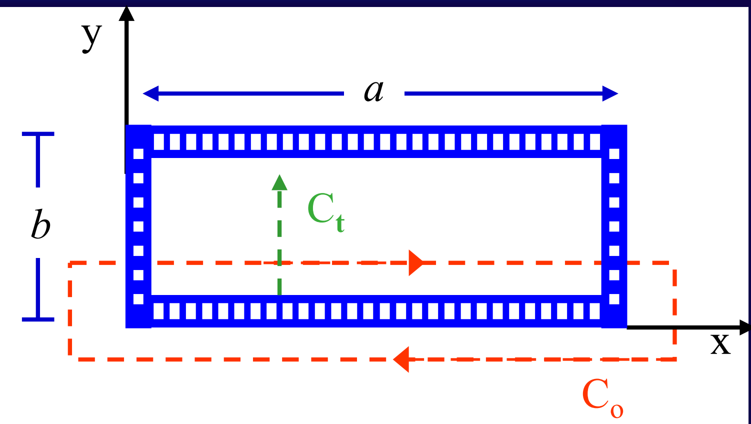
6.7

Example: What is the ratio b/a for an air-filled coax and 50[Ω] line?

$$50 = \frac{377}{2\pi} \ln(b/a)$$

$$\Rightarrow \frac{b}{a} = 2.3$$

Rectangular Waveguide (Dominant TE_{10} mode)



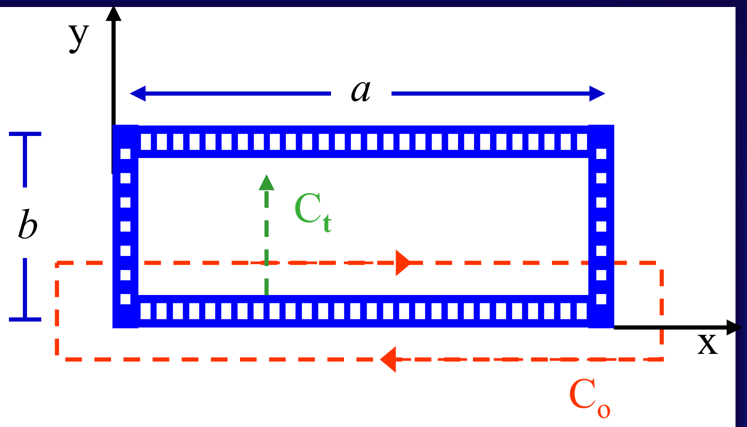
The Electromagnetic fields in a Rectangular Waveguide for the are TE_{10} mode are

$$E_y = E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8a$$

$$H_x = -\frac{k_z}{\omega\mu} E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8b$$

$$H_z = j\frac{\pi/a}{\omega\mu} E_0 \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z} \qquad 6.8c$$

Rectangular Waveguide (Dominant TE_{10} mode)

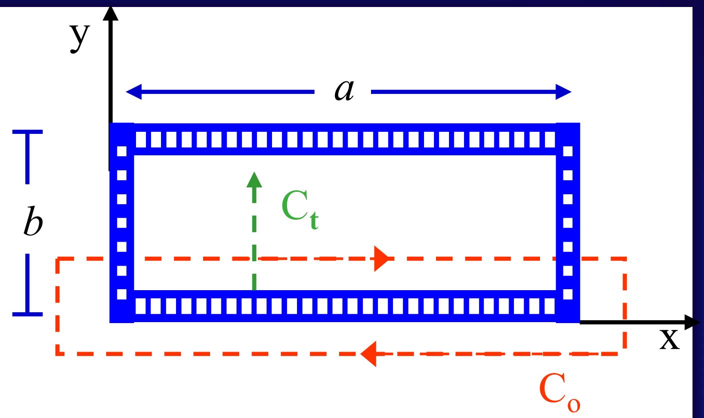


Using the General Definitions

$$V(z) = \alpha_1 \int_0^b E_y \Big|_{x=a/2} dy = \alpha_1 E_0 \, b \, e^{-jk_z z} \qquad 6.9a$$

$$I(z) = \alpha_2 \oint_{C_0} \hat{x} \bullet \mathbf{H} \, dx = \alpha_2 \frac{2E_0 a k_z}{\pi \omega \mu} e^{-jk_z z} \qquad 6.9b$$

Rectangular Waveguide (Dominant TE_{10} mode)



The time-average power transmitted is given by

$$P_t = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

equate

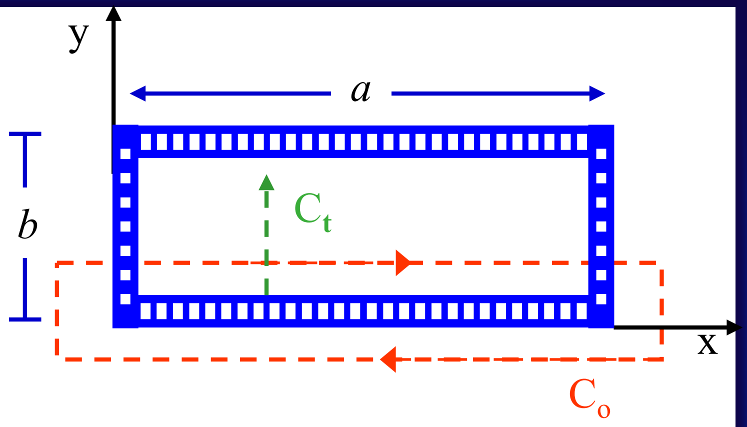
$$\frac{1}{2}\mathrm{Re}\left[V(z)I(z)^*\right] = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

$$\frac{1}{2}(\alpha_1 E_0 b)\left(\alpha_2\frac{2E_0 a k_z}{\pi\omega\mu}\right) = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2$$

$$\alpha_1\alpha_2\frac{ab}{\pi}\frac{k_z}{\omega\mu}E_0^2 = \frac{ab}{4}\frac{k_z}{\omega\mu}E_0^2 \Rightarrow \alpha_1\alpha_2 = \frac{\pi}{4}$$

choose $\alpha_1 = \sqrt{\dfrac{ak_z}{2b\omega\mu}}$ and $\alpha_2 = \sqrt{\dfrac{\pi^2 b\omega\mu}{8ak_z}} \Rightarrow Z_0 = 1$

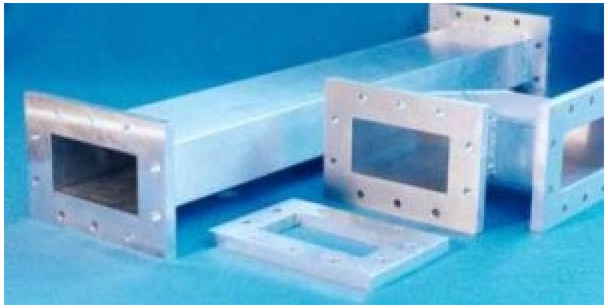
Rectangular Waveguide (Dominant TE_{10} mode)



$$V(z) = \sqrt{\frac{ak_z}{2b\omega\mu}} E_0 \, b \, e^{-jk_z z}$$

$$I(z) = \sqrt{\frac{\pi^2 b\omega\mu}{8ak_z}} \frac{2E_0 a k_z}{\pi\omega\mu} e^{-jk_z z}$$

$$Z_0 = 1 \ [\Omega]$$



http://www.fmmicrowave.com/_images/hohlleit.jpg

Transmission Line Equations
(Parallel plate waveguide)

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\frac{\partial}{\partial z} E_x = -j\omega\mu H_y$$

$$\mathrm{V}(z) = E_0 d e^{-jkz} = d\ E_x$$

$$\frac{\partial \mathrm{V}}{\partial z} = -j\omega \mathrm{LI}$$

$$\mathrm{L} = \frac{\mu d}{w}\left[\frac{\mathrm{H}}{\mathrm{m}}\right]$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

$$-\frac{\partial}{\partial z} H_y = j\omega\varepsilon E_x$$

$$\mathrm{I}(z) = \frac{E_0 w}{\eta} e^{-jkz} = w H_y$$

$$\frac{\partial \mathrm{I}}{\partial z} = -j\omega \mathrm{CV}$$

$$\mathrm{C} = \frac{\varepsilon w}{d}\left[\frac{\mathrm{F}}{\mathrm{m}}\right]$$

Transmission Line Equations
(Parallel plate waveguide)

$$\frac{\partial^2 V}{\partial z^2} + \omega^2 LCV = 0 \qquad \text{wave equation} \qquad 6.15$$

$$V = V_+ e^{-jkz} + V_- e^{+jkz} \qquad 6.16$$

$$I = \frac{1}{Z_0}\left[V_+ e^{-jkz} + V_- e^{+jkz}\right] \qquad 6.17$$

$$k = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad 6.18$$

Standing Waves on Terminated Transmission Lines

$$V(z) = V^{+} e^{-jkz} + V^{-} e^{+jkz} \qquad 6.22$$

$$I(z) = \frac{V^{+}}{Z_0} e^{-jkz} - \frac{V^{-}}{Z_0} e^{+jkz} \qquad 6.23$$

Impedance $\quad Z(z) = \dfrac{V(z)}{I(z)}$

$$Z_n(z) = \frac{Z(z)}{Z_0} = \frac{V^{+} e^{-jkz} + V^{-} e^{+jkz}}{V^{+} e^{-jkz} - V^{-} e^{+jkz}} \qquad 6.24$$

(note $Z(0) = Z_L$)

Standing Waves on Terminated Transmission Lines

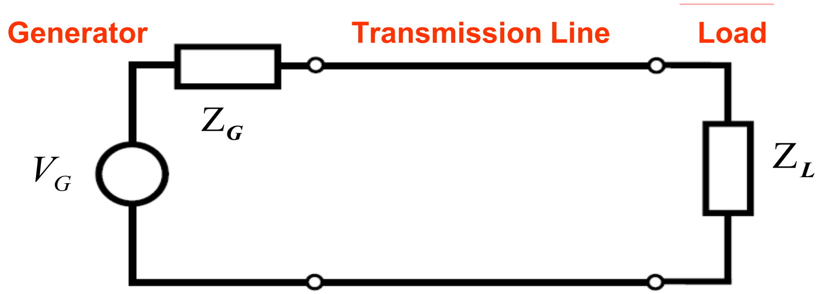
Reflection coefficient

$$\Gamma_L = \frac{V^-}{V^+}$$

With the reflection coefficient now defined, we can rewrite 6.24 as

$$Z_n(z) = \frac{e^{-jkz} + \Gamma_L e^{+jkz}}{e^{-jkz} - \Gamma_L e^{+jkz}}$$

Standing Waves on Terminated Transmission Lines



Evaluating Z_n at the Load ($z = 0$)

$$Z_{nL}(z=0) = \frac{Z(z=0)}{Z_0} = \frac{1+\Gamma_L}{1-\Gamma_L} \qquad \left(\text{or } \Gamma_L = \frac{Z_{nL}-1}{Z_{nL}+1}\right) \qquad 6.25$$

Evaluating Z_n at a point along line ($z = -l$)

$$Z_n(z=-l) = \frac{Z(z=-l)}{Z_0} = \frac{Z_L + jZ_0 \tan kl}{Z_0 + jZ_L \tan kl} \qquad 6.27$$

Standing Waves on Terminated Transmission Lines

$$\left|V_{(z)}\right| = \left|V^{+}\right|\left|1+\Gamma_L e^{j2kz}\right|$$

with $\Gamma_L = \left|\Gamma_L\right| e^{j\varphi}$

$$\left|V_{(z)}\right| = \left|V^{+}\right|\left|1+\left|\Gamma_L\right| e^{j(\varphi+2kz)}\right|$$

V_{max} when $\varphi + 2kz = 0\ ,\ -2\pi, \ldots$

V_{min} when $\varphi + 2kz = -\pi\ ,\ -3\pi, \ldots$

Standing Wave Pattern

$$\left|V_{(z)}\right| = \left|V^{+}\right| \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1 + \left|\Gamma_L\right| e^{j(\varphi + 2kz)}\right|$$

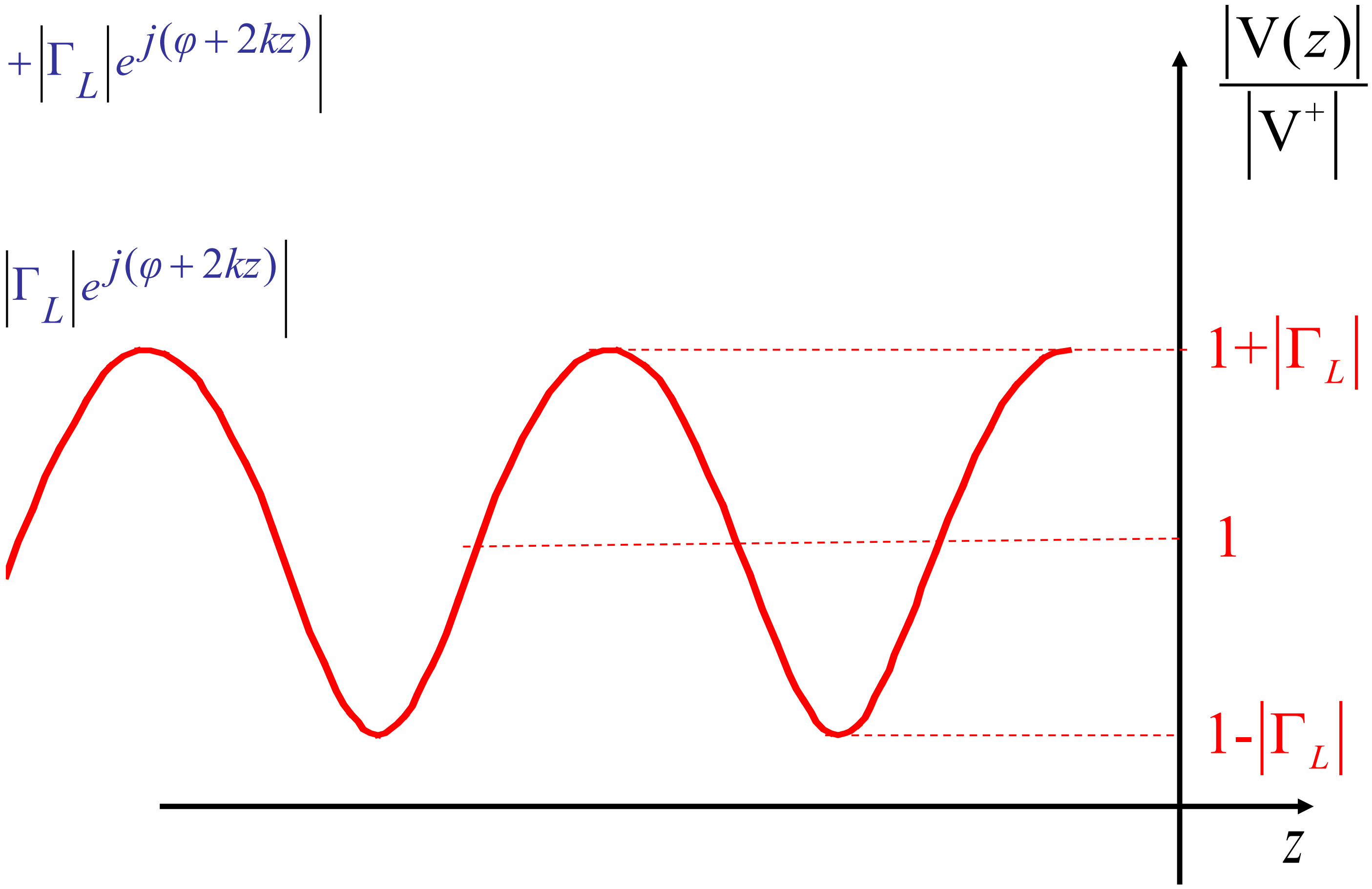


Fig. 6.6

Crank Diagram

$$\left|V_{(z)}\right| = \left|V^{+}\right|\left|1+\left|\Gamma_L\right|e^{j(\varphi+2kz)}\right|$$

$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1+\left|\Gamma_L\right|e^{j(\varphi+2kz)}\right|$$

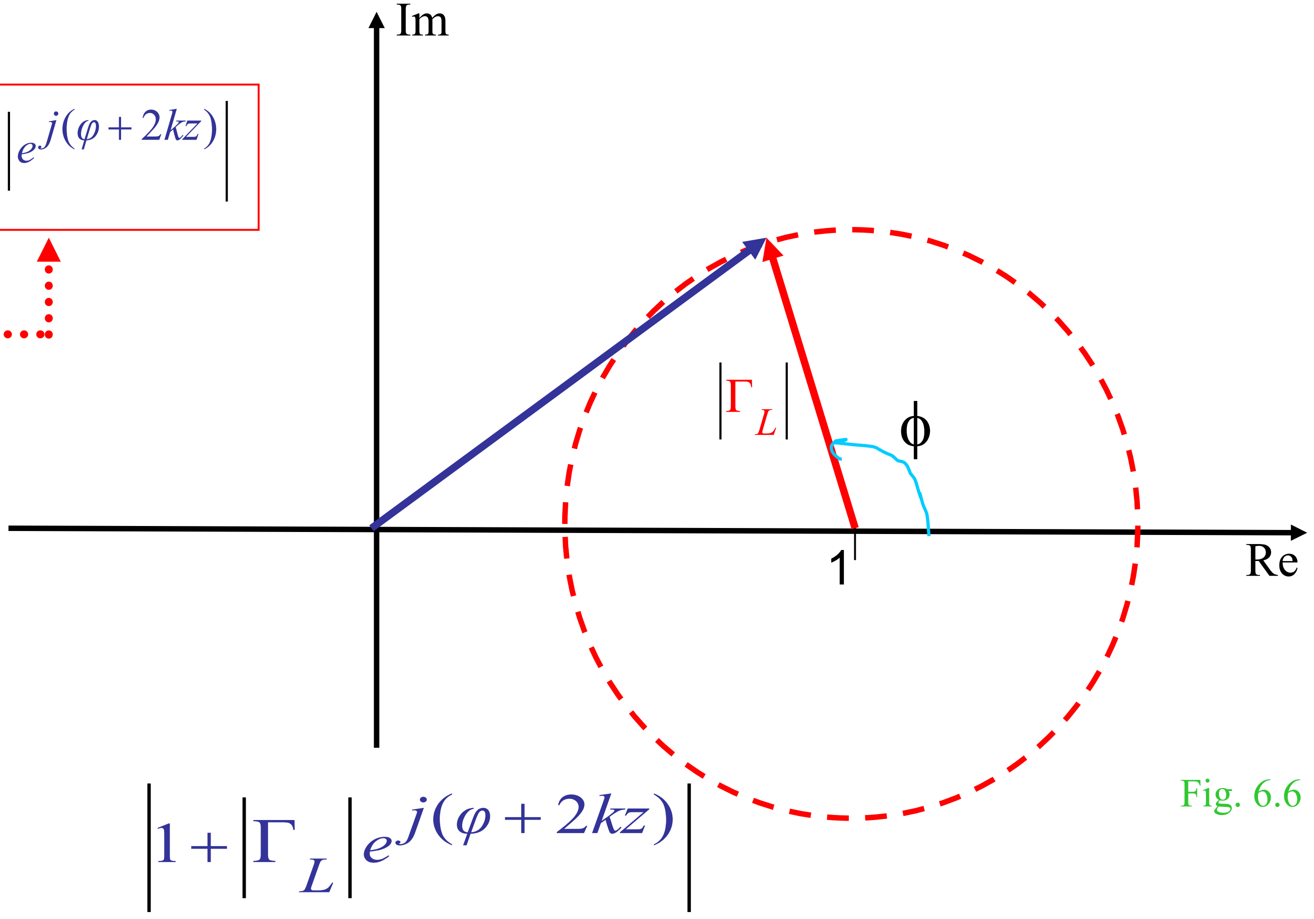
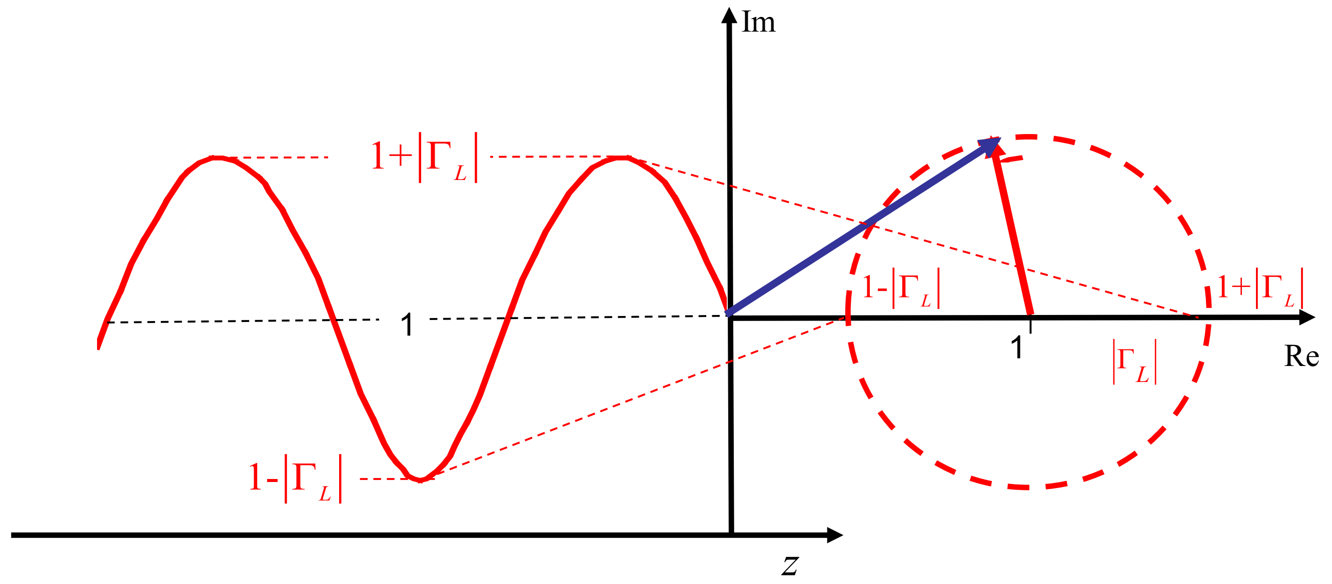


Fig. 6.6

Crank Diagram



$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1+\left|\Gamma_L\right|e^{j(\varphi+2kz)}\right|$$

Standing Wave Ratio (SWR or VSWR)

$$[VSWR] = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$[VSWR] = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \qquad 6.29$$

$$|\Gamma_L| = \frac{[VSWR]-1}{[VSWR]+1} \qquad 6.30$$

To determine $(\angle\Gamma_L)$ or Z_L also need position of V_{min}

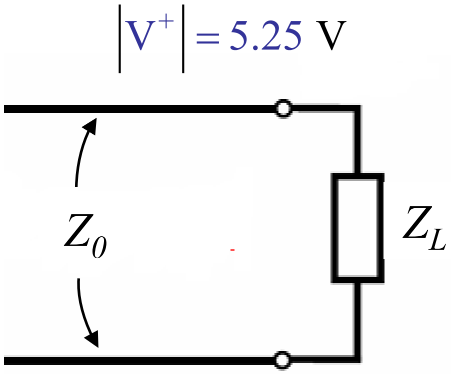
Note :

Power $\sim V^2 \Rightarrow \frac{P^-}{P^+} = |\Gamma_L|^2$

Power to load $\sim 1-|\Gamma_L|^2$

Ex. 6.6

$Z_L = 17.4 - j30 [\Omega]$ and $Z_0 = 50 [\Omega]$



$$\Gamma_L = \frac{Z_{nL} - 1}{Z_{nL} + 1} = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.24 - j.55$$

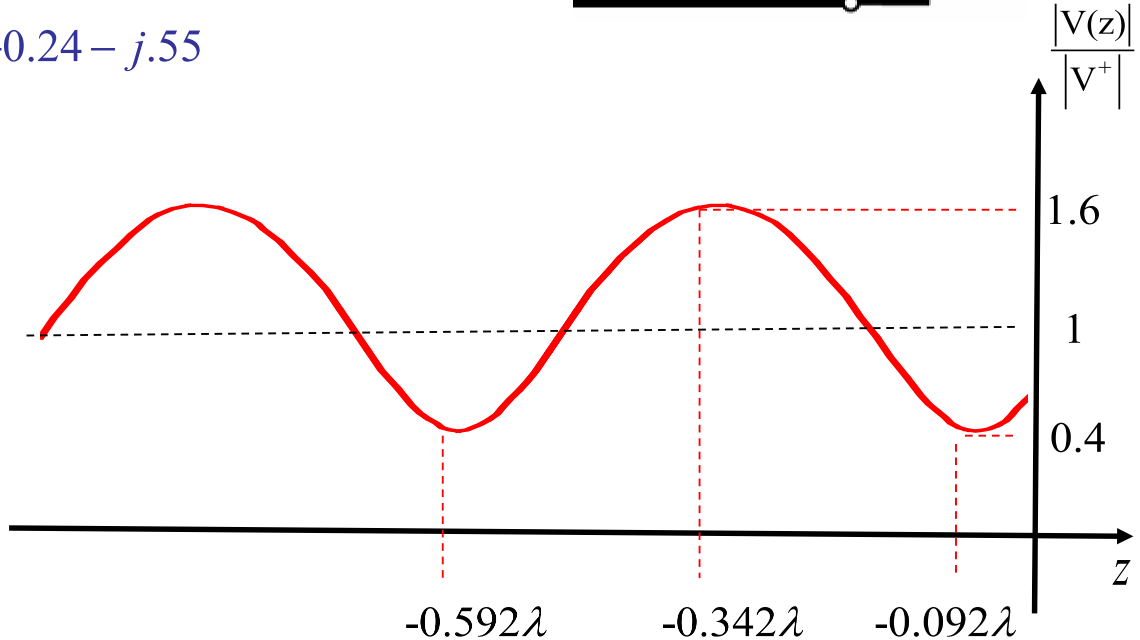
or

$$\Gamma_L = 0.6 e^{-j1.99}$$

where

$$|\Gamma_L| = 0.6$$

$$\phi = \angle \Gamma_L = \text{-1.99 [rad]}$$



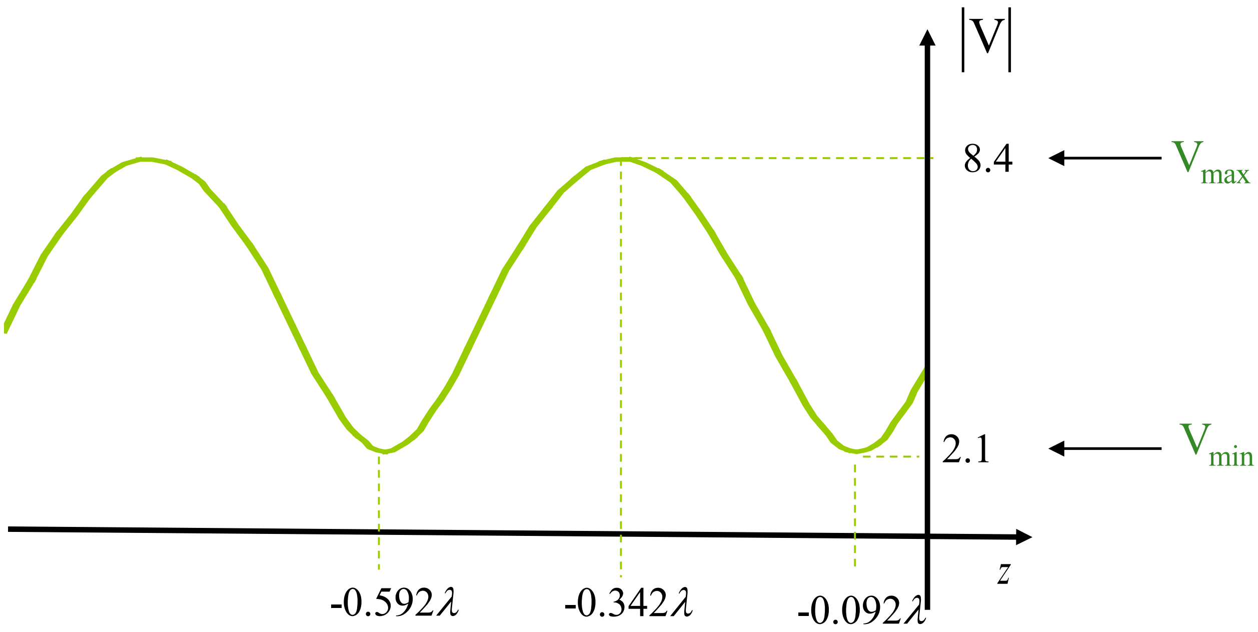
V_{max} when $\varphi + 2kz = 0, -2\pi, \ldots$

V_{min} when $\varphi + 2kz = -\pi, -3\pi, \ldots$

Ex. 6.6

V_{min} where $\phi + 2k\, z_m = -\pi$

$$z_m = \frac{-\pi - \phi}{2k} = \frac{(-\pi + 1.99)\lambda}{2(2\pi)} = -.092\lambda$$



Ex. 6.6

Reverse problem

$$VSWR = \frac{\text{Vmax}}{\text{Vmin}} = \frac{8.4}{2.1} = 4.0$$

$$\left|\Gamma_L\right| = \frac{VSWR\text{-}1}{VSWR+1} = \frac{4-1}{4+1} = 0.6$$

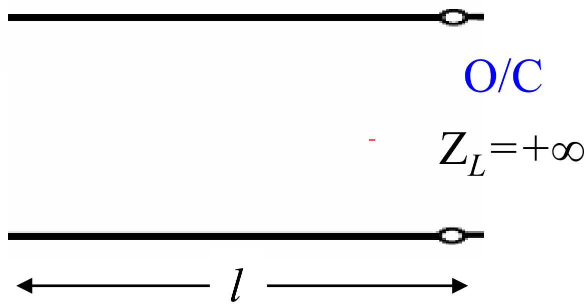
To determine ($\angle\Gamma_L$) or Z_L also need position of V_{min}

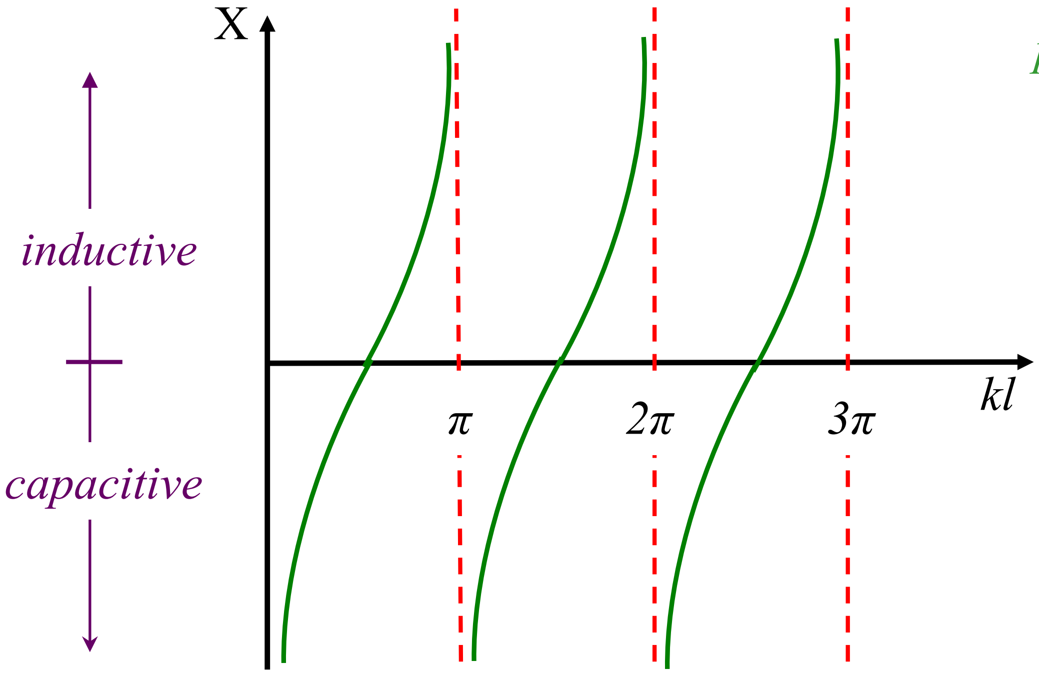
$$\angle\Gamma_L = \phi = -\pi - 2k\,z_m = -\pi - 2\left(\frac{2\pi}{\lambda}\right)(-.092\lambda) = -1.99$$

$$Z_L = Z_0\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] = \boxed{17.4 - j30 \quad \Omega}$$

Open Circuit

$$Z(-l) = Z_0 \frac{Z_L + jZ_0 \tan kl}{Z_0 + jZ_L \tan kl} = Z_0 \frac{1}{j \tan kl}$$

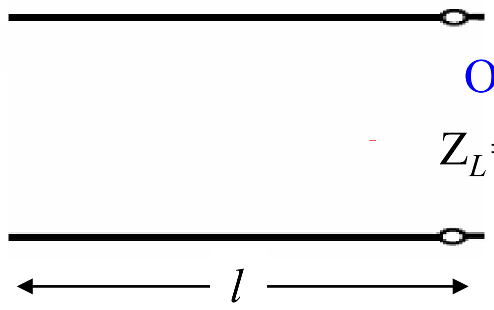




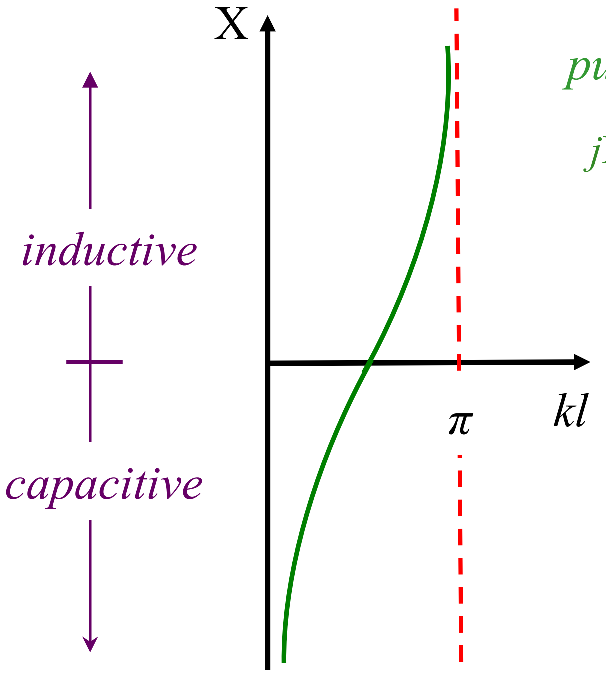
pure imaginary

$$jX = -jZ_0 \cot kl$$

If the impedance is purely imaginary, Z = R+jX becomes Z = jX, where X is called the reactance.



$\lim kl << 1$ (small l or low freq)



pure imaginary

$jX = -jZ_0 \cot kl$

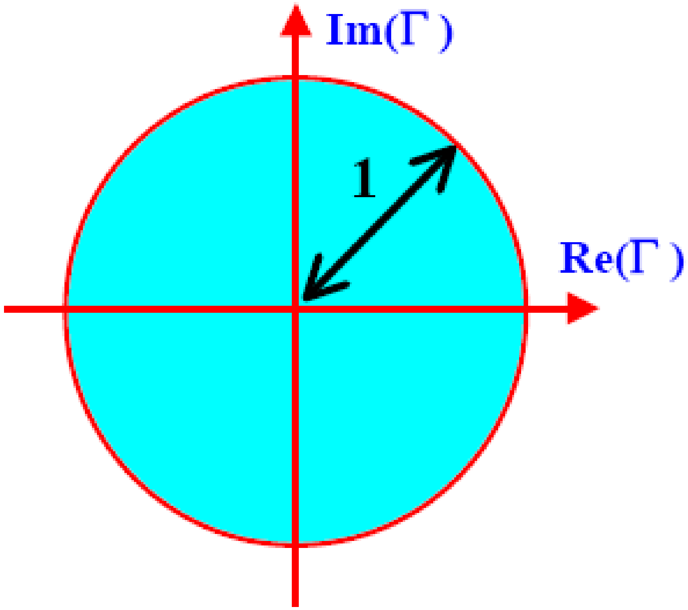
Note: at $kl = \frac{\pi}{2} \Rightarrow \left(l = \frac{\lambda}{4} \right)$

$Z\left(-\lambda/4\right) = 0$ (short circuit)

near $kl \leq \pi$ Z is inductive

near $kl \geq 0$ Z is capacitive

Philip Smith



• Shows the entire universe of complex impedances in one convenient circle.

• Invented at Bell Labs by Philip Smith in 1937.

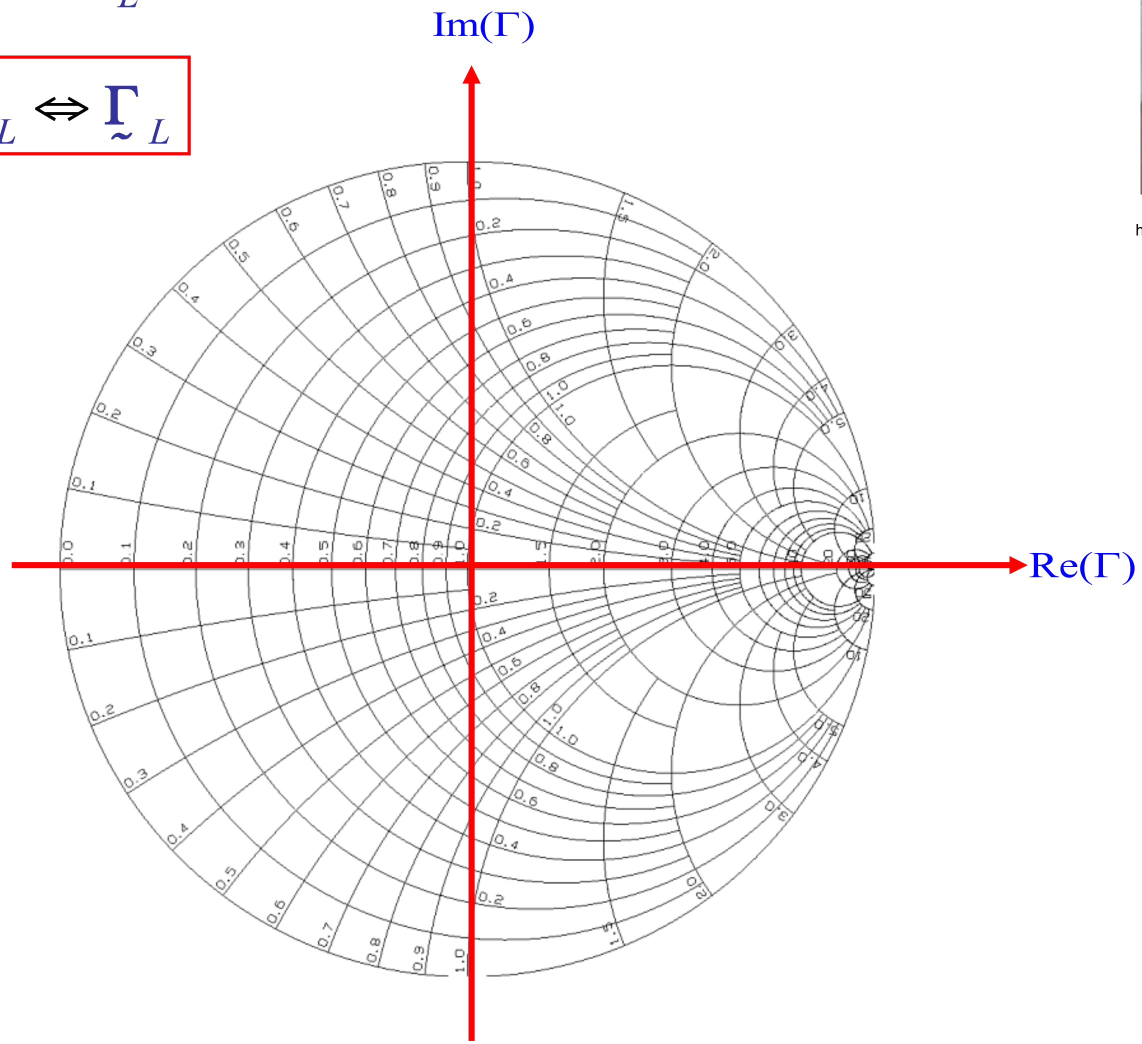
• By 1975 about 9 million copies of his chart sold to microwave engineers all over the world.

• Its usefulness continues to this day as a method of displaying measured and calculated data produced by computer software and modern measurement instruments.

Smith Chart

$$Z_{nL} = \frac{1+\Gamma_L}{1-\Gamma_L} = R + jX$$

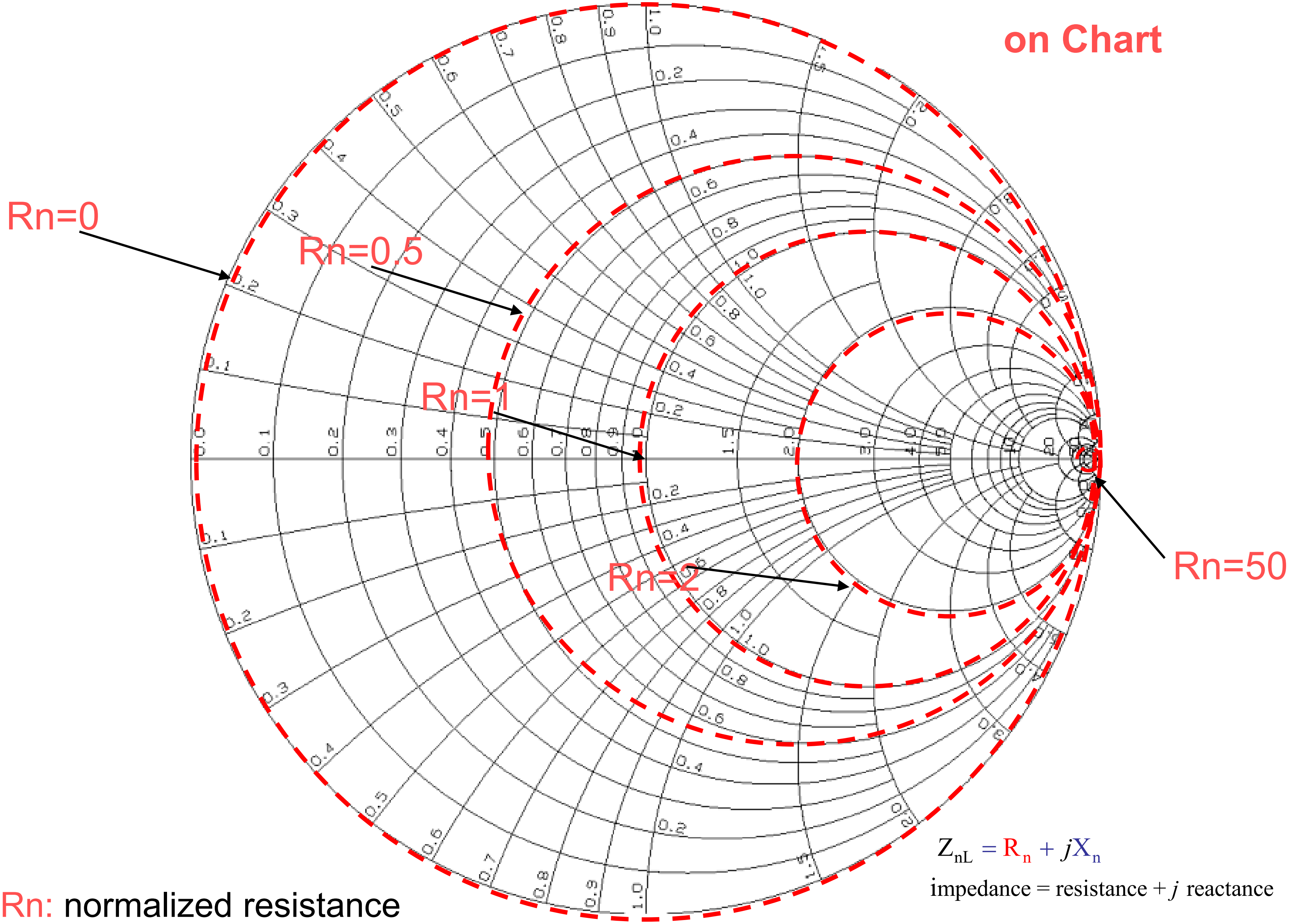
$$\underset{\sim}{Z}_{nL} \Leftrightarrow \underset{\sim}{\Gamma}_L$$



http://my.ece.ucsb.edu/sanabria/images/chris_tatoo2.jpg

Z Chart

Constant R Circles on Chart



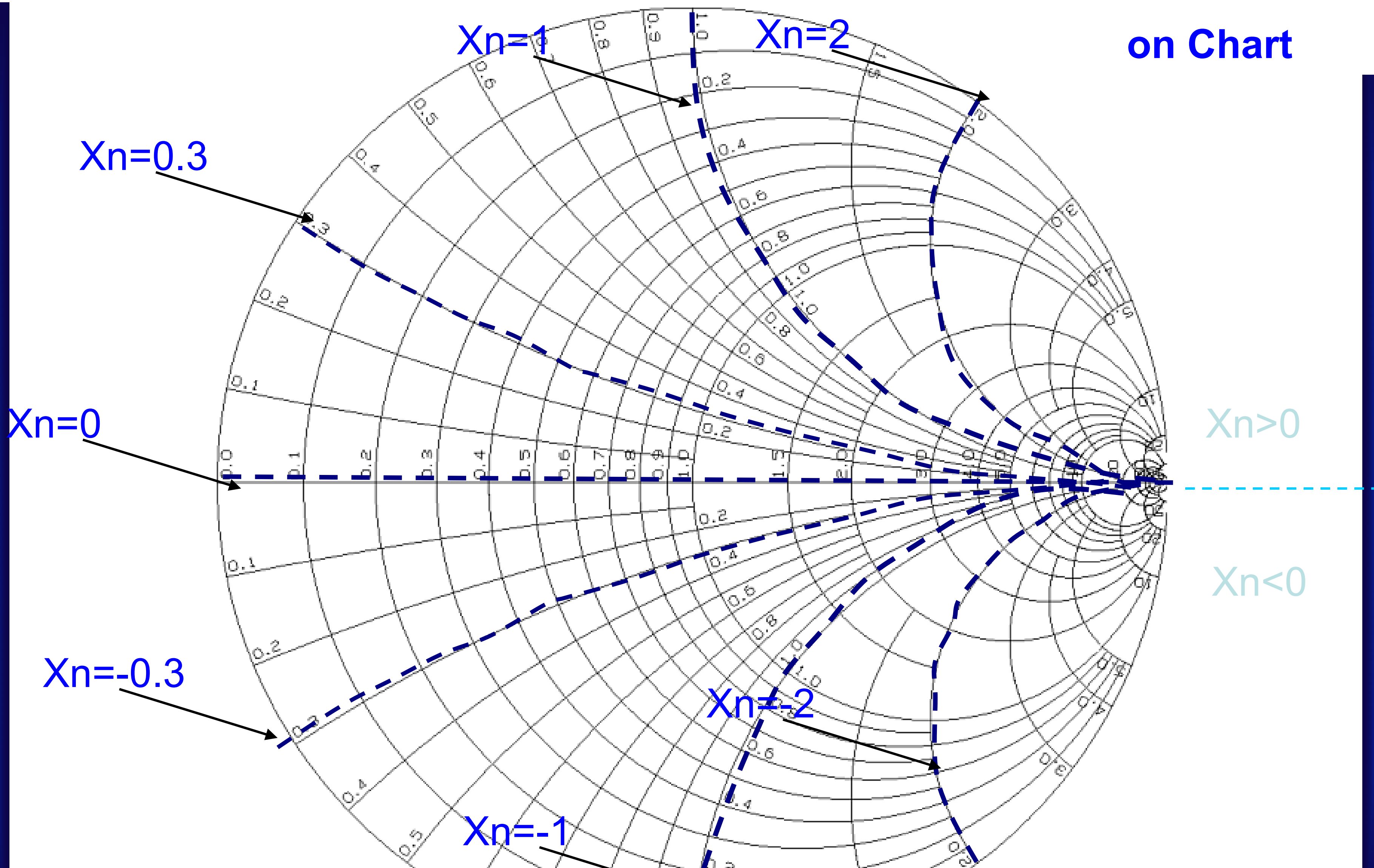
$$Z_{nL} = R_n + jX_n$$

impedance = resistance + j reactance

Rn: normalized resistance

Z Chart

Constant X Arcs on Chart

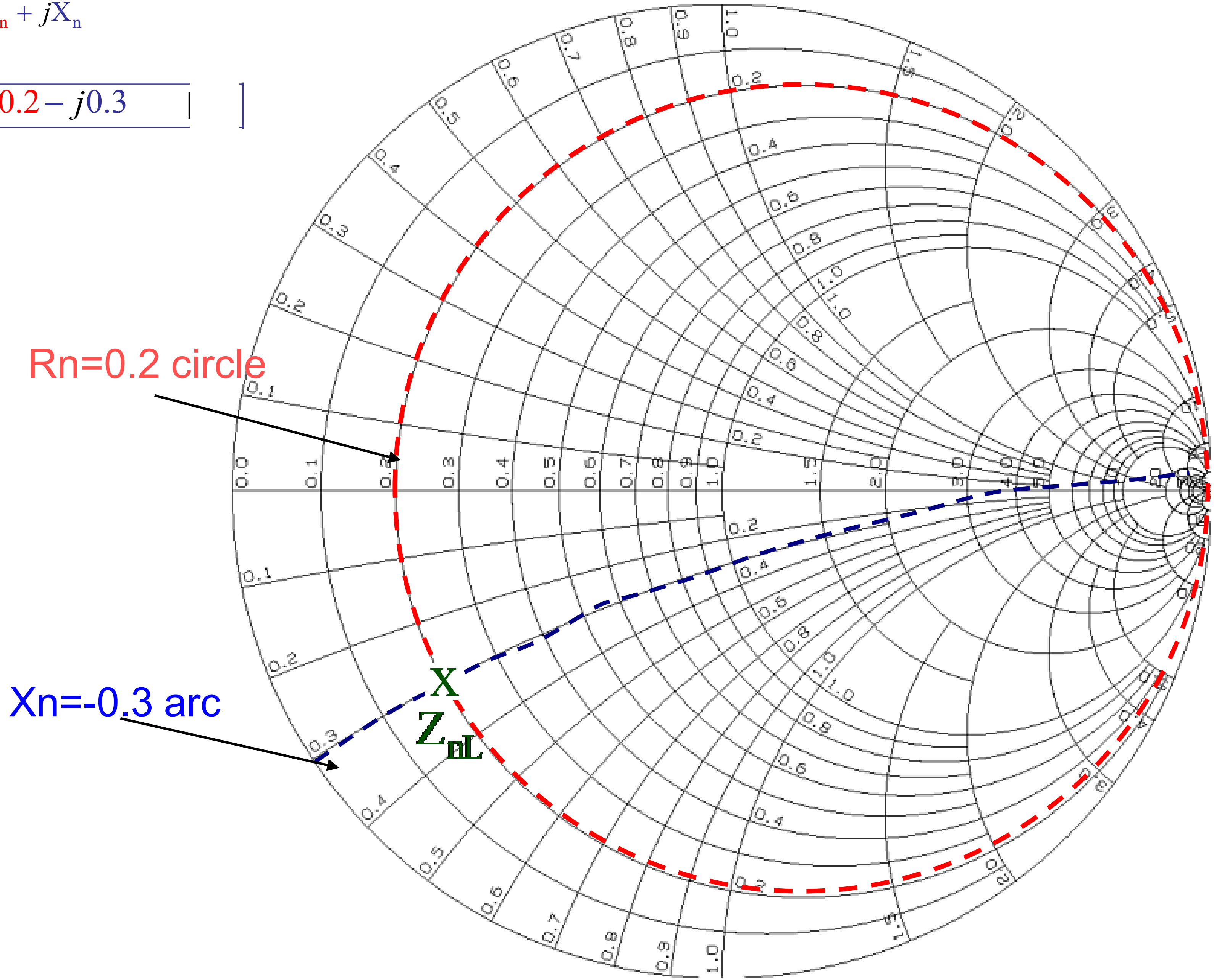


$Z_{nL} = R_n + jX_n$

impedance = resistance + *j* reactance

$Z_{nL} = R_n + jX_n$

$Z_{nL} = 0.2 - j0.3$

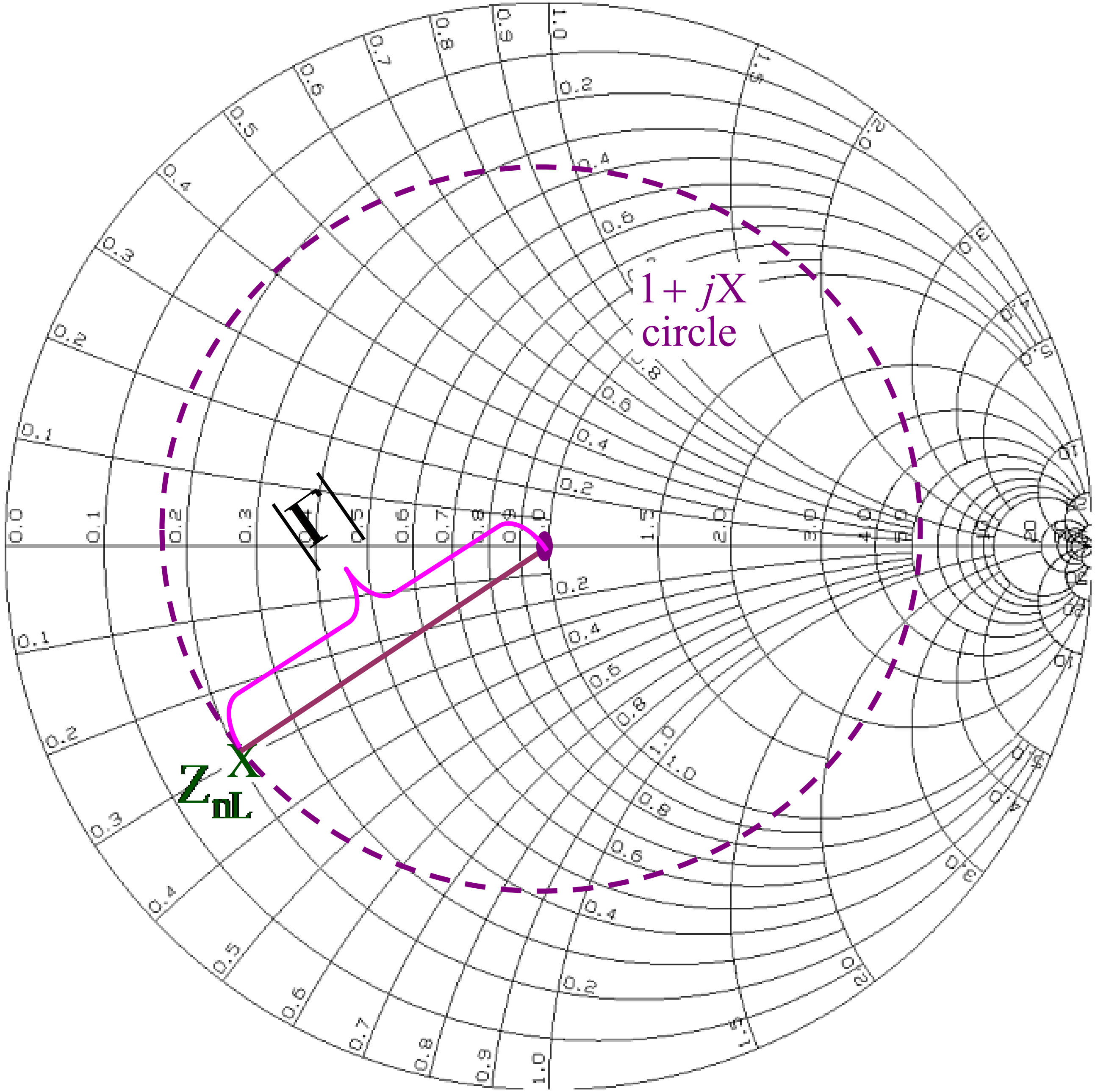


$Z_{nL} = R_n + jX_n \quad [\Omega]$

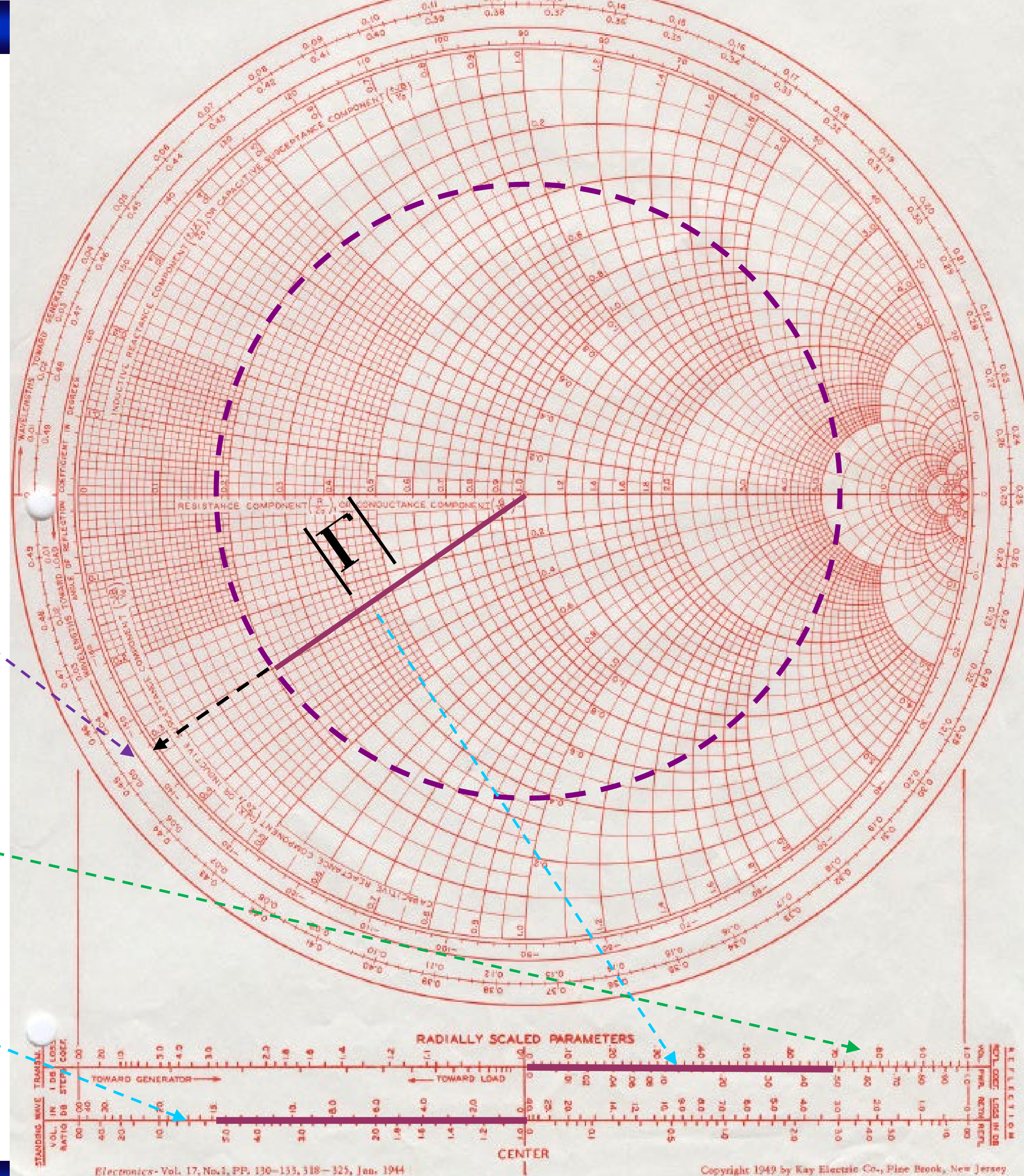
$Z_{nL} = 0.2 - j0.3 \quad [\Omega]$

Using your compass draw the 0.2-j0.3 circle with center at z_n=1.

(NOTE: this circle is not on the chart until you draw it.)



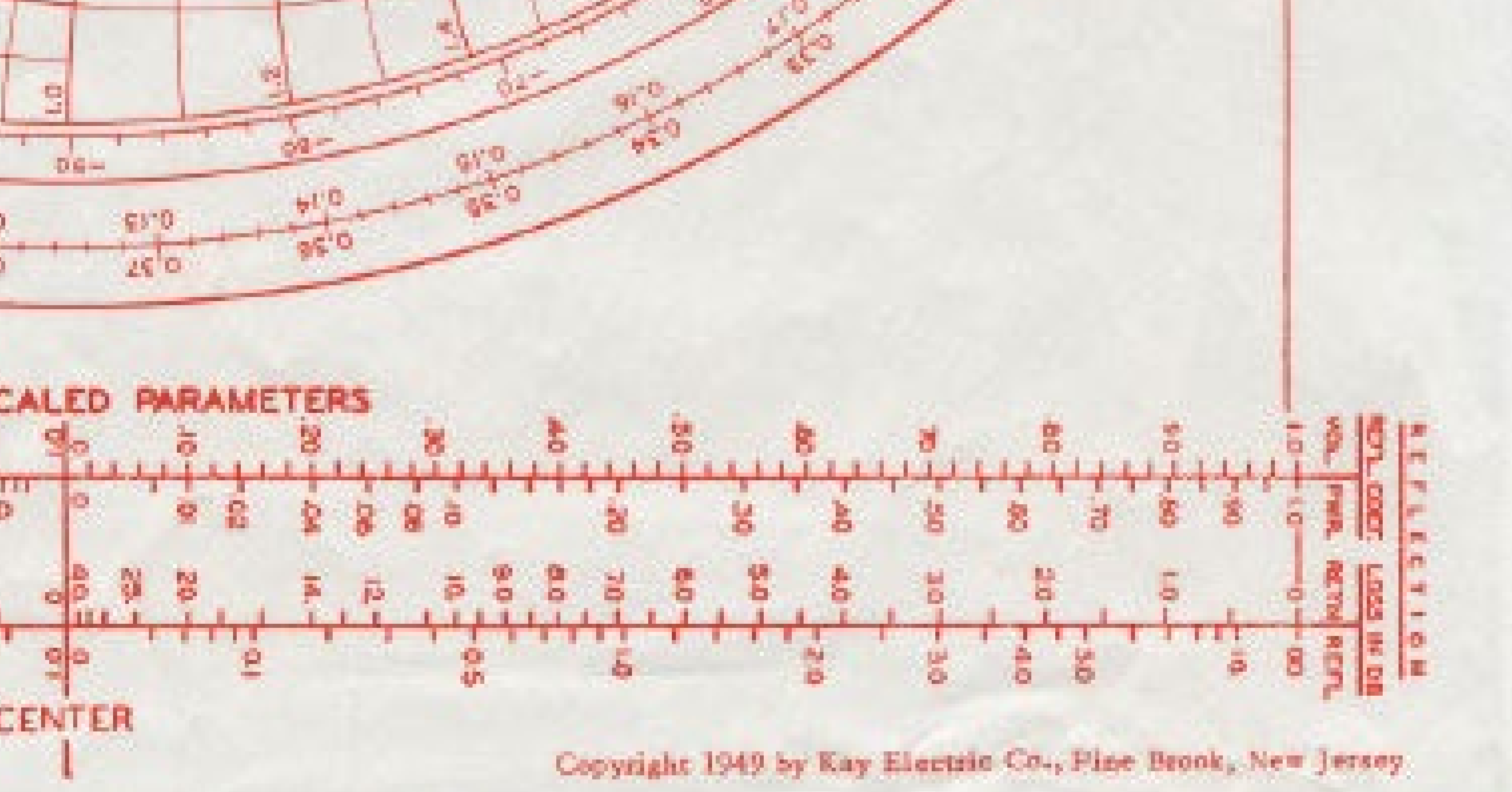
Find the value of reflection coefficient for ZnL = 0.2 – j0.3

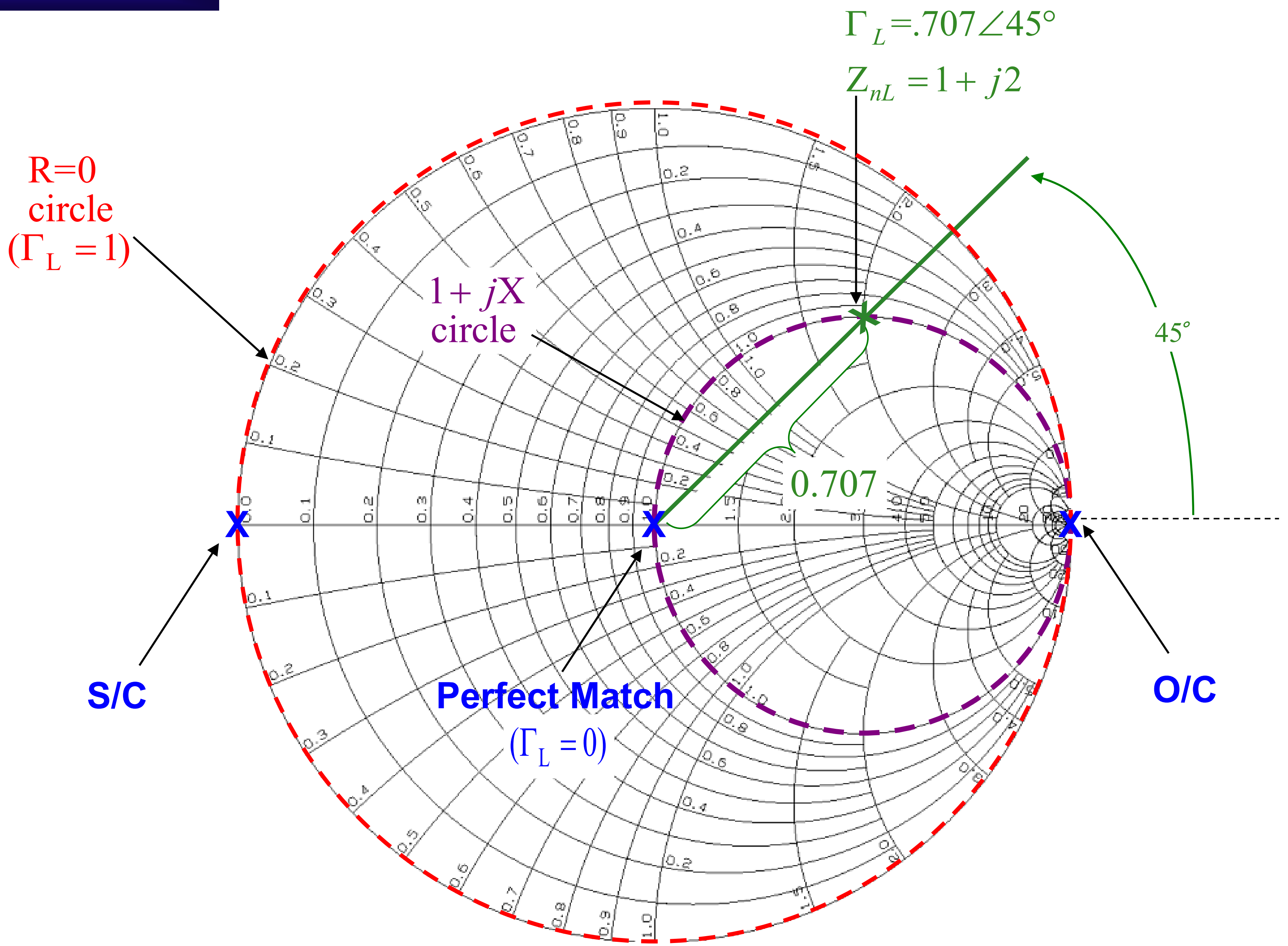


Reflection Coefficient PHASE = -145.4 degrees

Reflection Coeff MAGNITUDE = 0.691

VSWR =5.47


CALED PARAMETERS
CENTER
Copyright 1949 by Kay Electric Co., Pine Brook, New Jersey

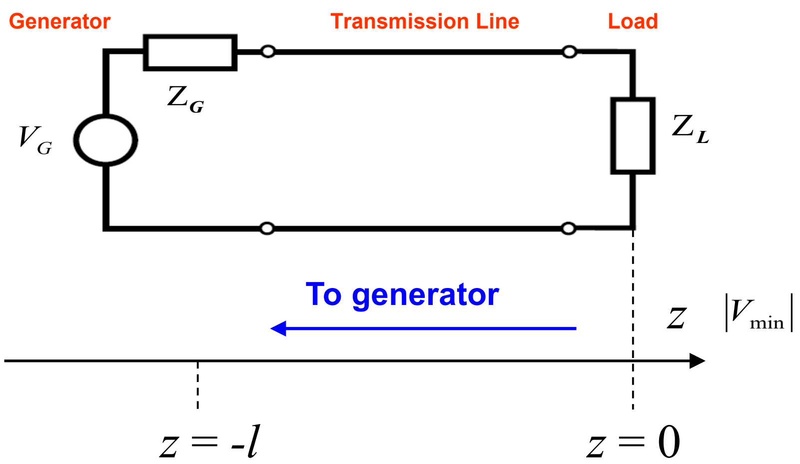


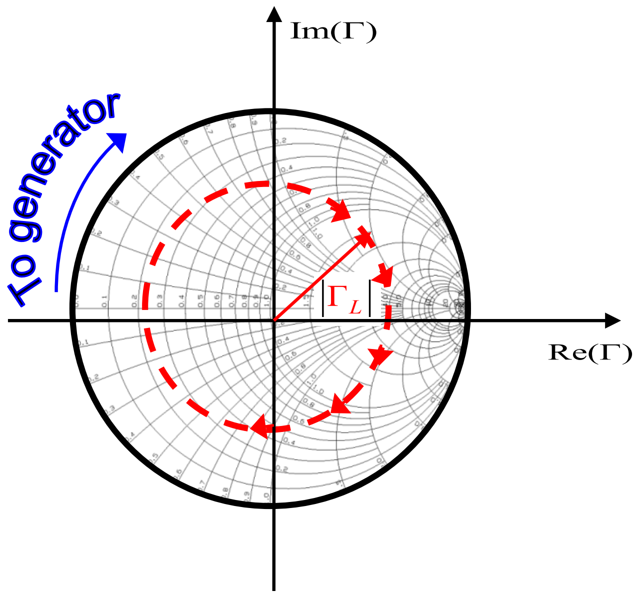
Γ L =.707∠45°
Z nL = 1 + j2
R=0
circle
(Γ L = 1)
1 + jX
circle
0.707
45°
S/C
Perfect Match
(Γ L = 0)
O/C

fig 6.13

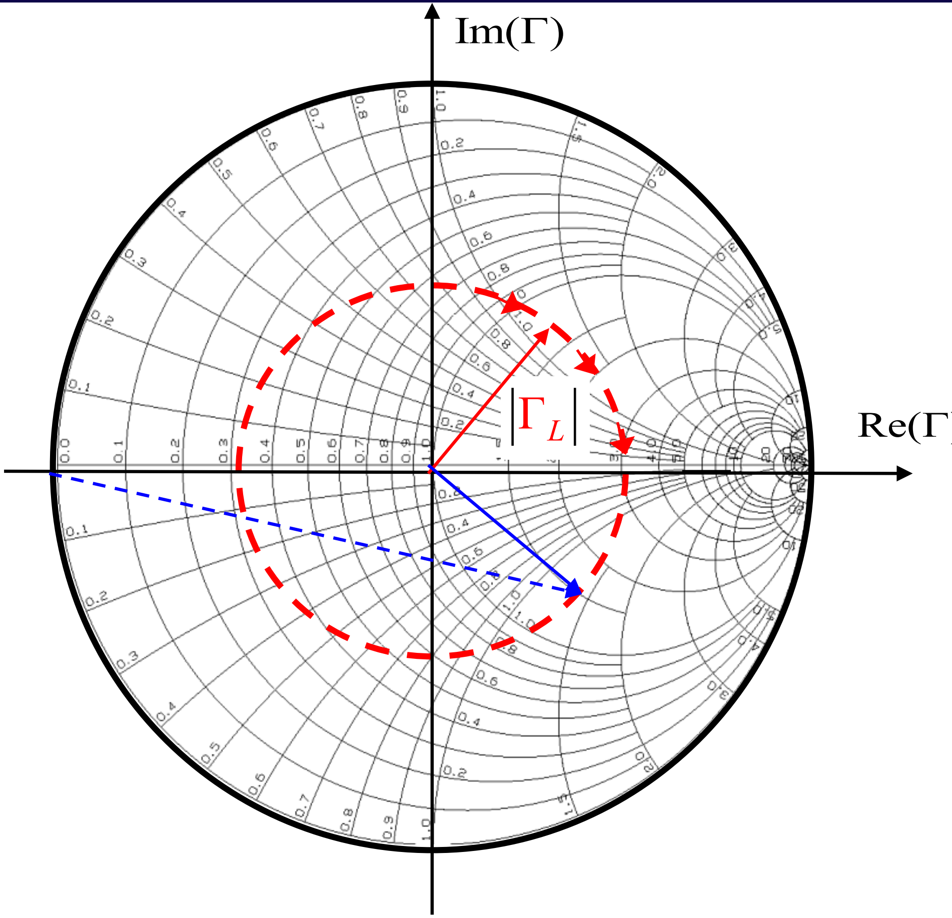
$$Z(z) = Z_0 \frac{e^{-jkz} + \Gamma_L e^{+jkz}}{e^{-jkz} - \Gamma_L e^{+jkz}} = Z_0 \frac{1 + \Gamma_L e^{+2jkz}}{1 - \Gamma_L e^{+2jkz}}$$

movement in negative $\hat{z}$ direction (toward generator) $\Leftrightarrow$ clockwise motion on circle of constant $|\Gamma_L|$





$$\frac{\left|V_{(z)}\right|}{\left|V^{+}\right|} = \left|1+\Gamma_L e^{j2kz}\right|$$



Note:

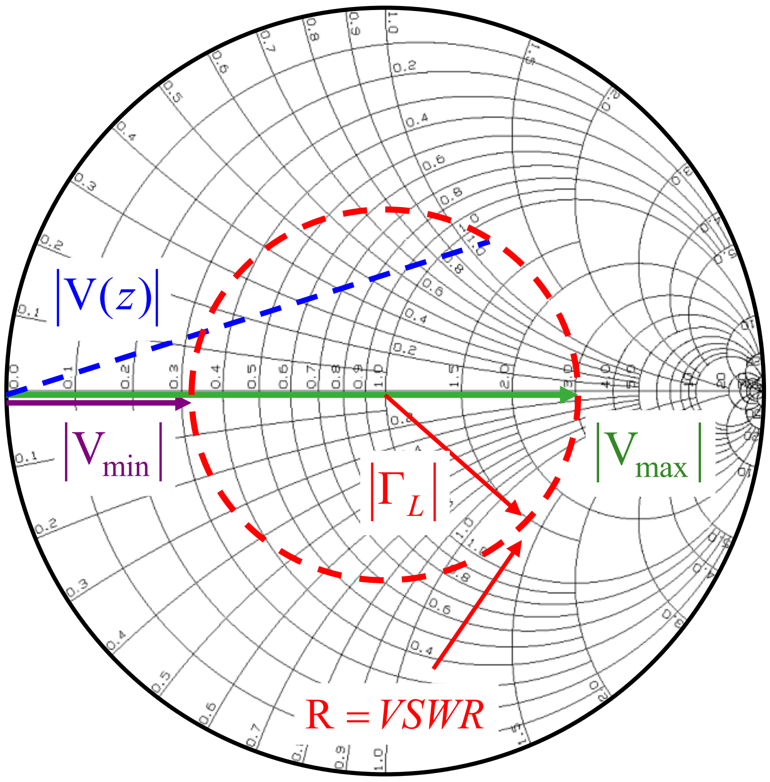
complete circle $(360° = 2\pi \text{ rad}) = \frac{\lambda}{2}$

$Y_n(z) = \frac{1}{Z_n(z)} \Rightarrow$ can just replace Γ_L by $(-\Gamma_L)$

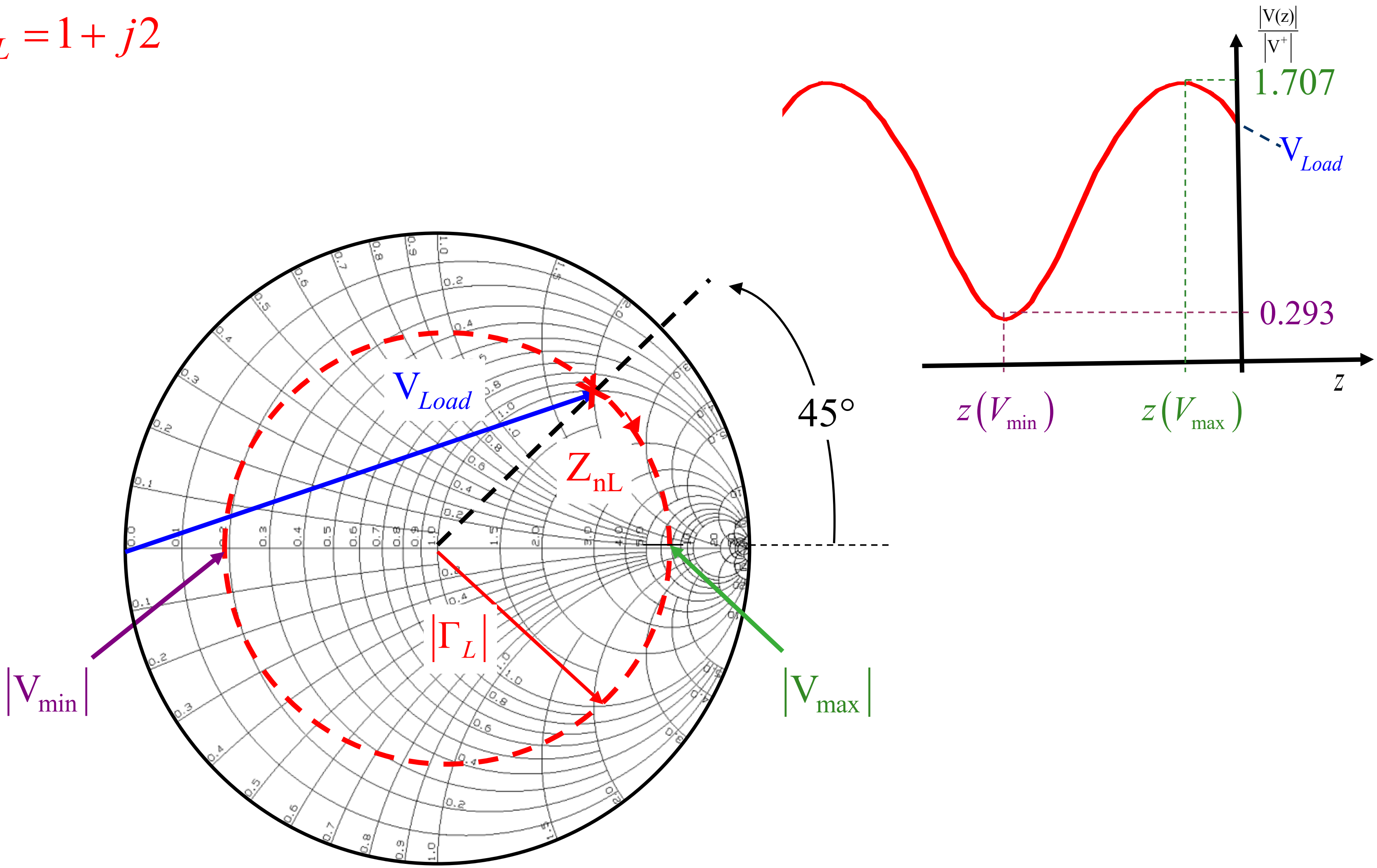
$$\left|V_{max}\right| = 1 + \left|\Gamma_L\right| \text{ (RH Real axis)}$$

$$\left|V_{min}\right| = 1 - \left|\Gamma_L\right| \text{ (LH Real axis)}$$

$$\frac{\left|V(z)\right|}{\left|V^+\right|} = \left|1 + \Gamma_L e^{2jkz}\right|$$



$$Z_{nL} = 1 + j2$$

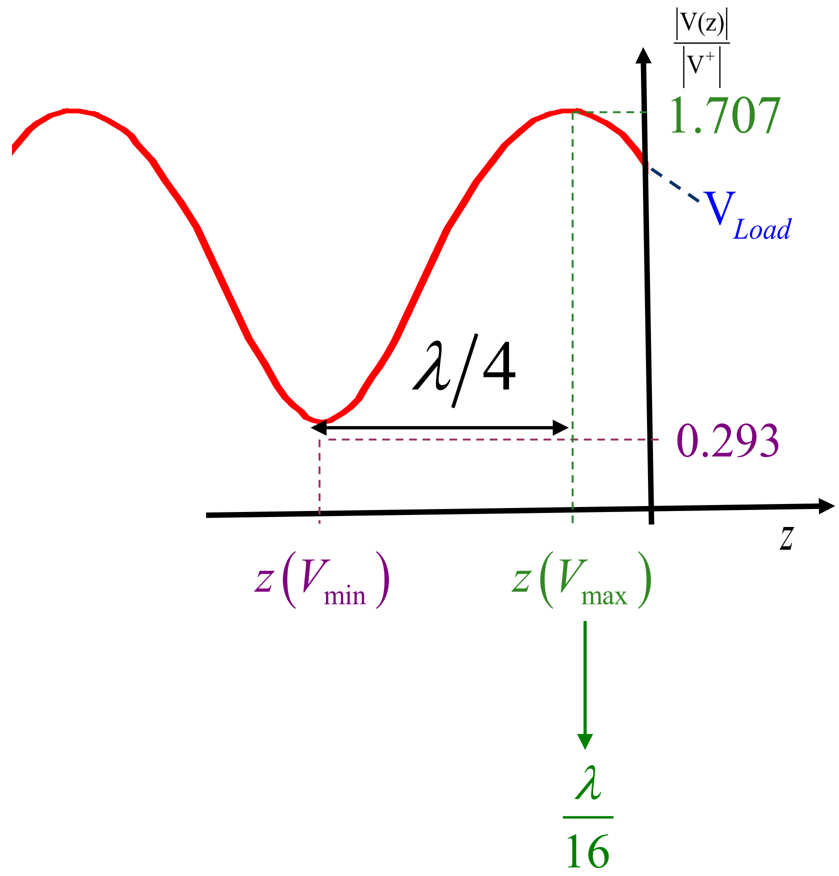


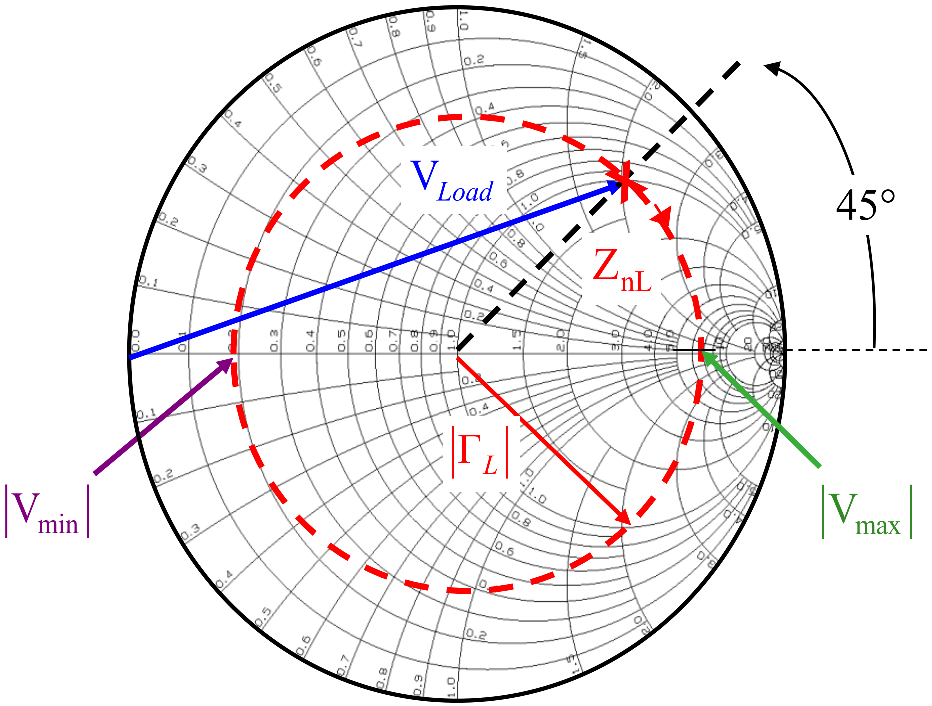
$Z_{nL} = 1 + j2$

$\Gamma = .707\angle 45°$

$360° \text{ on Smith Chart} = \dfrac{\lambda}{2}$

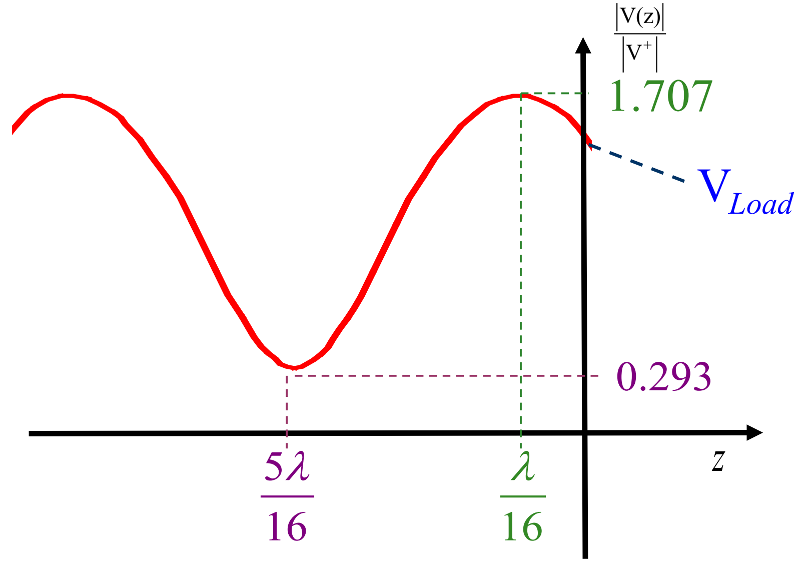
$180° \text{ on Smith Chart} = \dfrac{\lambda}{4}$

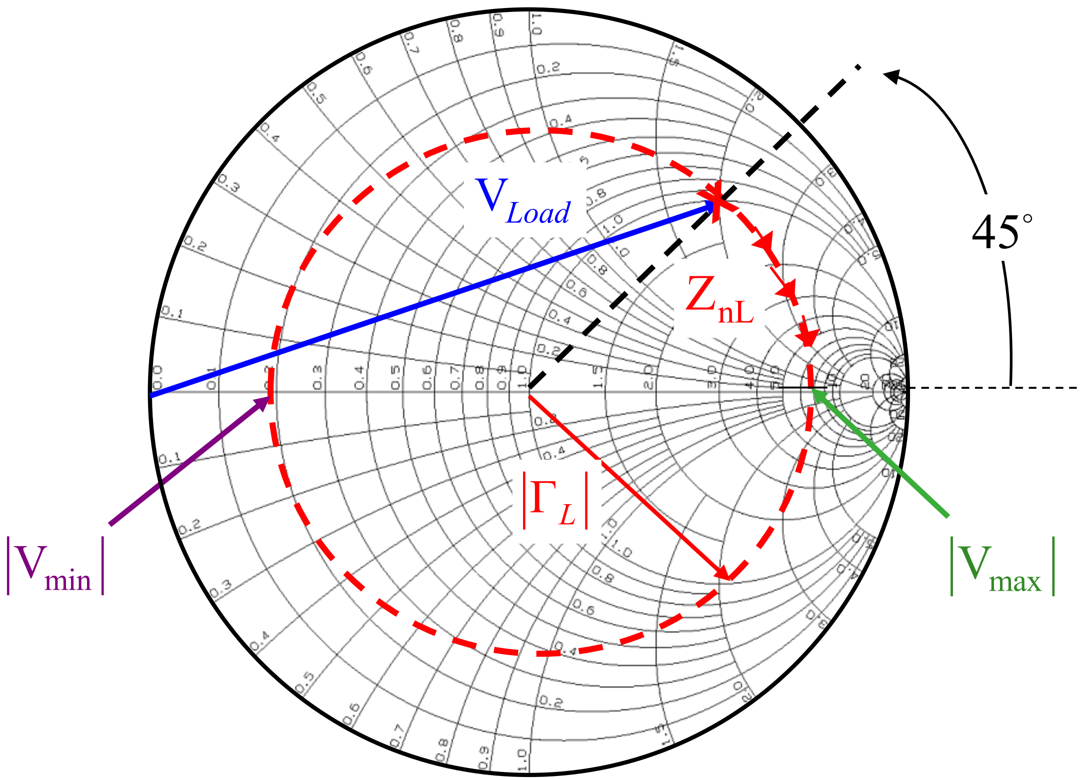




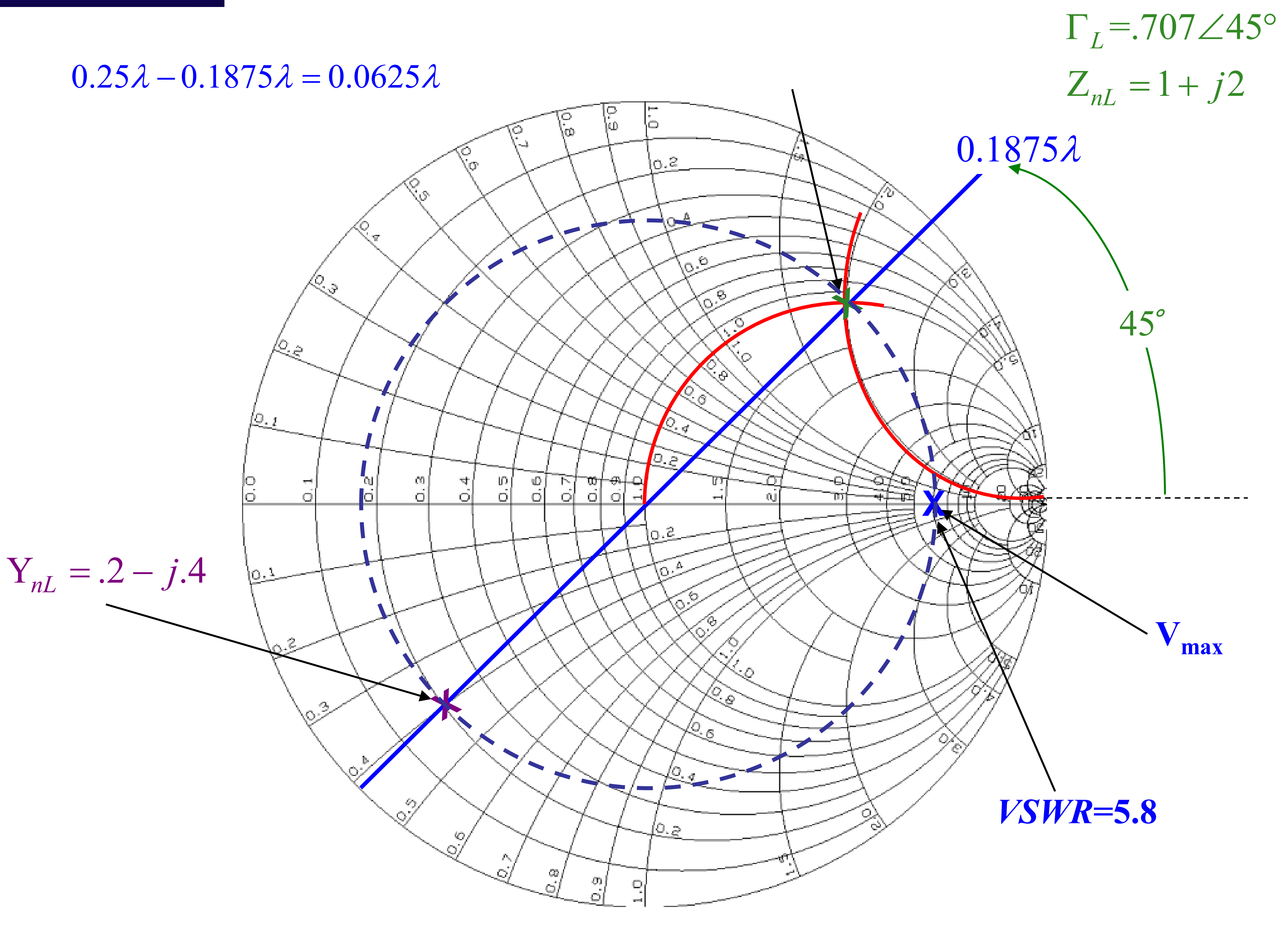
$Z_{nL} = 1 + j2$

$\Gamma = .707\angle 45^\circ$





Smith Chart



Γ_L =.707∠45°
Z_nL = 1 + j2
0.25λ − 0.1875λ = 0.0625λ
0.1875λ
45°
Y_nL = .2 − j.4
V_max
VSWR=5.8

Usually want power to be absorbed by load (minimize $|\Gamma_L|^2$).
To do so one adds pure reactances (or susceptances) to tune or match the network.

$$Z = R + jX \qquad [\Omega]$$

impedance = resistance + j reactance

$$Y = G + jB \qquad [S]$$

admittance = conductance + j susceptance

Note: It is physically easier to add a shunt susceptance than series reactance.

Example: Given $Z_{nL} = 2 + j2 \Rightarrow \Gamma = 0.62\angle 30°$

$$|\Gamma|^2 = 0.62^2 = 38\%$$ power reflected

change from $Z_{nL} \Rightarrow Y_{nL} = G_{nL} + jB_{nL}$

$$Y_{nL} = \frac{1}{Z_{nL}} = \frac{1}{2 + j2} = 0.25 - j0.25$$

could add $+j.25$ at load

$$Y_{nL} = 0.25 \Rightarrow \Gamma = 0.6\angle 0°$$

$$|\Gamma|^2 = 0.6^2 = 36\%$$ power reflected

instead rotate toward generator to $1+jB$ circle and add a $-jB$ there.

Smith Chart

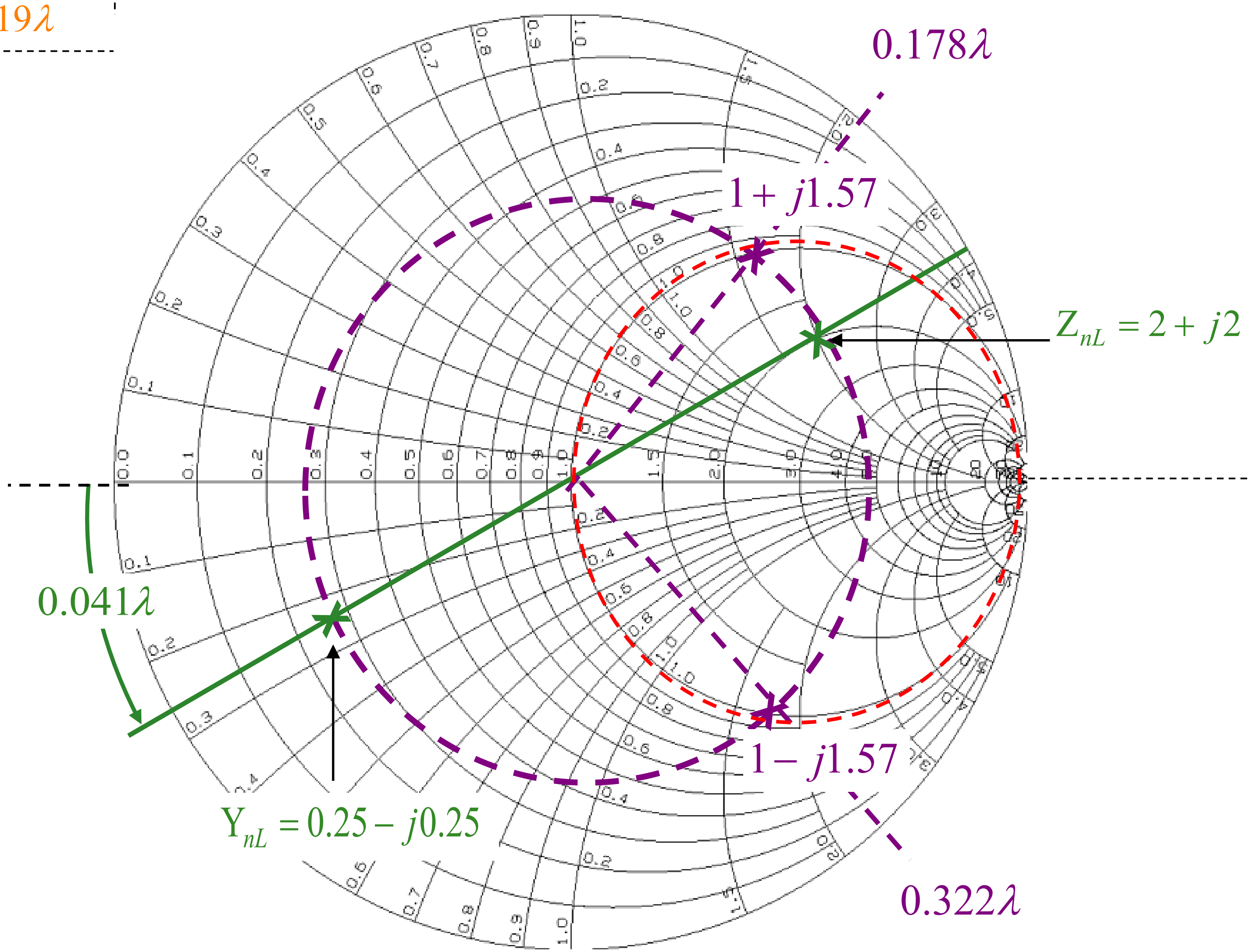
Solution:

Add $+j1.57$ at 0.362λ

Or $-j1.57$ at 0.219λ

$$0.041\lambda + 0.178\lambda = 0.219\lambda$$

$$0.041\lambda + 0.322\lambda = 0.363\lambda$$



Smith Chart

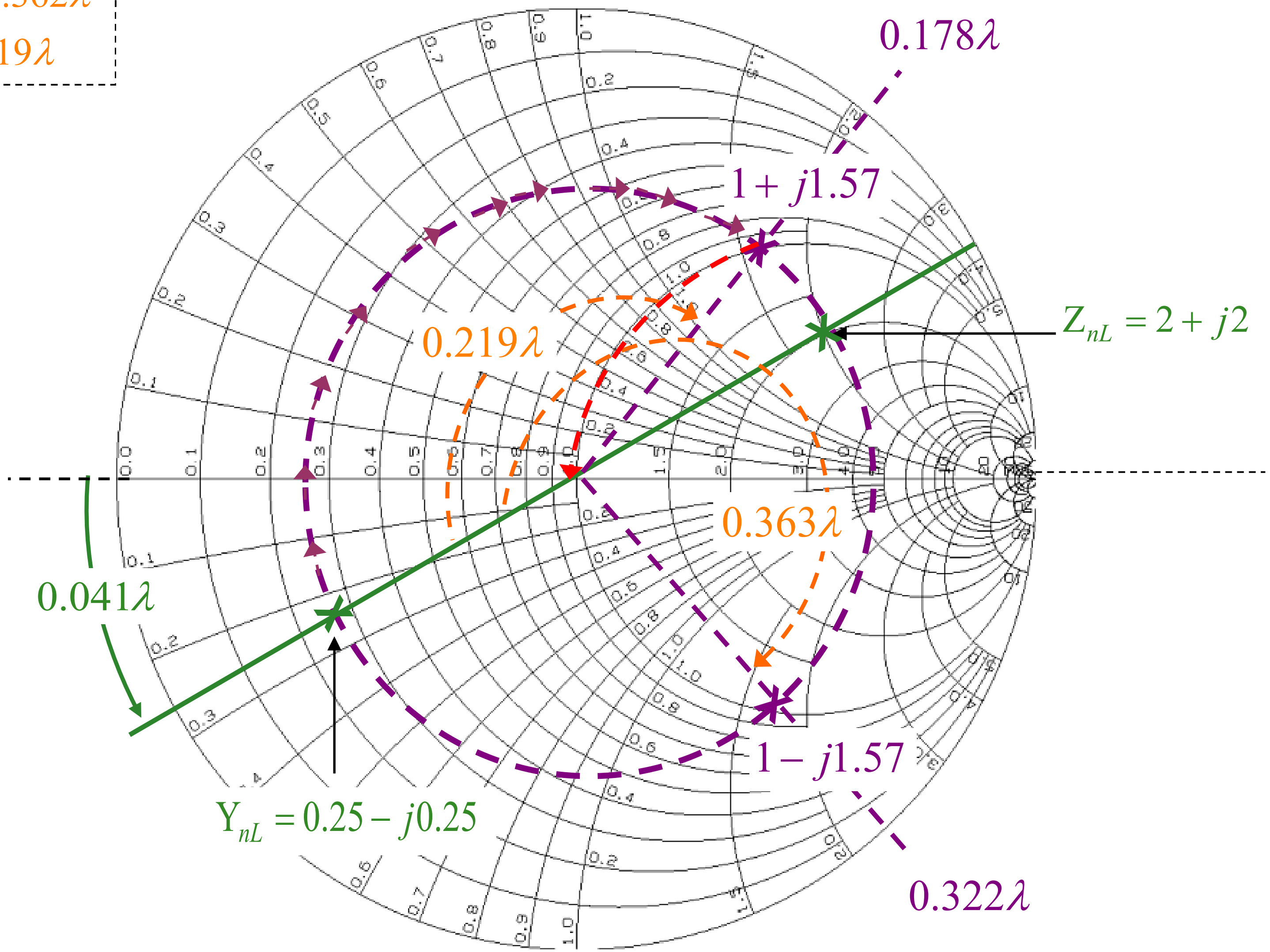
Solution:

Add $+j1.57$ at 0.362λ

Or $-j1.57$ at 0.219λ

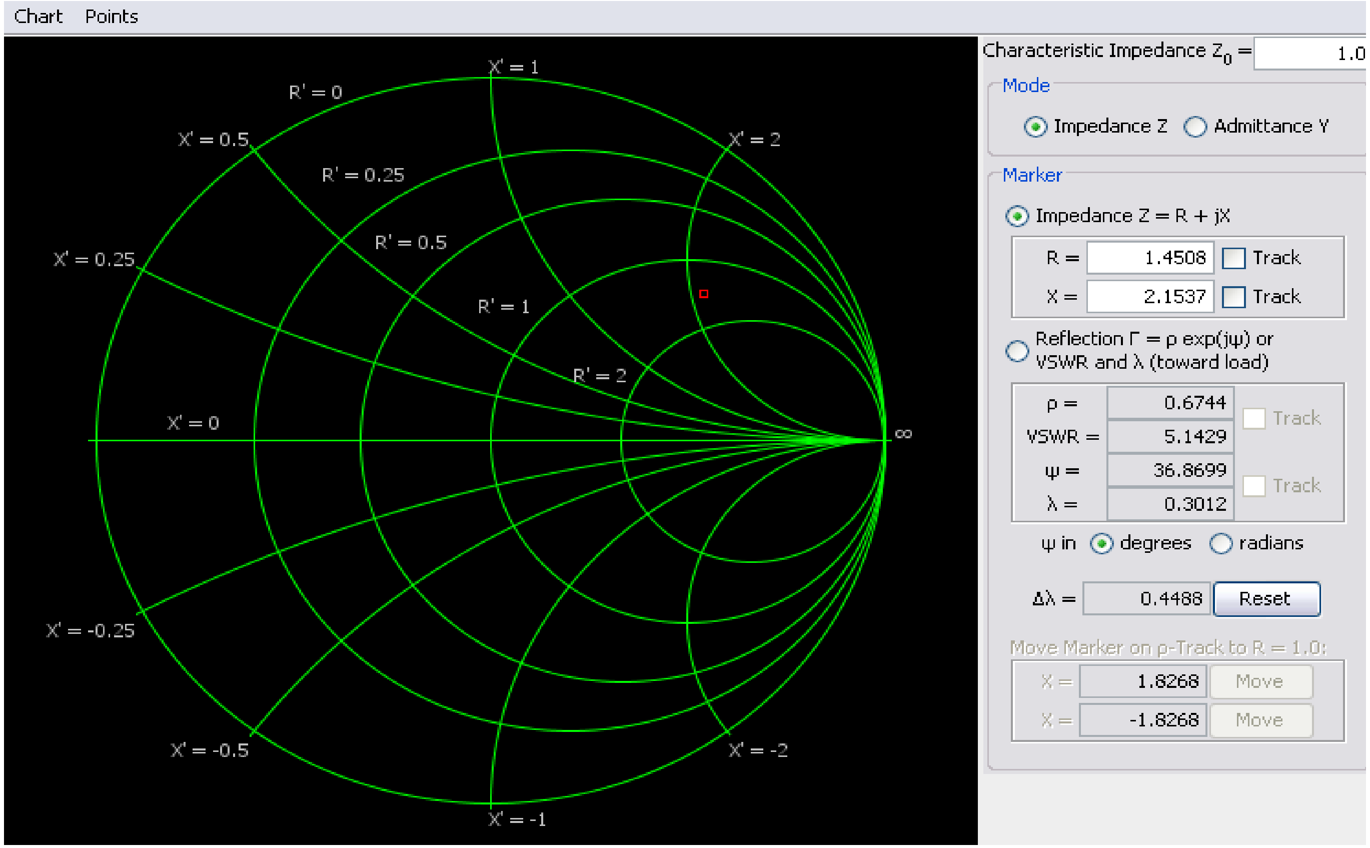
$$0.041\lambda + 0.178\lambda = 0.219\lambda$$

$$0.041\lambda + 0.322\lambda = 0.363\lambda$$

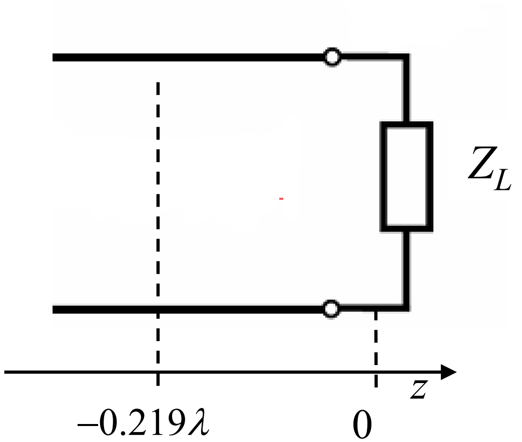


Smith Chart Applet

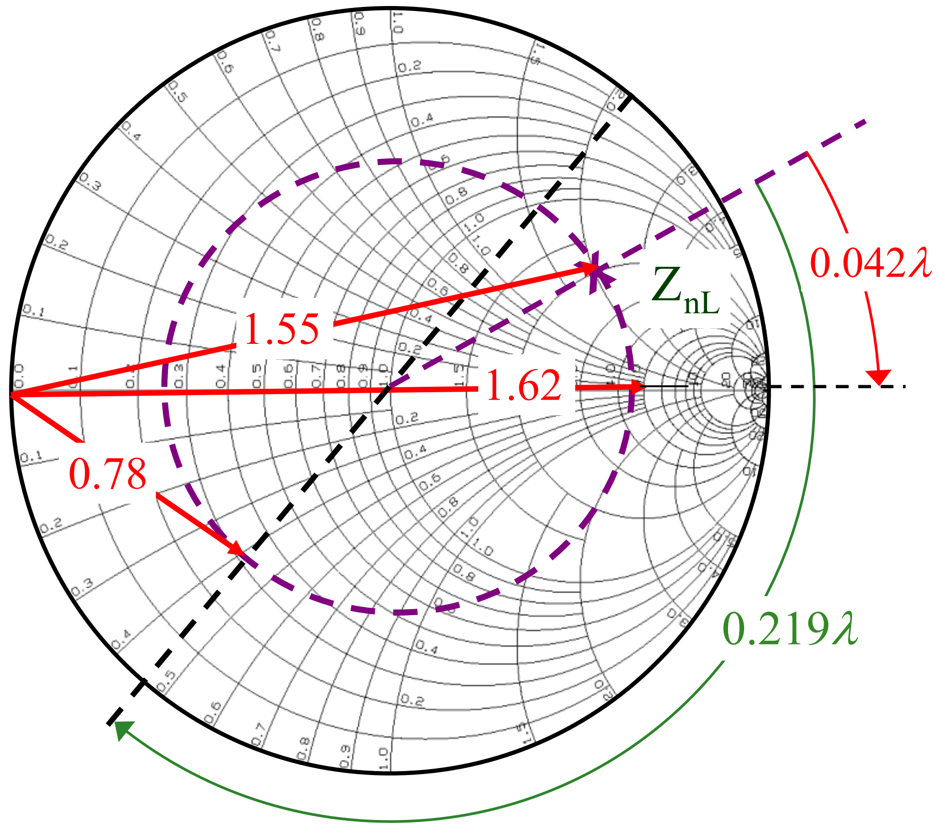
http://www.ocf.berkeley.edu/~joydip/smithchart/index.html



Standing Wave Pattern



Voltages: Unmatched Line



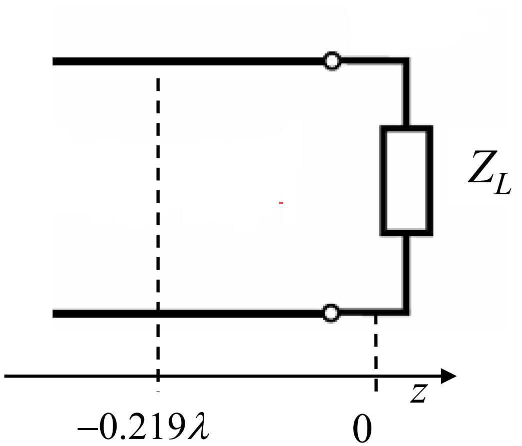
$$V_{load} = V(0) = 1.55$$

$$V_{max} = 1.62$$

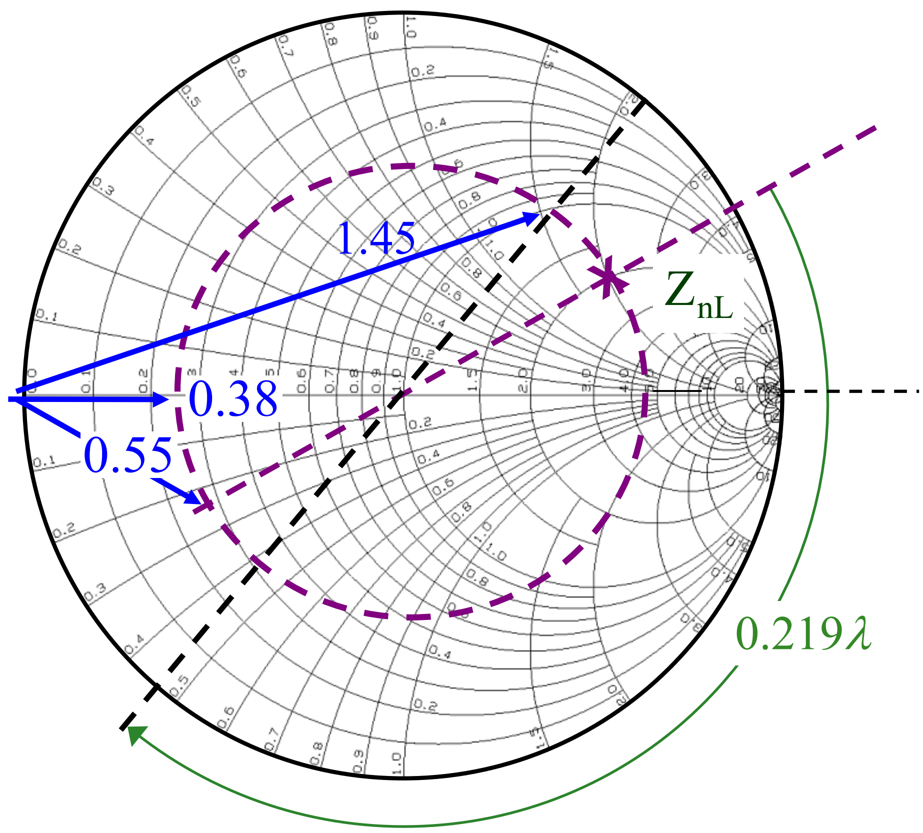
$$V_{min} = 0.38$$

$$V(-0.219\lambda) = 0.78$$

Standing Wave Pattern



Currents: Unmatched Line



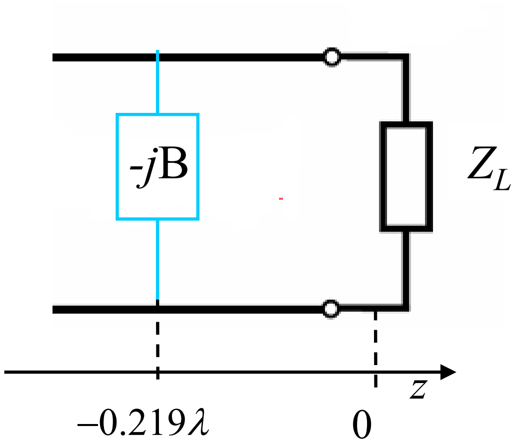
$$I_{load} = 0.55$$

$$I_{max} = 1.62$$

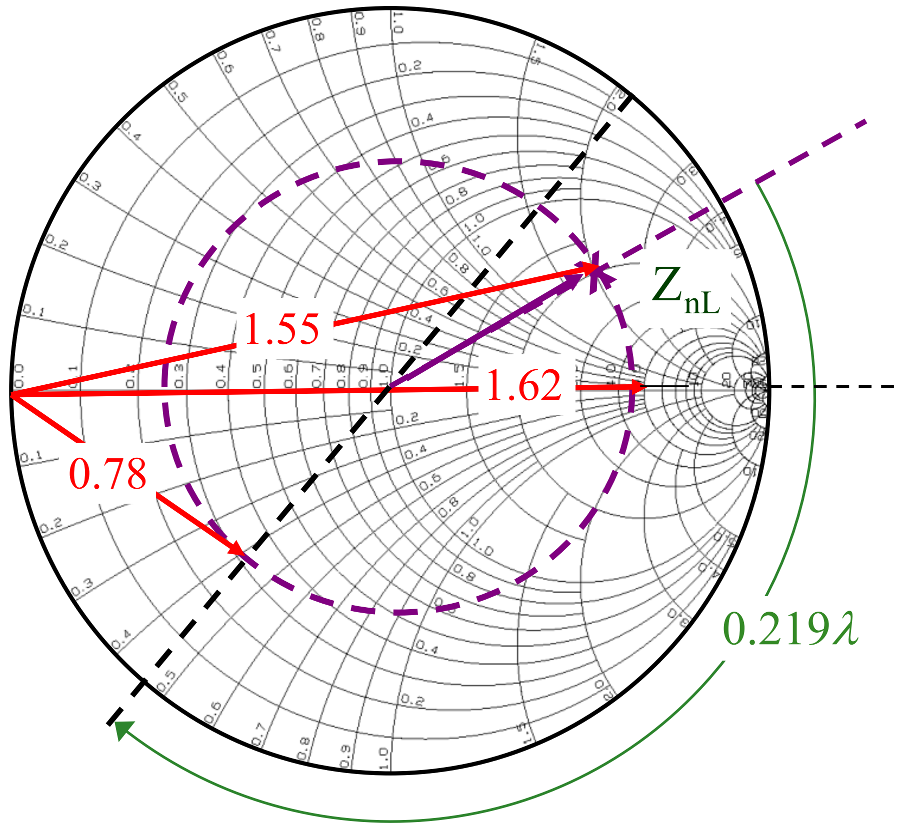
$$I_{min} = 0.38$$

$$I(-0.219\lambda) = 1.45$$

Standing Wave Pattern



Voltages: Matched Line



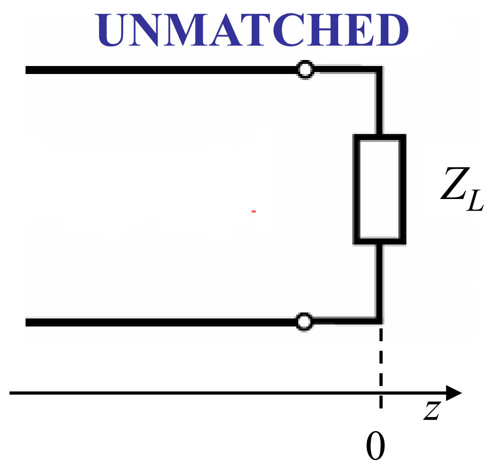
$$V_{load} = 1.55$$

$$V_{max} = 1.62$$

$$V_{min} = 0.38$$

$$V(-0.219\lambda) = 0.78$$

Standing Wave Pattern



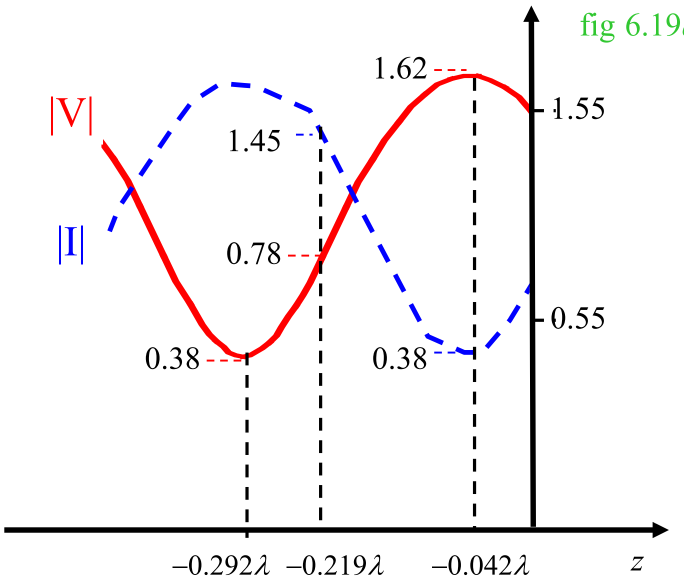


fig 6.19a

$V_{load} = 1.55$

$V_{max} = 1.62$

$V_{min} = 0.38$

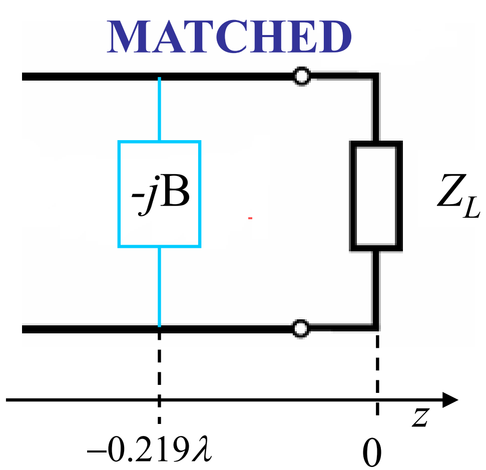
$V(-0.219\lambda) = 0.78$

$I_{load} = 0.55$

$I_{max} = 1.55$

$I_{min} = 0.38$

$I(-0.219\lambda) = 1.45$



For tuned line

V,I between load and shunt are

greater than those for unmatched line

by a factor of $\frac{1}{0.78}$

for the same incident power.

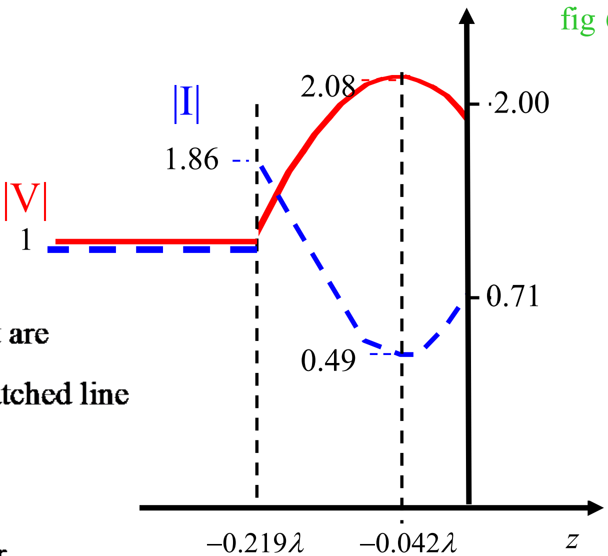
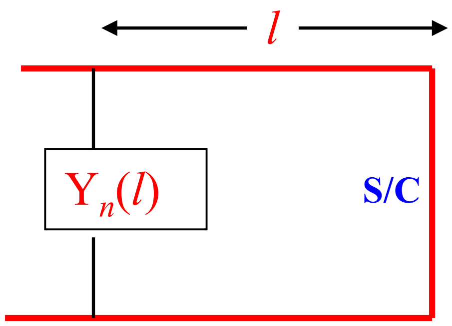


fig 6.19b

How to Add Shunt Susceptance

For a transmission line ⇒ usually add a short circuit section of a line placed perpendicular to the main line.



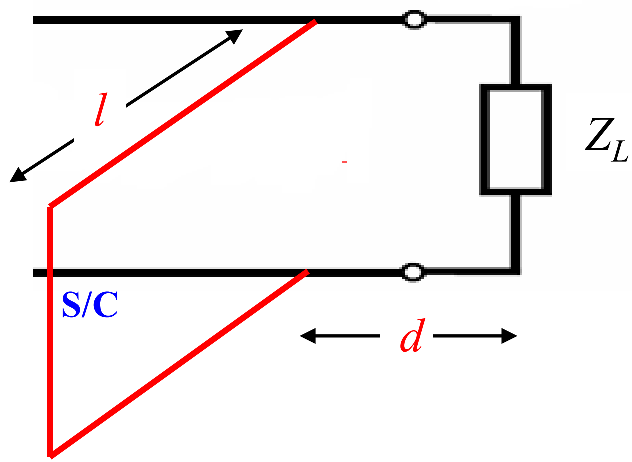
$Y_n(l) = -j \cot kl$

6.36

for kl varying from 0 to π

⇓

$0 \le l \le \frac{\lambda}{2}$



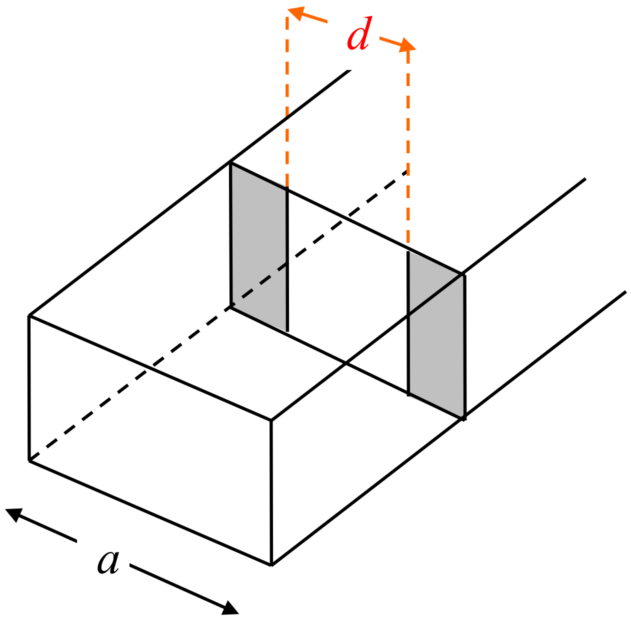
all possible values

$-j\infty \le jB \le +j\infty$

fig 6.20

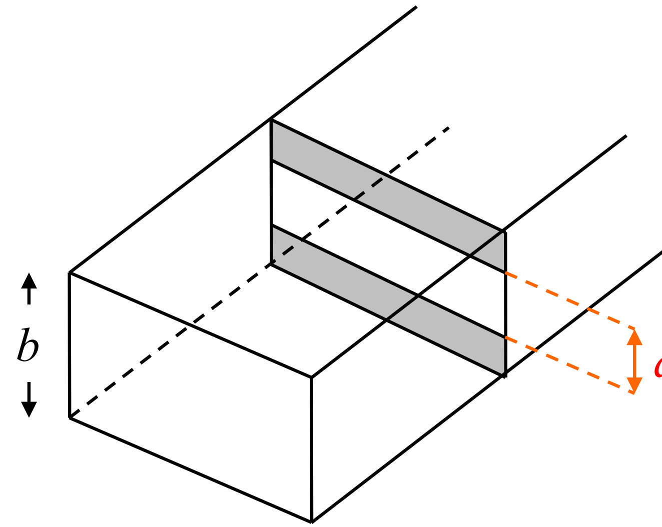
How to Add Shunt Susceptance

For a rectangular waveguide $\Rightarrow$ usually insert a metal iris inside the waveguide.



Inductive Iris

$$B_0 = \frac{-\lambda_g}{a}\cot^2\left(\frac{\pi d}{2a}\right) \quad 6.37$$

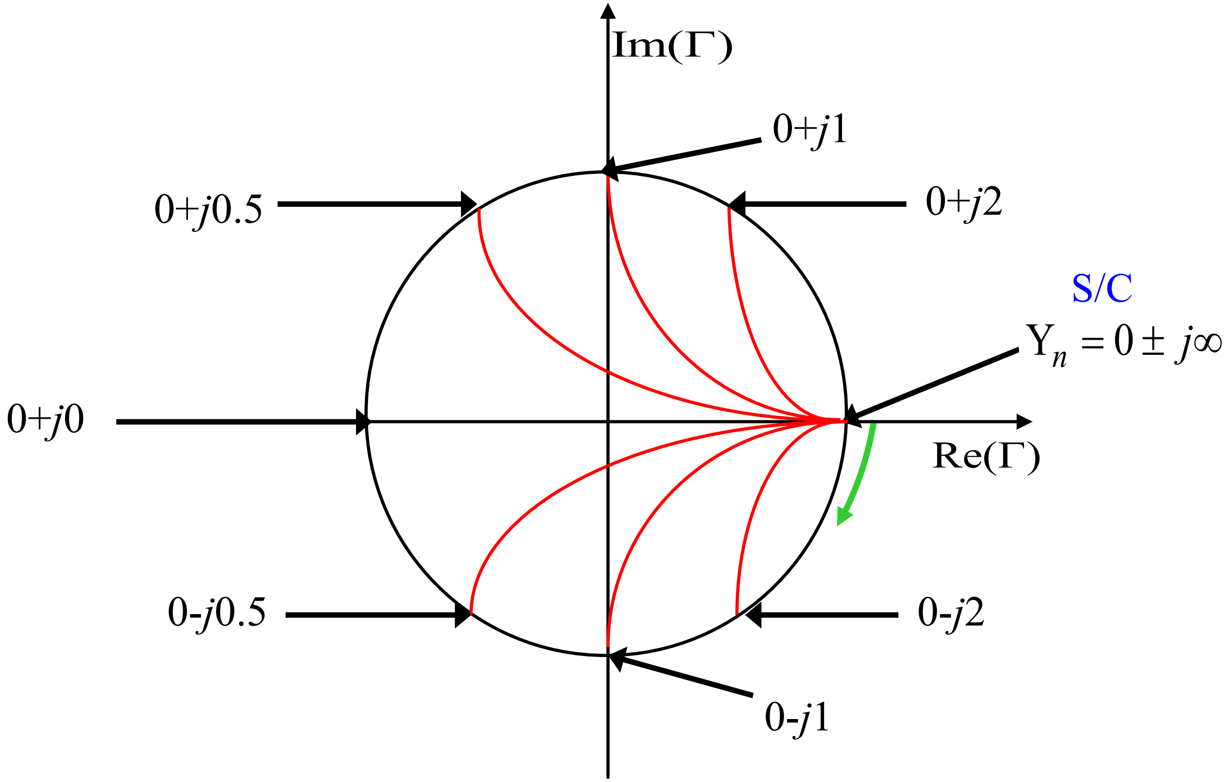


Capacitive Iris

$$B_0 = \frac{4b}{\lambda_g}\ln\left[\csc\left(\frac{\pi d}{2b}\right)\right] \quad 6.38$$

S/C Stub – Smith Chart Solution

Rotate clockwise from S/C to desired *j*B



S/C Stub – Smith Chart Solution

Example: To add

$$jB = -j1.57$$

analytically

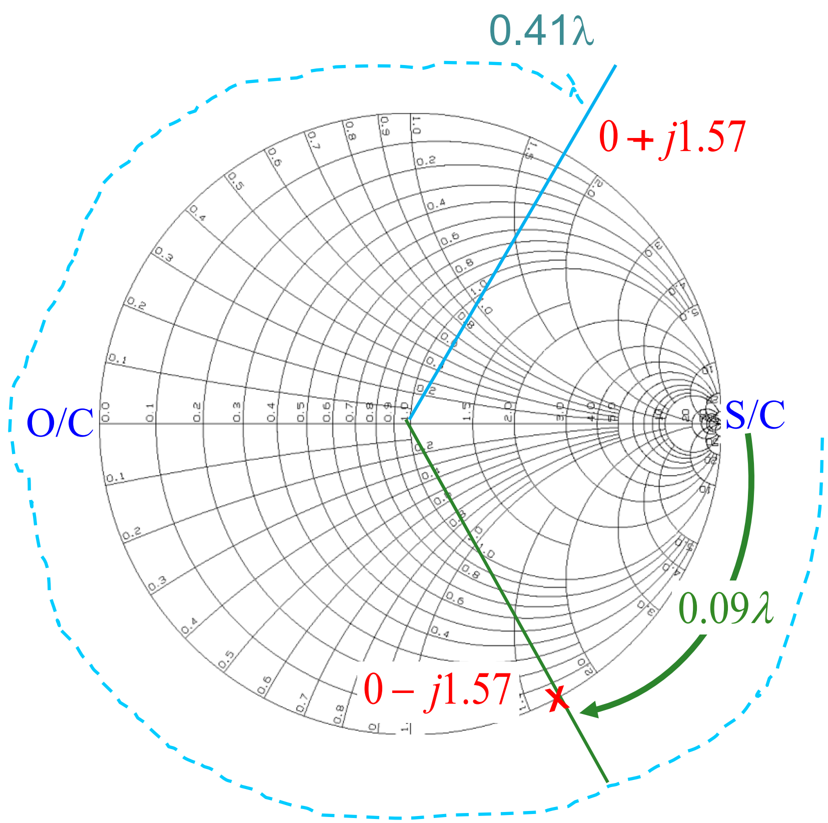
$$Y_n = -j\cot kl$$

$$-j1.57 = -j\cot kl$$

$$\cot kl = 1.57 \quad ; \quad \tan kl = \frac{1}{1.57}$$

$$kl = \frac{2\pi}{\lambda} l = 0.567 \text{ [radians]}$$

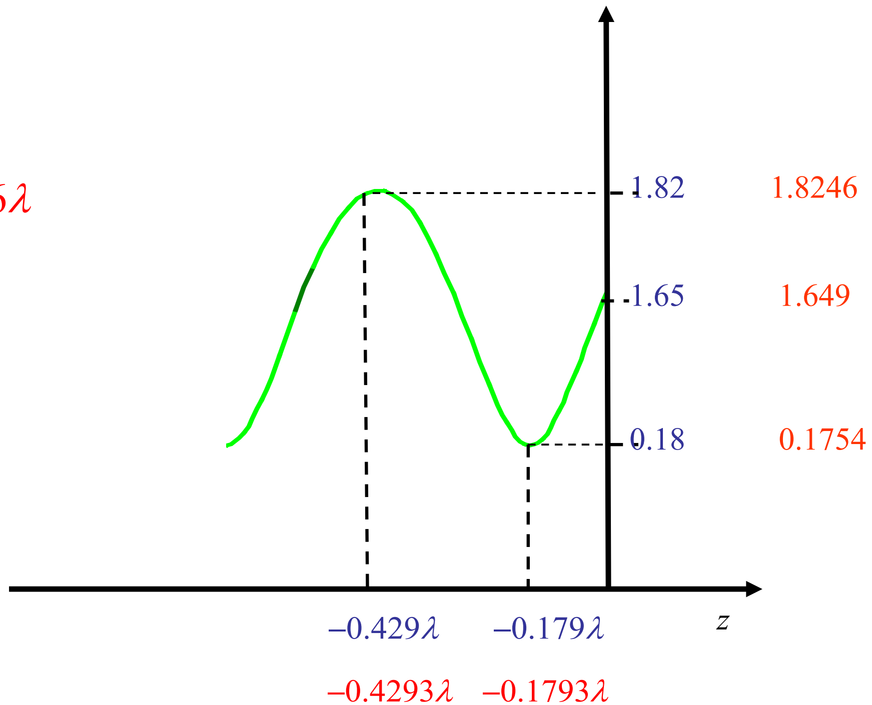
$$l = 0.0903\lambda$$

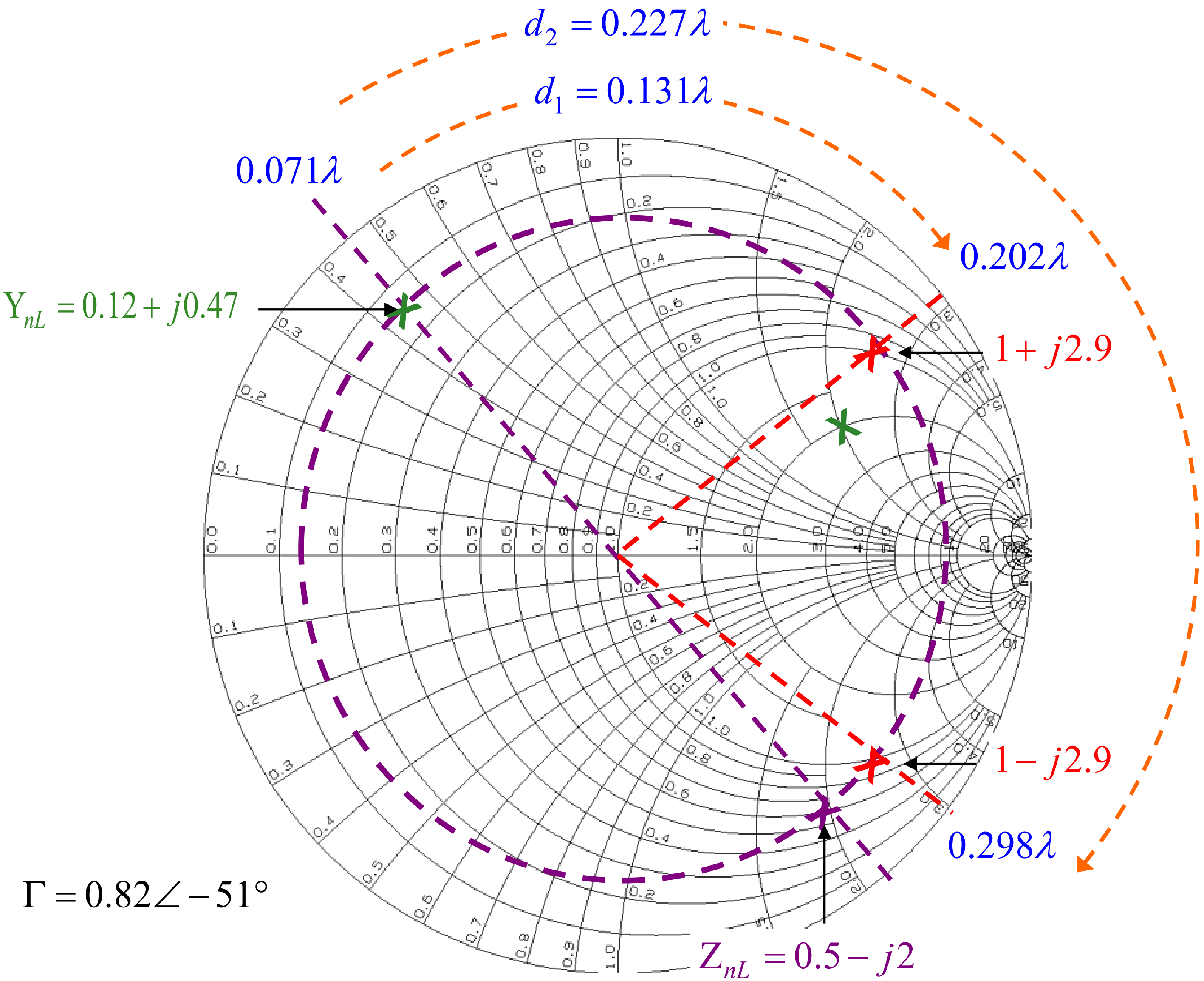


S/C Stub – Smith Chart Solution

Example: Given Z_{nL}=0.5-j2 find everything

Smith chart	Analytically
$\Gamma = 0.82\angle\text{-}51°$	$\Gamma = 0.8246\angle\text{-}50.9°$
$Y_n = 0.12 + j0.47$	$Y_n = 0.1176 + j0.4706$
$SWR = 10.5$	$SWR = 10.46$
$d_1 = 0.131\lambda$	$d_1 = 0.1315\lambda$
$B_1 = -2.9$	$B_1 = -2.915$
$l_1 = 0.053\lambda$	$l_1 = 0.0526\lambda$
$d_2 = 0.227\lambda$	
$B_2 = +2.9$	
$l_2 = 0.447\lambda$	





$d_2 = 0.227\lambda$
$d_1 = 0.131\lambda$
0.071λ
0.202λ
$Y_{nL} = 0.12 + j0.47$
$1 + j2.9$
$1 - j2.9$
0.298λ
$\Gamma = 0.82\angle -51°$
$Z_{nL} = 0.5 - j2$

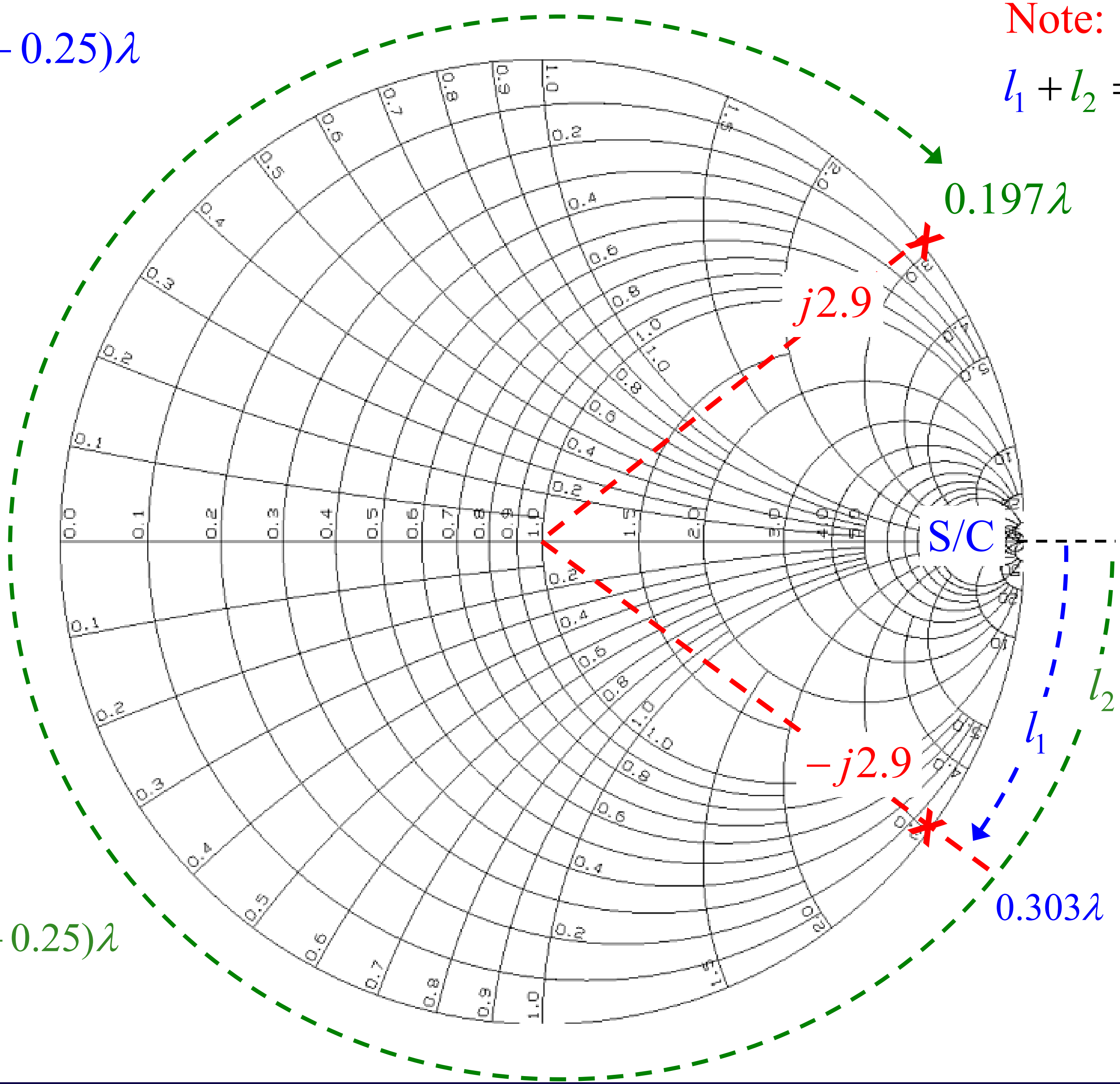
fig 6.18

$$l_1 = (0.303 - 0.25)\lambda$$

$$l_1 = 0.053\lambda$$

Note:

$$l_1 + l_2 = 0.5\lambda$$



$$l_2 = (0.197 + 0.25)\lambda$$

$$l_2 = 0.447\lambda$$