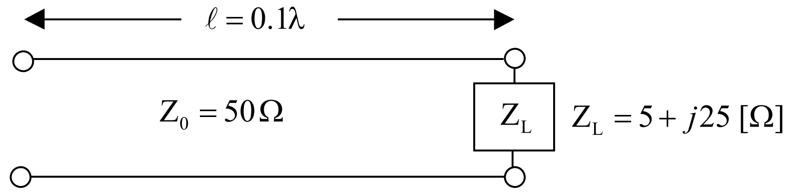
Smith Chart Problems



1. The 0.1λ length line shown has a characteristic impedance of 50Ω and is terminated with a load impedance of $Z_L = 5 + j25\Omega$.

   (a) Locate $z_L = \dfrac{Z_L}{Z_0} = 0.1 + j0.5$ on the Smith chart.

   See the point plotted on the Smith chart.

   (b) What is the impedance at $\ell = 0.1\lambda$?

   Since we want to move away from the load (i.e., toward the generator), read read 0.074λ on the WAVELENGTHS TOWARD GENERATOR scale and add $\ell = 0.1\lambda$ to obtain 0.174λ on the WAVELENGTHS TOWARD GENERATOR scale. A radial line from the center of the chart intersects the constant reflection coefficient magnitude circle at $z = 0.38 + j1.88$. Hence $Z = zZ_0 = 50(0.38 + j1.88) = 19 + 94\Omega$.

   (c) What is the VSWR on the line?

   Find VSWR $= z_{\text{max}} = 13$ on the horizontal line to the right of the chart's center. Or use the SWR scale on the chart.

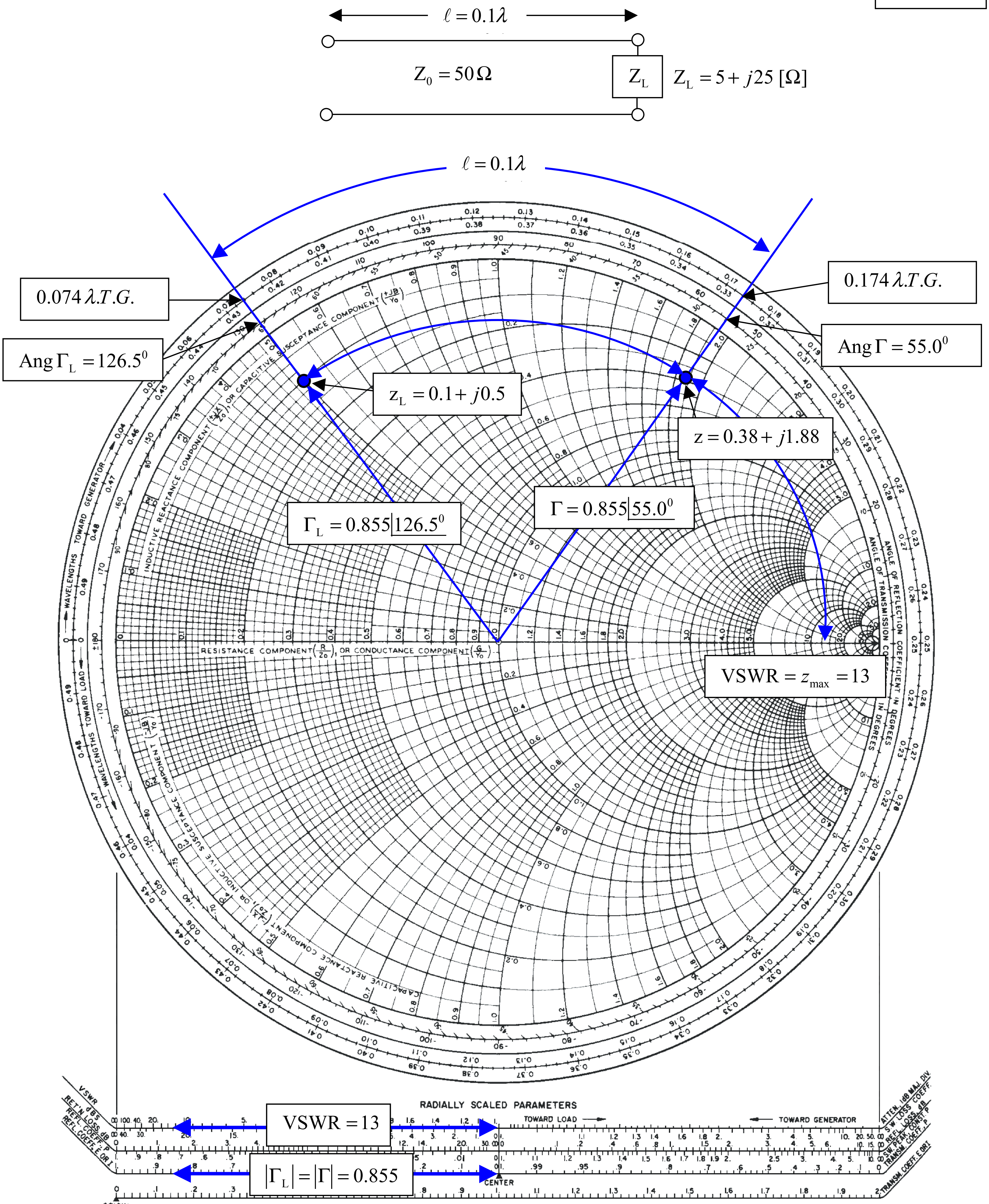
   (d) What is Γ_L?

   From the REFLECTION COEFFICIENT scale below the chart, find $|\Gamma_L| = 0.855$. From the ANGLE OF REFLECTION COEFFICIENT scale on the perimeter of the chart, find the angle of $\Gamma_L = 126.5°$. Hence $\Gamma_L = 0.855e^{j126.5°}$.

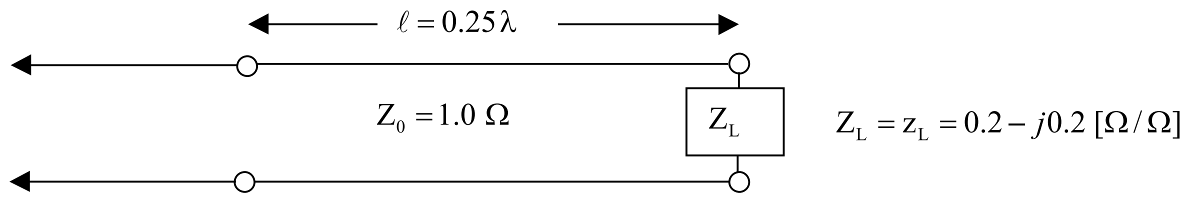
   (e) What is Γ at $\ell = 0.1\lambda$ from the load?

   Note that $|\Gamma| = |\Gamma_L| = 0.855$. Read the angle of the reflection coefficient from the ANGLE OF REFLECTION COEFFICIENT scale as $55.0°$. Hence $\Gamma = 0.855e^{j55.0°}$.

Problem 1



$\ell = 0.1\lambda$
$Z_0 = 50\,\Omega$
Z_L
$Z_L = 5 + j25\ [\Omega]$
$\ell = 0.1\lambda$
$0.074\,\lambda.T.G.$
$0.174\,\lambda.T.G.$
$\text{Ang}\,\Gamma_L = 126.5^0$
$\text{Ang}\,\Gamma = 55.0^0$
$z_L = 0.1 + j0.5$
$z = 0.38 + j1.88$
$\Gamma_L = 0.855\angle 126.5^0$
$\Gamma = 0.855\angle 55.0^0$
$\text{VSWR} = z_{max} = 13$
RADIALLY SCALED PARAMETERS
$\text{VSWR} = 13$
$|\Gamma_L| = |\Gamma| = 0.855$



2. A transmission line has $Z_0 = 1.0$, $Z_L = z_L = 0.2 - j0.2\Omega$.

   (a) What is z at $\ell = \dfrac{\lambda}{4} = 0.25\lambda$?

   From the chart, read 0.467λ from the WAVELENGTHS TOWARD GENERATOR SCALE. Add 0.25λ to obtain 0.717λ on the WAVELENGTHS TOWARD GENERATOR SCALE. This is not on the chart, but since it repeats every half wavelength, it is the same as $0.717\lambda - 0.500\lambda = 0.217\lambda$. Drawing a radial line from the center of the chart, we find an intersection with the constant reflection coefficient magnitude circle at $z = Z = 2.5 + j2.5$.

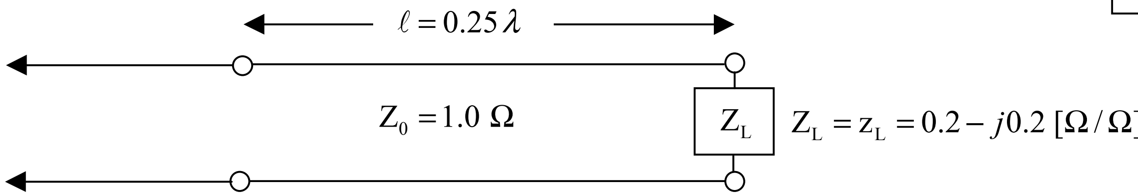
   (b) What is the VSWR on the line?

   From the intersection of the constant reflection coefficient circle with the right hand side of the horizontal axis, read VSWR $= z_{\text{max}} = 5.3$.

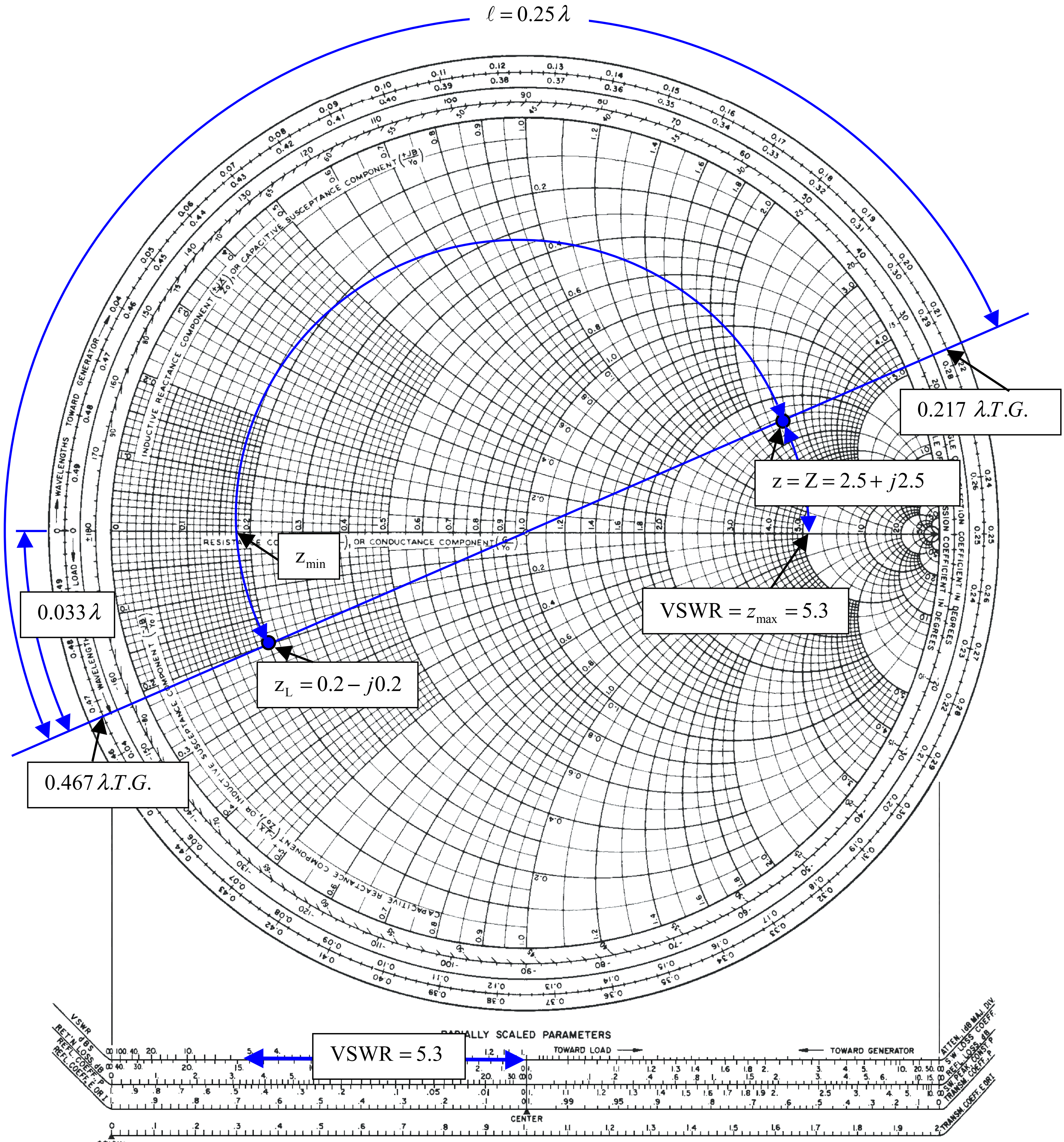
   (c) How far from the load is the first voltage minimum? The first current minimum?

   The voltage minimum occurs at z_{min} which is at a distance of $0.500\lambda - 0.467\lambda = 0.033\lambda$ from the load. Or read this distance directly on the WAVELENGTHS TOWARD LOAD scale.
   The current minimum occurs at z_{max} which is a quarter of a wavelength farther down the line or at $0.033\lambda + 0.25\lambda = 0.283\lambda$ from the load.

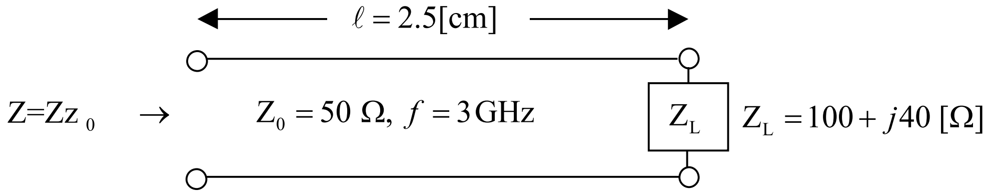
Problem 2



$\ell = 0.25\,\lambda$
$Z_0 = 1.0\ \Omega$
Z_L
$Z_L = z_L = 0.2 - j0.2\ [\Omega/\Omega]$



$\ell = 0.25\,\lambda$
$0.217\ \lambda.T.G.$
$z = Z = 2.5 + j2.5$
z_{min}
$0.033\,\lambda$
VSWR $= z_{max} = 5.3$
$z_L = 0.2 - j0.2$
$0.467\ \lambda.T.G.$
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
VSWR = 5.3
CENTER
ORIGIN



3. The air-filled two-wire line has a characteristic impedance of 50Ω and is operated at $f = 3$ GHz. The load is $Z_L = 100 + j40\Omega$.

   (a) For the line above, find z_L on the chart.

   The normalized load is $z_L = \dfrac{Z_L}{Z_0} = \dfrac{100 + j40}{50} = 2.0 + j0.8$.
   See the Smith chart for location of point.

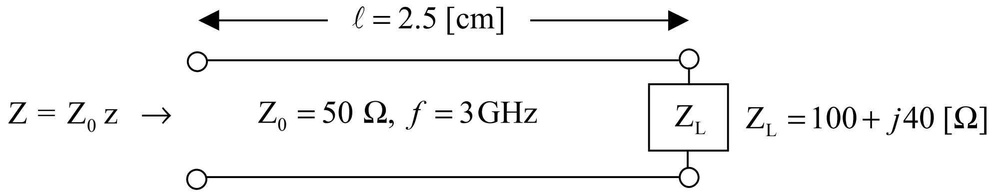
   (b) What is the line impedance 2.5 cm from the load?

   Note that $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^{10} \text{ cm/sec}}{3 \times 10^9 \text{ Hz}} = 10$ cm. Since we are going to move toward the generator (away from the load), at the normalized load position, first read 0.217λ on the WAVELENGTHS TOWARD GENERATOR scale. Then add 2.5 cm/10 cm $= 0.25\lambda$ to this value to obtain 0.467λ on the WAVELENGTHS TOWARD GENERATOR scale. A radial line from the center at this point intersects the constant reflection coefficient magnitude circle at $z = 0.43 - j0.17$, so $Z = zZ_0 = 50(0.43 + j0.17) = 21.5 - j8.5\Omega$.

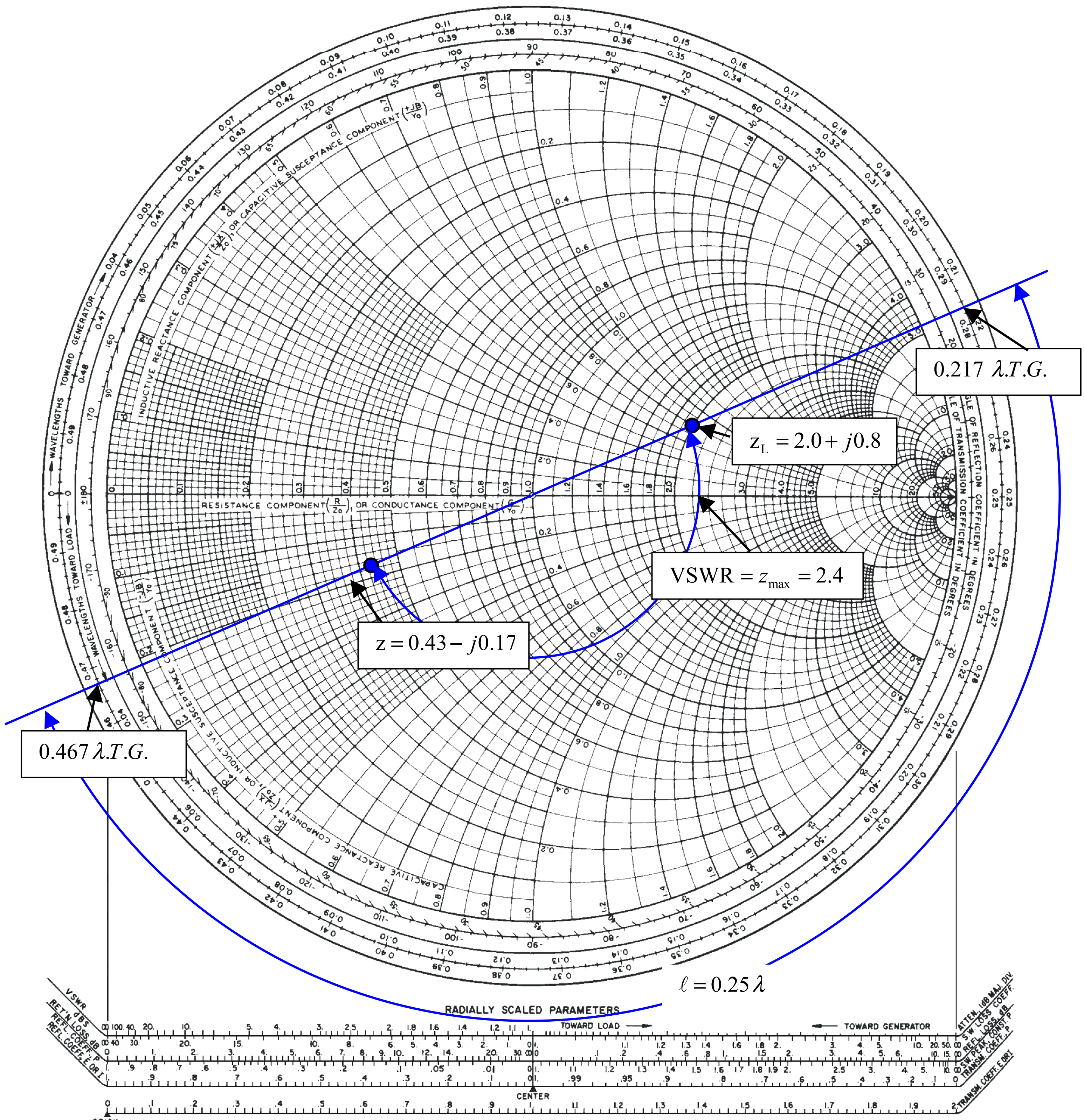
   (c) What is the VSWR on the line?

   From the intersection of the reflection coefficient circle and the horizontal axis on the right hand side of the chart, read VSWR = 2.4. Or use the SWR scale below the chart.

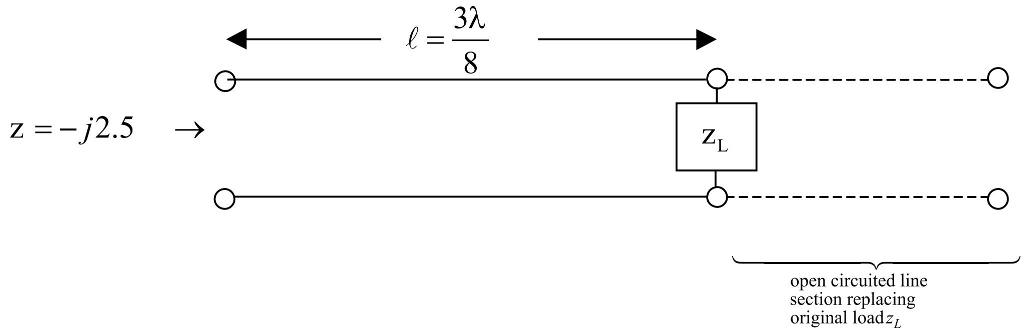
Problem 3



$\ell = 2.5$ [cm]
$Z = Z_0\,z \rightarrow$
$Z_0 = 50\ \Omega,\ f = 3\,\mathrm{GHz}$
Z_L
$Z_L = 100 + j40\ [\Omega]$



$0.217\ \lambda T.G.$
$z_L = 2.0 + j0.8$
$\mathrm{VSWR} = z_{max} = 2.4$
$z = 0.43 - j0.17$
$0.467\ \lambda T.G.$
$\ell = 0.25\lambda$
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



4. The line shown is $3/8\lambda$ long and its normalized input impedance is $z = -j2.5$.

   (a) What is the normalized receiving or load end impedance, z_L?

   Since we must move toward the load to find the load impedance, locate $z = -j2.5$ on the chart and find the intersection on the WAVELENGTHS TOWARD LOAD scale at 0.189λ. Add the distance $3/8\lambda = 0.375\lambda$ to get 0.064λ on the WAVELENGTHS TOWARD LOAD scale. At the intersection of the line from the center of the chart and the constant reflection coefficient circle, read $z_L = -j0.425$.

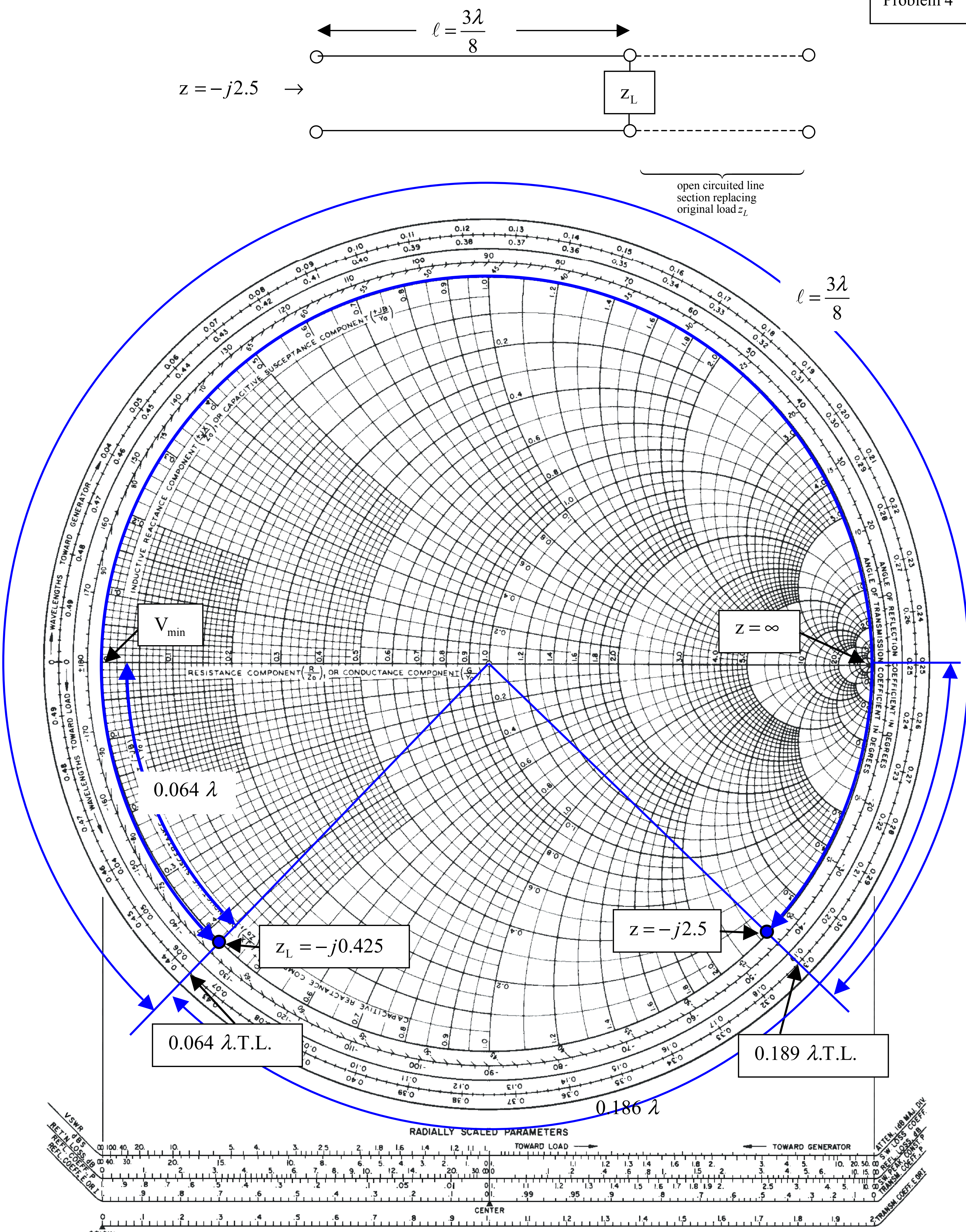
   (b) What is the distance from the load to the first voltage minimum?

   The voltage minimum occurs at the impedance minimum on the horizontal line to the left of the chart's center. This distance can be read directly off the WAVELENGTHS TOWARD LOAD scale (why?) as 0.064λ. Note that the line impedance at the voltage minimum would be that of a short circuit.

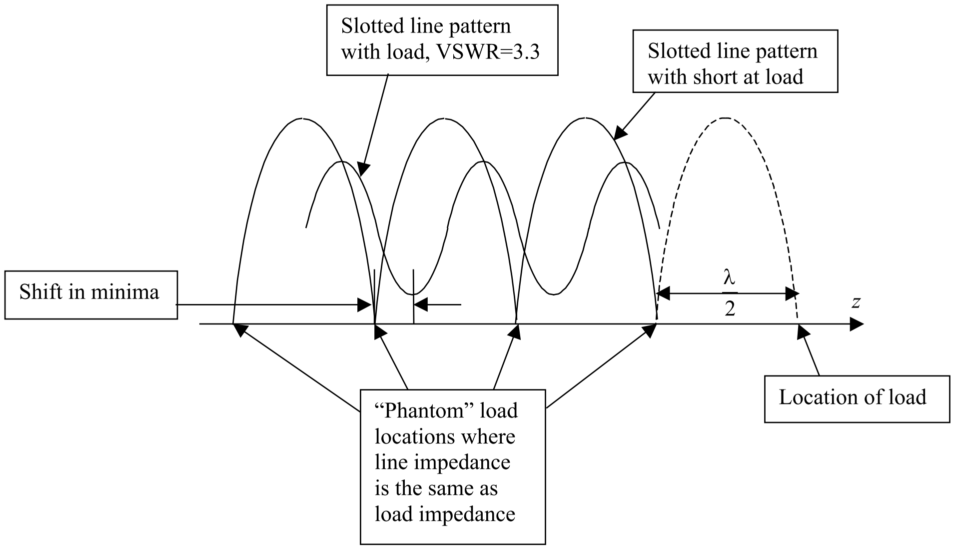
   (c) What length of open-circuited line could be used to replace z_L?

   From the position of z_L, we want to move toward the new load (i.e., toward the end of the open-circuited line) until we reach the open circuit condition $z = \infty$. This occurs at 0.25λ on the WAVELENGTHS TOWARD LOAD scale and the total distance moved is hence $0.25\lambda - 0.064\lambda = 0.186\lambda$, which is the length of open-circuited line needed to replace z_L.

Problem 4



ℓ = 3λ/8
z = −j2.5 →
z_L
open circuited line section replacing original load z_L
ℓ = 3λ/8
V_min
z = ∞
0.064 λ
z_L = −j0.425
z = −j2.5
0.064 λ.T.L.
0.189 λ.T.L.
0.186 λ
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



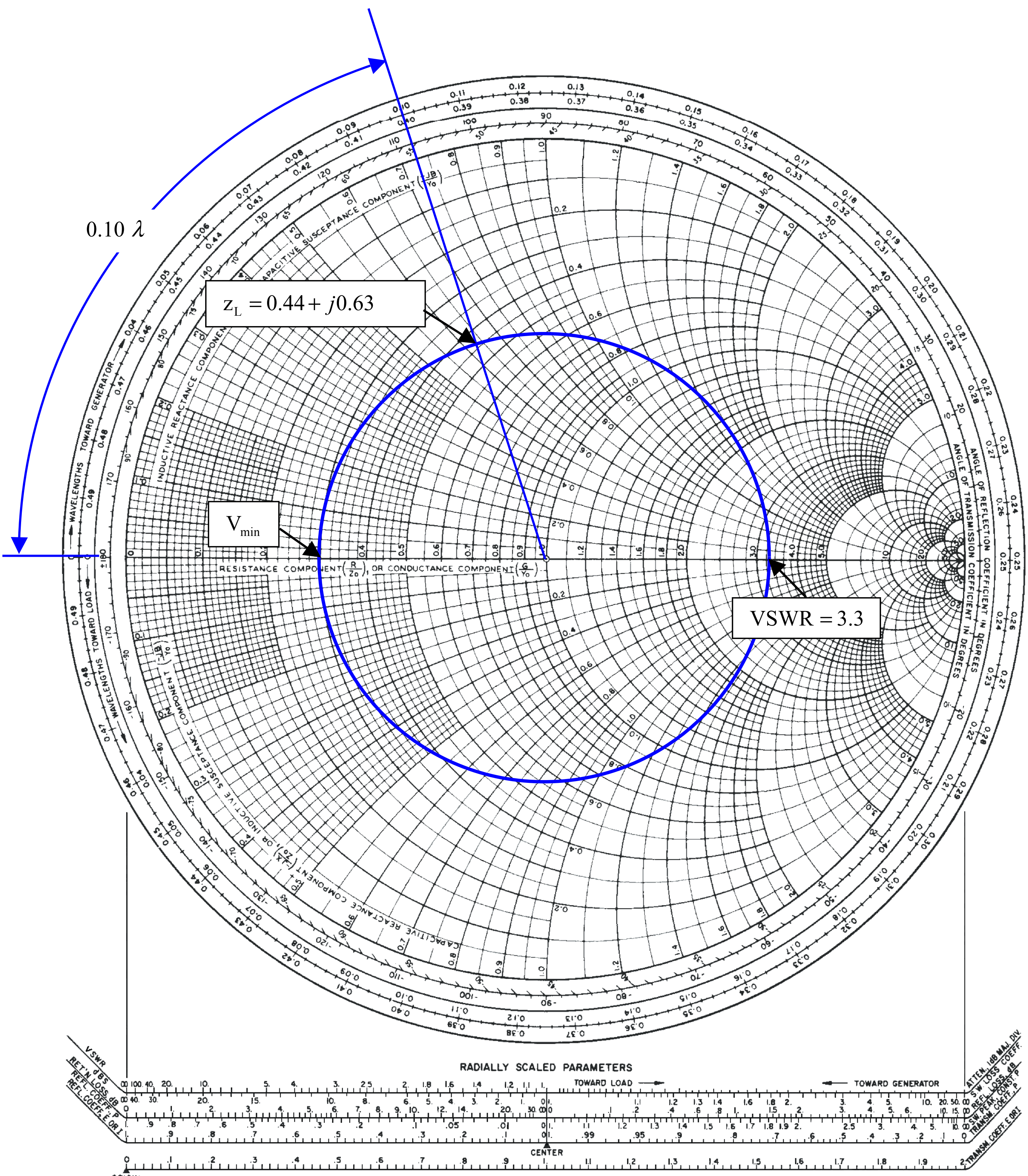
5. An air-filled 50Ω coaxial line has a loaded VSWR of 3.3 at a frequency of 3GHz. Replacing the load with a short causes the voltage minimum to move 1.0 cm towards the generator. What is the normalized load impedance?

For an air-filled line, the wavelength is

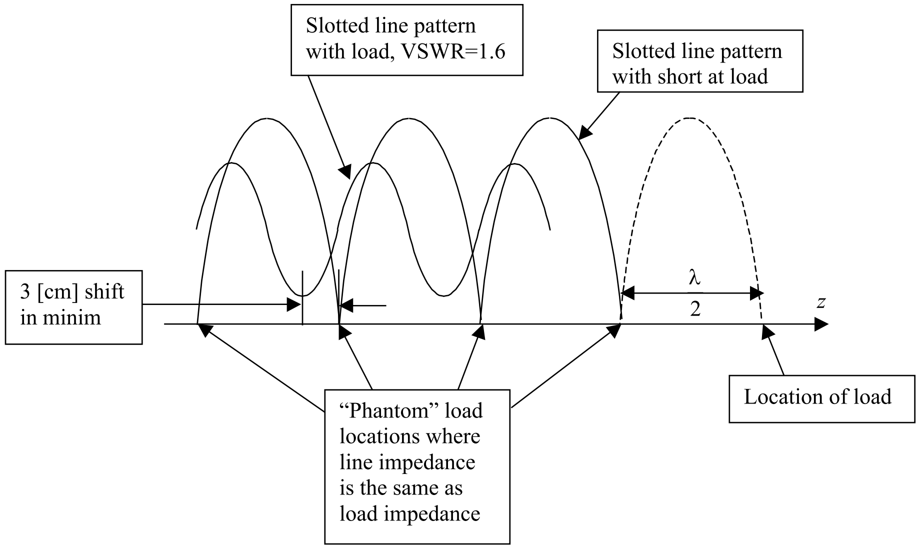
$$\lambda = \frac{c}{f} = \frac{3 \times 10^{10} \text{ cm/sec}}{3 \times 10^{9} \text{ Hz}} = 10 \text{ cm}.$$

With the VSWR located at $z_{\text{max}} = 3.3$, draw the circle of constant reflection coefficient magnitude. With the load in place, our only information is the location of a voltage minimum (equivalent to an impedance minimum). But thinking of the Smith chart as a crank diagram, we know that the voltage minimum must be at the intersection of the constant reflection coefficient circle and the horizontal axis on the left hand side of the chart between $z = 0$ and $z = 1.0$, (i.e., at $z_{\text{min}} = 0.3$). With this information, we have thus established a correspondence between a location on the line and a point on the chart. Now, move from this minimum voltage point on the chart $\frac{1.0 \text{ cm}}{10.0 \text{ cm}} = 0.1\lambda$ toward the generator to the location where the voltage minimum was measured in the shorted line measurement. Since it was obtained from a shorted line measurement, the location of this point must be an integral number of half-wavelengths from the load (and hence must have the same line impedance as at the load). The normalized impedance here is therefore $z_L = 0.44 + j0.63$. For a line with a 50Ω characteristic impedance, the load impedance is thus $Z_L = 50(0.44 + j0.63) = 22 + j31.5$.

Problem 5



0.10 λ
zL = 0.44 + j0.63
Vmin
VSWR = 3.3
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



6. A slotted line measurement on an air-filled TEM line yields a VSWR of 1.6 at a frequency of 1 GHz. When the load is replaced by a short, the voltage minimum moves 3 cm towards the load. Find the normalized load impedance.

   Locate the reflection coefficient circle by its intersection with $z_{\mathrm{max}} = \mathrm{VSWR} = 1.6$. The wavelength for an air-filled TEM line is

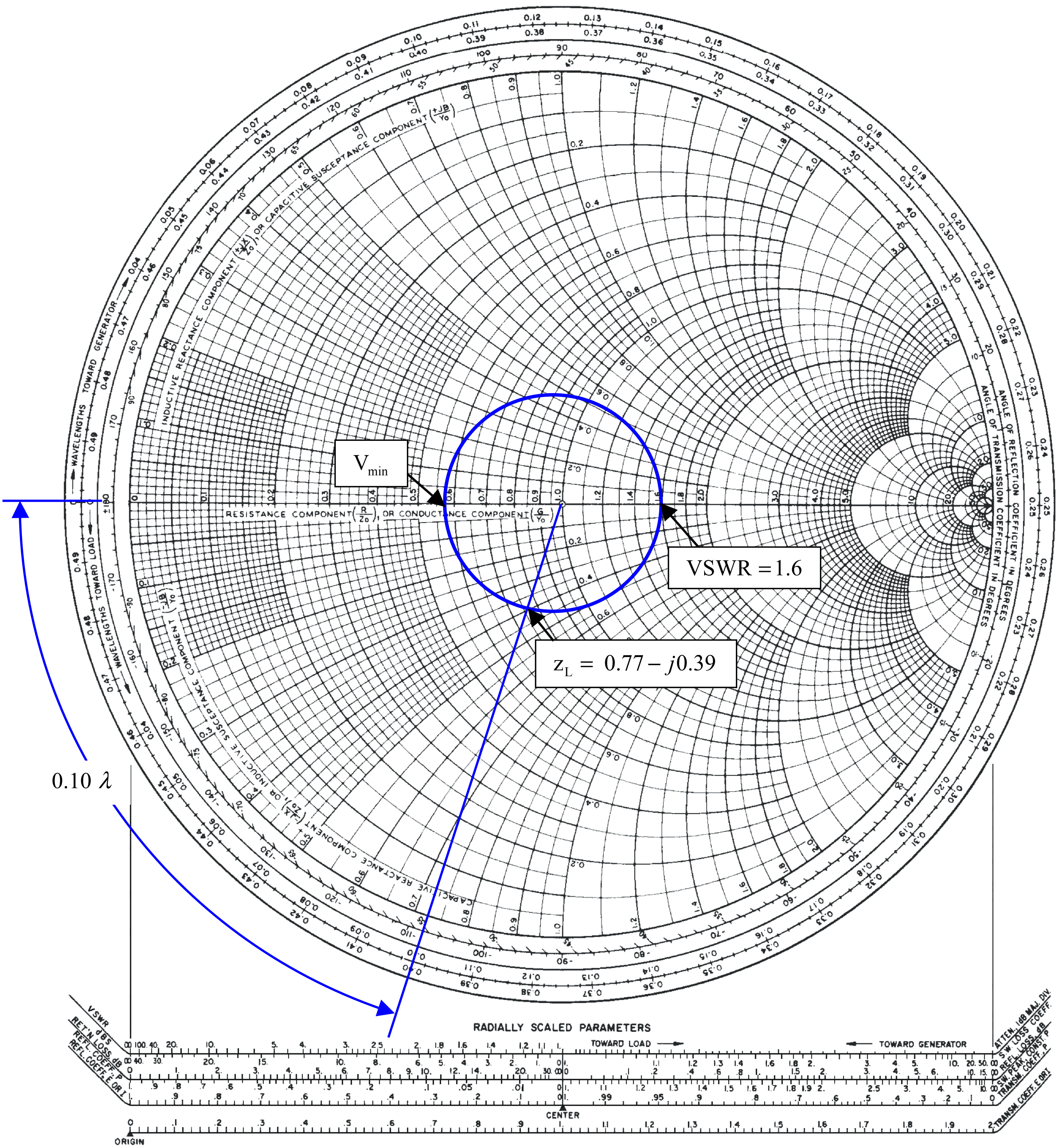
   $$\lambda = c/f = \frac{3 \times 10^{10} \text{ cm/s}}{10^9 \text{ Hz}} = 30 \text{ cm}.$$

   Hence the shift in the voltage minimum is

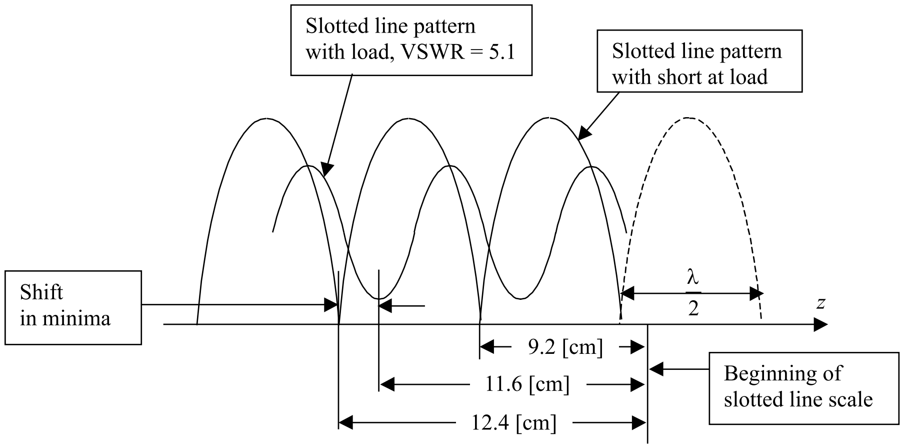
   $$\frac{3 \text{ cm}}{30 \text{ cm}/\lambda} = 0.1\lambda.$$

   Locate the voltage minimum at the impedance minimum, $z_{\mathrm{min}} = 0.625$ and move 0.1λ toward the load to a position an integral number of half-wavelengths from the load. The impedance there, $z_L = 0.77 - j0.39$, is the same as that of the load.

Problem 6



V_min
VSWR = 1.6
z_L = 0.77 − j0.39
0.10 λ
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (−jX/Zo), OR INDUCTIVE SUSCEPTANCE COMPONENT (−jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



7. A slotted line measurement yields the following parameter values:

   (a) Voltage minima at 9.2 cm and 12.4 cm measured away from the load with the line terminated in a short.

   (b) VSWR = 5.1 with the line terminated in the unknown load; a voltage minimum is located 11.6 cm measured away from the load.

   What is the normalized line impedance?

   Note that this data could have come from either a waveguide or a TEM line measurement. If the transmission system is a waveguide, then the wavelength used is actually the *guide* wavelength. From the voltage minima on the shorted line, the (guide) wavelength may be determined:

$$\frac{\lambda_g}{2} = 12.4 \text{ cm} - 9.2 \text{ cm} = 3.2 \text{ cm}$$

   or

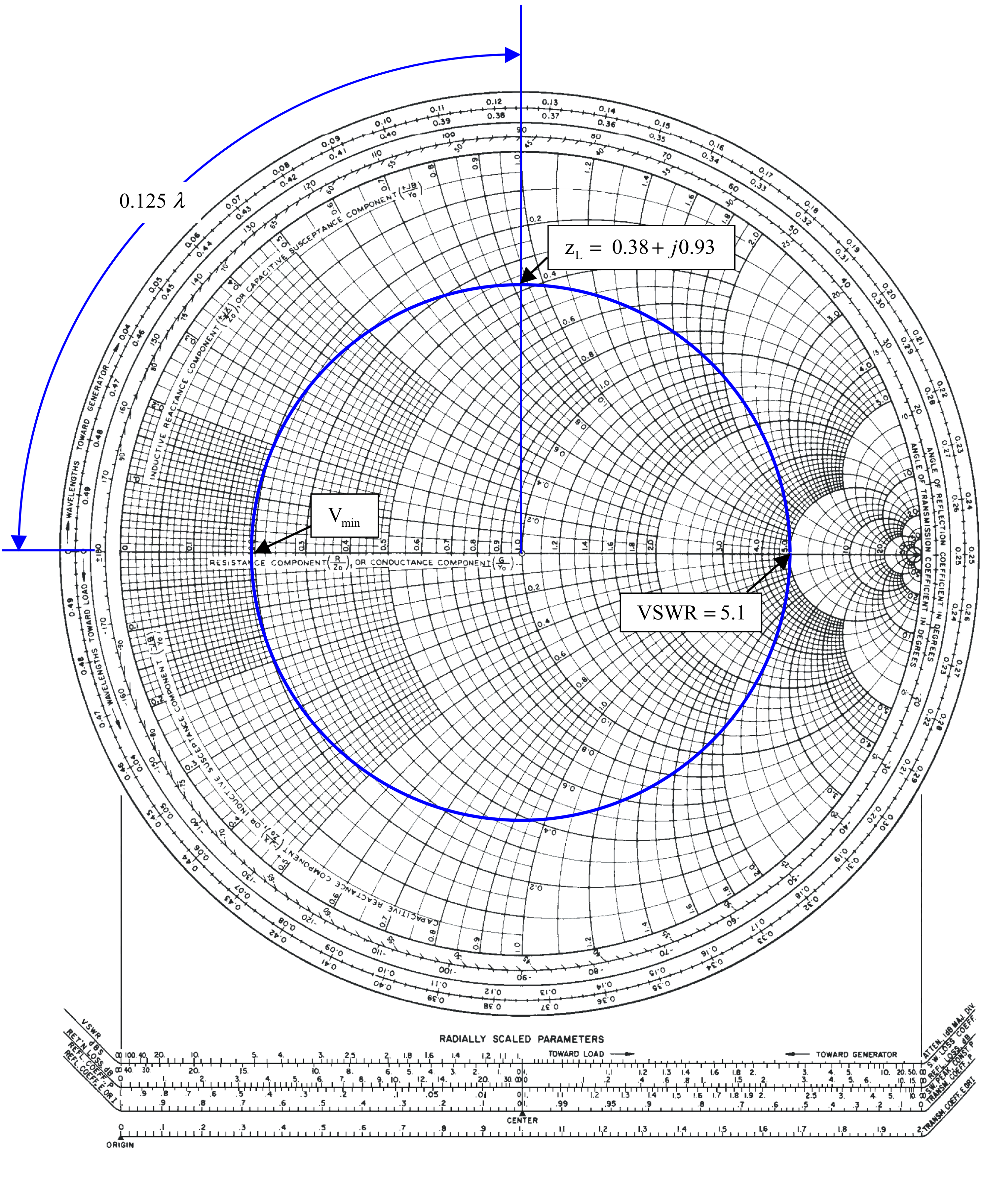
$$\lambda_g = 6.4 \text{ cm}.$$

   Hence the shift in the voltage minimum when the load is replaced by a short is

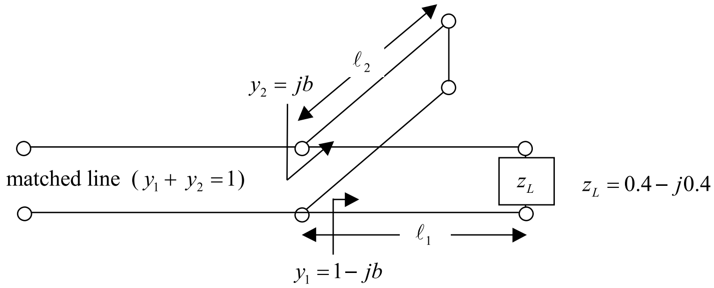
$$\frac{12.4 \text{ cm} - 11.6 \text{ cm}}{6.4 \text{ cm}/\lambda_g} = 0.125\lambda_g$$

   toward the generator. Locate the reflection coefficient magnitude circle by its intersection with $z_{\mathrm{max}} = \mathrm{VSWR} = 5.1$ on the horizontal axis. Then from the voltage minimum opposite z_{max}, move $0.125\lambda_g$ toward the generator to find a position an integral number of half-wavelengths from the load. The impedance there is the same as that of the load, $z_L = 0.38 + j0.93$. Alternatively, move $0.5\lambda_g - 0.125\lambda_g = 0.375\lambda_g$ toward the load to locate the same value.

Problem 7



0.125 λ
z_L = 0.38 + j0.93
V_min
VSWR = 5.1
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



8. Use a single parallel stub tuner to match the above line to its load. Use a shorted stub and find its distance from the load, ℓ_1, and its length ℓ_2.

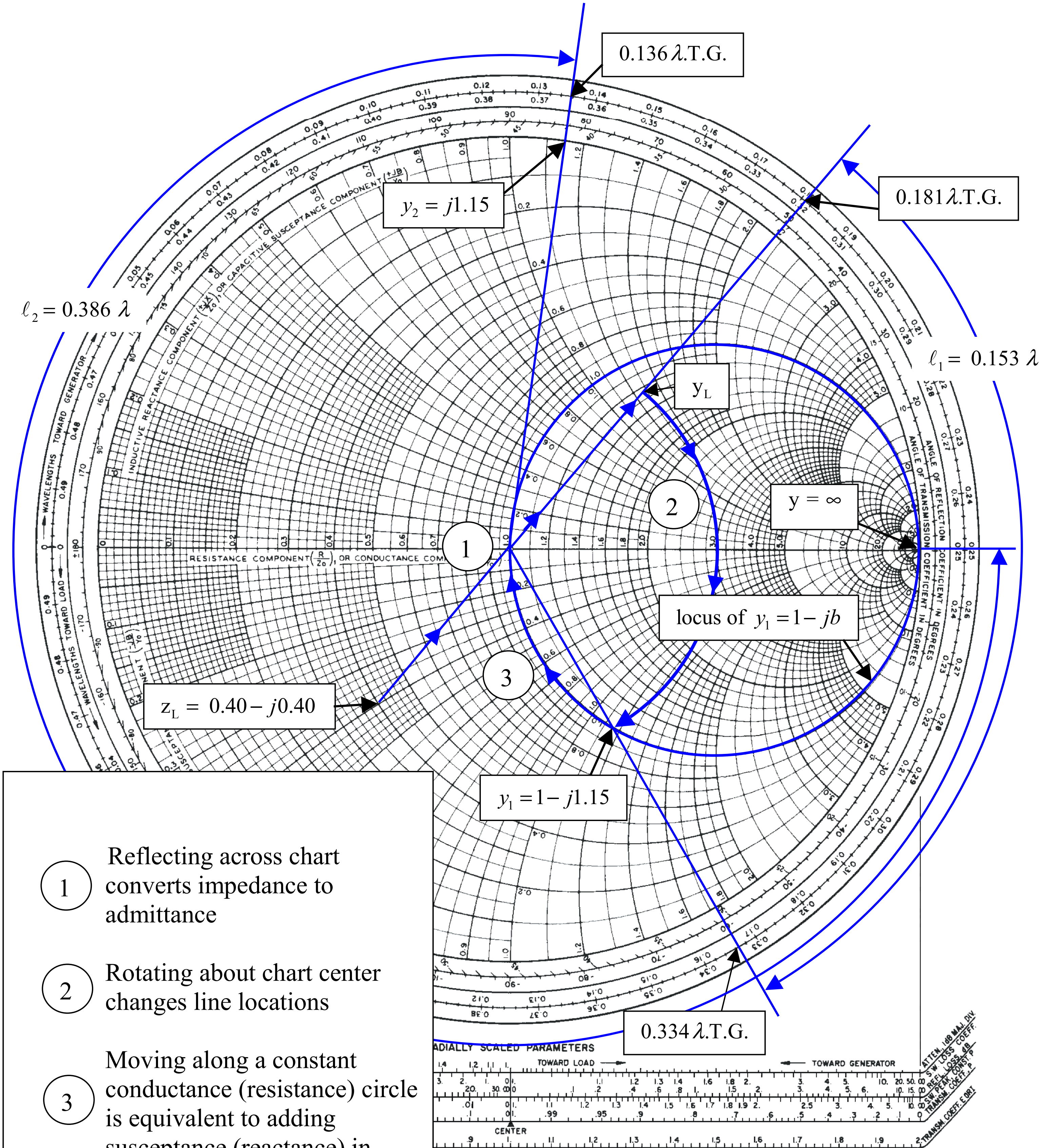
   First, locate the load impedance $z_L = 0.4 - j0.4$ on the chart and reflect it to obtain the load admittance, $y_L = 1.24 + j1.24$. To match the line, the parallel combination of the load and stub line sections must present a unit (normalized) admittance at the junction point, i.e. $y_1 + y_2 = 1.0$. Since the stub section is lossless, its input admittance must be purely susceptive, i.e. of the form $y_2 = jb$, and hence the input admittance of the loaded section must have the form $y_1 = 1.0 - jb$. The locus of y_1 is thus the unit conductance circle.

   To determine the unknown susceptive part of y_1, we move from the point on the chart corresponding to y_L toward the generator until we reach the intersection with the unit conductance circle, yielding $y_1 = 1.0 - j1.15$ and thus $y_2 = j1.15$. The electrical distance moved between the points y_L and y_1 is $\ell_1 = 0.334\lambda - 0.181\lambda = 0.153\lambda$ measured on the WAVELENGTHS TOWARD GENERATOR scale.

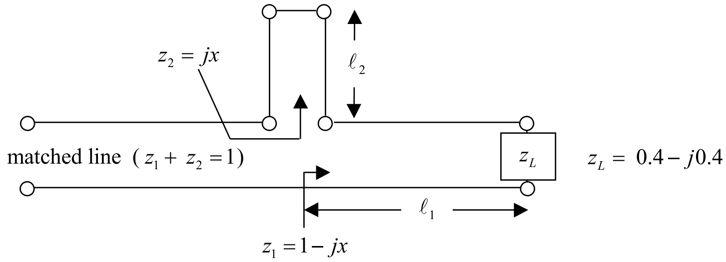
   To find ℓ_2, locate $y_2 = j1.15$ and move toward the load to the infinite admittance point on the chart, corresponding to a shorted stub. The shorted stub length is therefore $\ell_2 = 0.386\lambda$, the sum of 0.25λ measured on the WAVELENGTHS TOWARD LOAD scale and 0.136λ measured on the WAVELENGTHS TOWARD GENERATOR scale (Why two different scales?).

   Note that a second solution may be obtained by rotating past the first intersection to the second intersection with the unit conductance circle at $y_1 = 1.0 + j1.15$. This will yield a longer section of line ℓ_1. Also, we will have $y_2 = -j1.15$ and the corresponding length of the shorted stub section ℓ_2 will be shorter than in the previous solution. In this case it is not clear which solution is preferable. In general, however, if one solution results in *both* ℓ_1 and ℓ_2 being shorter, then that configuration normally has the greater bandwidth.

Problem 8



1. Reflecting across chart converts impedance to admittance
2. Rotating about chart center changes line locations
3. Moving along a constant conductance (resistance) circle is equivalent to adding susceptance (reactance) in parallel (series).



9. Use a single series stub tuner to match the above line to its normalized load. Use a shorted stub and find its distance from the load, ℓ_1, and its length ℓ_2.

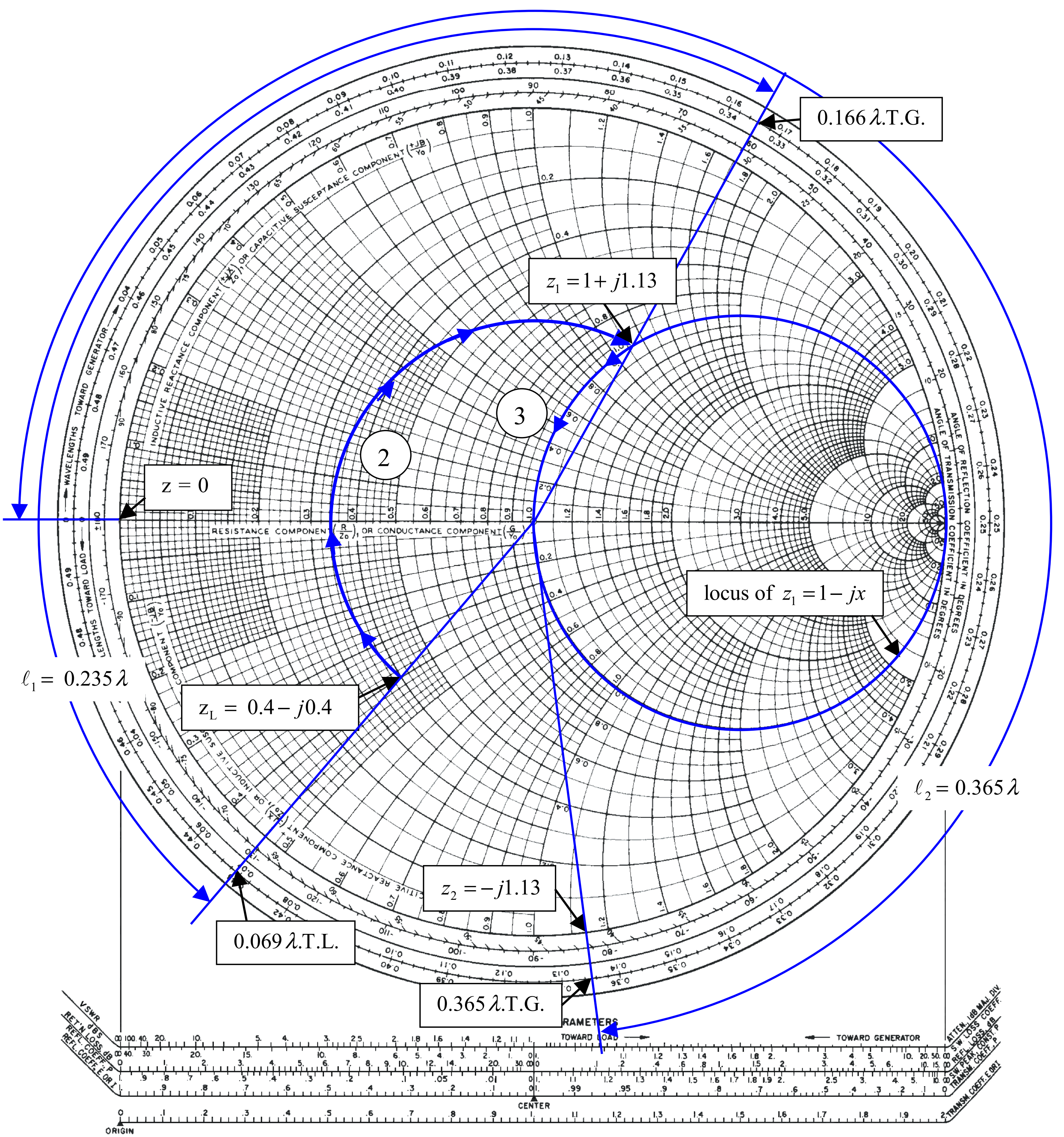
This problem is similar to the parallel stub tuner except that, since the stub is in series with the line, constructions are done using impedances rather than admittances. First, locate the load impedance $z_L = 0.4 - j0.4$ on the chart. To match the line, the series combination of the load and stub line sections must present a unit (normalized) impedance at the junction point, i.e. $z_1 + z_2 = 1.0$. Since the stub section is lossless, its input impedance must be purely reactive, i.e. of the form $z_2 = jx$, and hence the input admittance of the loaded section must have the form $z_1 = 1.0 - jx$. The locus of z_1 is thus the unit resistance circle.

The unknown reactive part of z_1 is determined by moving from the point on the chart corresponding to z_L toward the generator until the intersection with the unit resistance circle is reached, yielding $z_1 = 1.0 + j1.13$ and thus $z_2 = -j1.13$. The electrical distance moved between the points z_L and z_1 is $\ell_1 = 0.235\lambda$, the sum of 0.069λ measured on the WAVELENGTHS TOWARD LOAD scale and 0.166λ measured on the WAVELENGTHS TOWARD GENERATOR scale (Why different scales?).

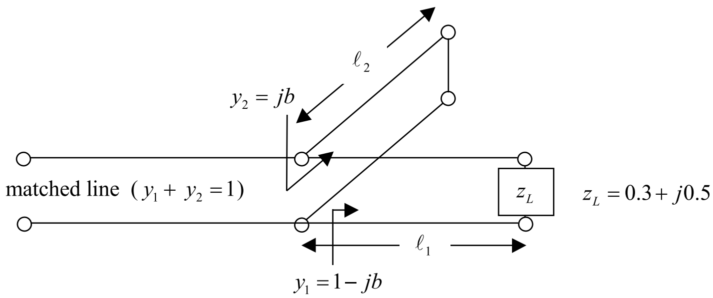
To find ℓ_2, locate $z_2 = -j1.13$ and move toward the load until the impedance is zero, corresponding to a shorted stub. The shorted stub length is therefore $\ell_2 = 0.365\lambda$, which can be measured directly on the WAVELENGTHS TOWARD GENERATOR scale.

A second solution may be found by rotating past the first intersection to the second intersection with the unit resistance circle, yielding $z_1 = 1.0 - j1.13$ and a longer section of line ℓ_1. Then $z_2 = j1.13$ and the corresponding length of the shorted stub section ℓ_2 will be shorter than in the previous solution. In this case it is not clear which solution is preferable. In general, however, if one solution results in *both* ℓ_1 and ℓ_2 being shorter, then that configuration normally has the greater bandwidth.

Problem 9



0.166 λ.T.G.
z₁ = 1 + j1.13
3
2
z = 0
locus of z₁ = 1 − jx
ℓ₁ = 0.235 λ
z_L = 0.4 − j0.4
ℓ₂ = 0.365 λ
z₂ = −j1.13
0.069 λ.T.L.
0.365 λ.T.G.
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



10. Use a single parallel stub tuner to match the above line to its normalized load. Use a shorted stub and find its distance from the load, ℓ_1, and its length ℓ_2.

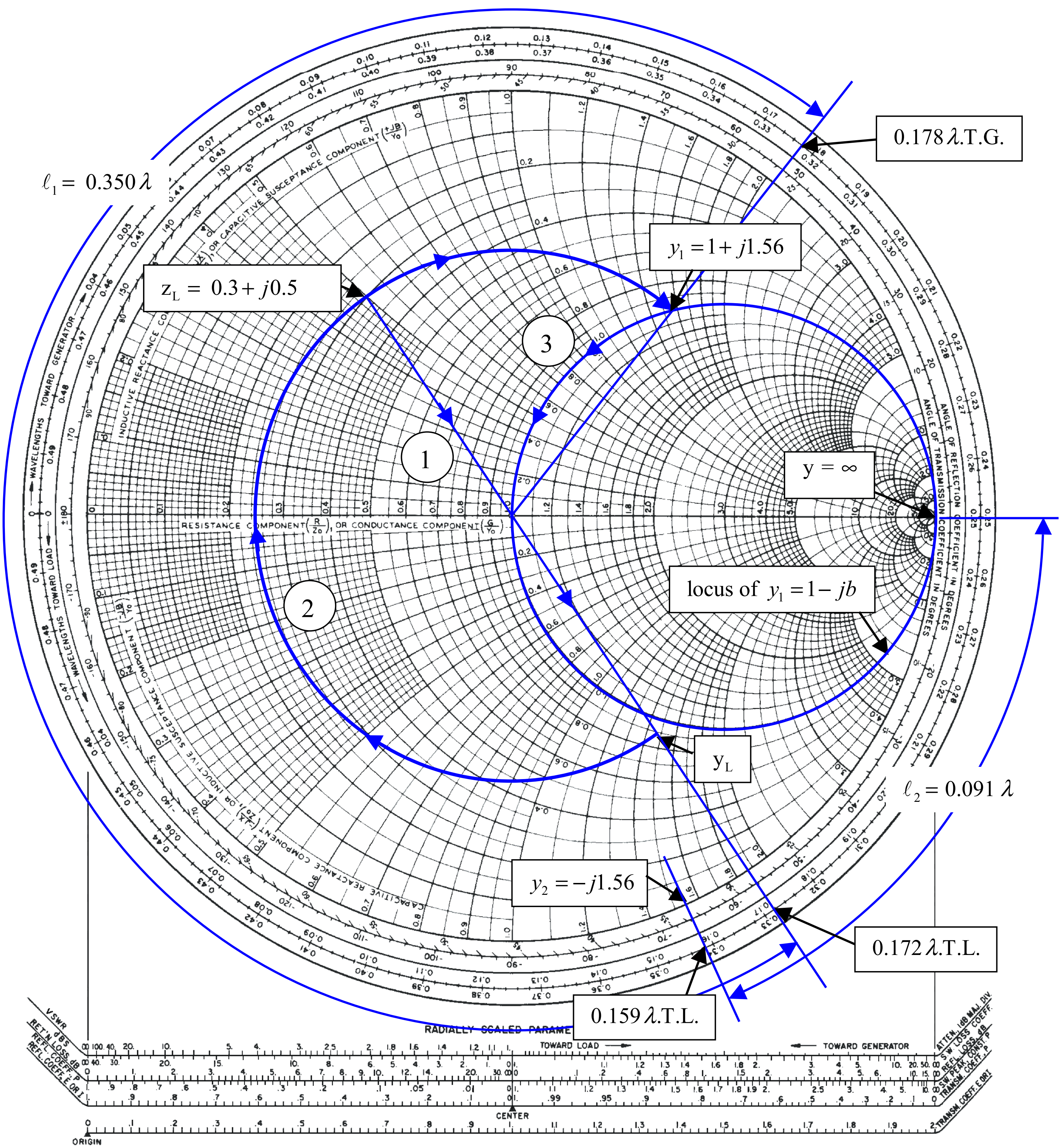
    First, locate the load impedance $z_L = 0.3 + j0.5$ on the chart and reflect it to obtain the load admittance, $y_L = 0.89 - j1.48$. To match the line, the parallel combination of the load and stub line sections must present a unit (normalized) admittance at the junction point, i.e. $y_1 + y_2 = 1.0$. Since the stub section is lossless, its input admittance must be purely susceptive, i.e. of the form $y_2 = jb$, and hence the input admittance of the loaded section must have the form $y_1 = 1.0 - jb$. The locus of y_1 is thus the unit conductance circle.

    To determine the unknown susceptive part of y_1, we move from the point on the chart corresponding to y_L toward the generator until we reach the intersection with the unit conductance circle, yielding $y_1 = 1.0 + j1.56$ and thus $y_2 = -j1.56$. The electrical distance moved between the points y_L and y_1 is $\ell_1 = 0.350\lambda$, the sum of 0.172λ measured on the WAVELENGTHS TOWARD LOAD scale and 0.178λ measured on the WAVELENGTHS TOWARD GENERATOR scale (Why different scales?).

    To find ℓ_2, locate $y_2 = -j1.56$ and move toward the load to the infinite admittance point on the chart, corresponding to a shorted stub. The shorted stub length is therefore $\ell_2 = 0.250\lambda - 0.159\lambda = 0.091\lambda$, where both distances are measured on the WAVELENGTHS TOWARD LOAD scale.

    As usual, a second solution may be obtained by rotating past the first intersection to the second intersection with the unit conductance circle at $y_1 = 1.0 - j1.56$. This will yield a longer section of line ℓ_1. Also, we will have $y_2 = j1.56$ with the corresponding length of the shorted stub section ℓ_2 also longer than in the previous solution. In this case one would generally choose the solution with both ℓ_1 and ℓ_2 shorter, that configuration normally having the greater bandwidth.

Problem 10



0.178 λ.T.G.
ℓ1 = 0.350 λ
y1 = 1 + j1.56
zL = 0.3 + j0.5
3
1
y = ∞
2
locus of y1 = 1 − jb
yL
ℓ2 = 0.091 λ
y2 = −j1.56
0.172 λ.T.L.
0.159 λ.T.L.