Blast From the Past!

Exam 1

Spring 2021

An electromagnetic surface wave propagates on a grounded dielectric slab as shown below. (The structure is infinite in the y direction.) In the air region above the slab ($z > 0$), the electric field in the phasor domain has the following form:

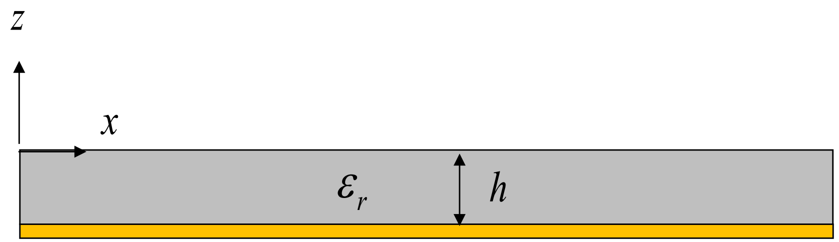
$$\underline{E}(x,z) = \underline{\hat{y}}\ e^{-jk_x x}\ e^{-\alpha_z z} \quad [\mathrm{V/m}]$$

where k_x and α_z are both positive real numbers. Note that there is no y variation of the fields of this surface wave.

a) Find the magnetic field vector $\underline{H}$ in the air region, in the phasor domain.

b) Find the complex Poynting vector in the air region.

c) Find the time average power in watts going (from left to right) through a surface S, which extends from the top of the slab ($z = 0$) to infinity in the vertical z direction, and is one meter wide in the y direction.



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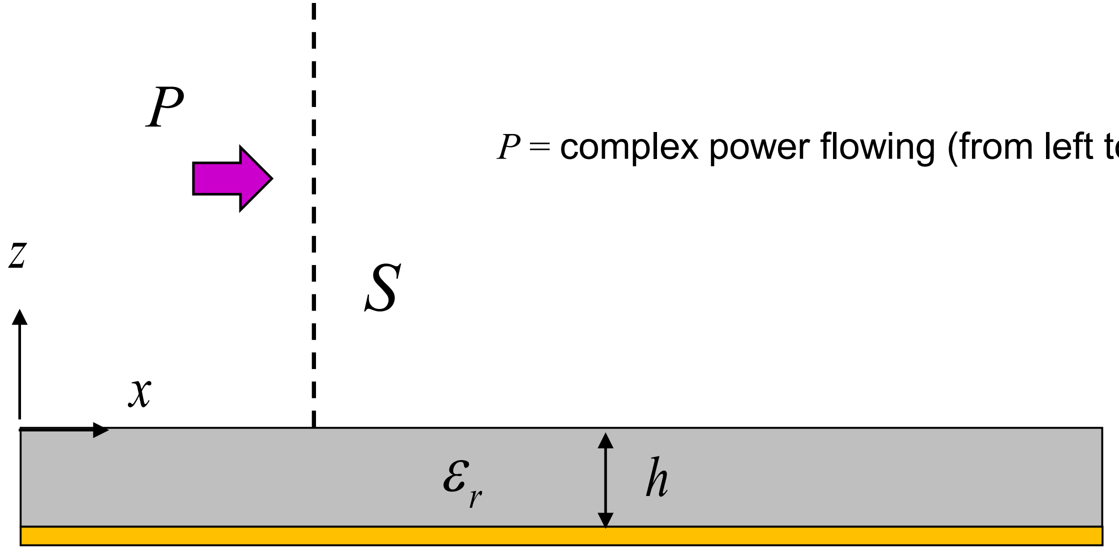
Spring 2017

$$\underline{E}(x,z) = \underline{\hat{y}}\; e^{-jk_x x}\, e^{-\alpha_z z} \quad \text{[V/m]}$$



P = complex power flowing (from left to right) through the surface S.

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$$\underline{E}(x,z) = \underline{\hat{y}}\, e^{-jk_x x}\, e^{-\alpha_z z} \quad [\text{V/m}]$$

Part (a)

$$\underline{H} = -\frac{1}{j\omega\mu_0} \nabla \times \underline{E}$$

$$\nabla \times \underline{E} = \underline{\hat{x}}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + \underline{\hat{y}}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + \underline{\hat{z}}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

so

$$\underline{H} = -\frac{1}{j\omega\mu_0}\left[\underline{\hat{x}}(\alpha_z) + \underline{\hat{z}}(-jk_x)\right] e^{-jk_x x}\, e^{-\alpha_z z} \quad [\text{A/m}]$$

Part (b)

$$\underline{S} = \frac{1}{2}\underline{E} \times \underline{H}^*$$

$$\underline{H}^* = +\frac{1}{j\omega\mu_0}\left[\underline{\hat{x}}(\alpha_z) + \underline{\hat{z}}(+jk_x)\right] e^{+jk_x x}\, e^{-\alpha_z z} \quad [\text{A/m}]$$

so

$$\underline{S} = \frac{1}{2\omega\mu_0} e^{-2\alpha_z z}\left[\underline{\hat{x}}(k_x) + j\underline{\hat{z}}(\alpha_z)\right] \quad \left[\text{VA/m}^2\right]$$

Part (c)

$$P = \int_0^\infty \underline{S} \cdot \underline{\hat{x}}\, dz$$

so

$$P = \frac{k_x}{2\omega\mu_0}\int_0^\infty e^{-2\alpha_z z}\, dz = \frac{k_x}{4\omega\mu_0\alpha_z} \quad [\text{VA}]$$

$$P_r = \text{Re}(P) = P$$ (P is already real.)

$$P_r = \frac{k_x}{4\omega\mu_0\alpha_z} \quad [\text{W}]$$