Blast From the Past!

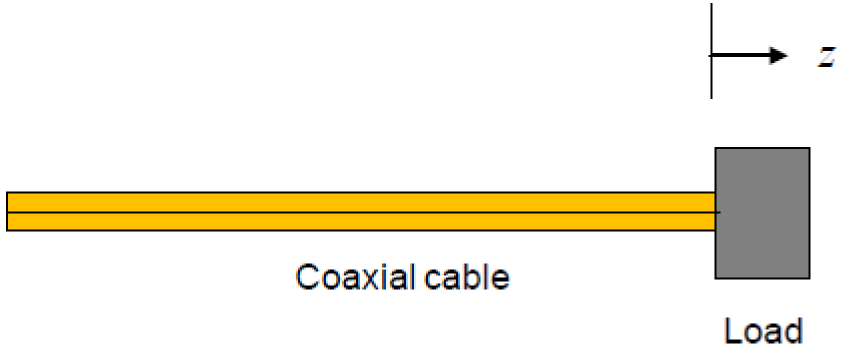
Exam 1
Fall 2018

Problem 3 (35 pts.)

A coaxial cable transmission line has a characteristic impedance of 50 [Ω]. The relative permittivity of the (nonmagnetic) Teflon filling the line is 2.1. It is found that a voltage minimum of 2.0 [V] occurs on the line at $z = -5$ [cm] and a voltage maximum of 4.0 [V] occurs on the line at $z = -15$ [cm]. (This voltage maximum is the one that is the closest to the voltage minimum.)

What is the unknown load impedance at $z = 0$?

Note: This is not supposed to be a Smith chart problem, so please do not use a Smith chart.



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Solution

For the phase of the load reflection coefficient ϕ, we have:

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

We have

$$\text{SWR} = \frac{4.0}{2.0} = 2$$

$$\text{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \Rightarrow |\Gamma_L| = \frac{\text{SWR-1}}{\text{SWR+1}}$$

$$\Rightarrow |\Gamma_L| = \frac{1}{3}$$

$$\phi + 2\beta z_{\min} = \pm\pi, \pm 3\pi, \ldots$$

$$\Rightarrow \phi + 2\left(\frac{2\pi}{\lambda_d}\right) z_{\min} = -\pi$$

(We can add any multiple of 2π to the final answer for ϕ.)

$$z_{\min} = -5\,[\text{cm}]$$

$$\frac{\lambda_d}{4} = -5\,[\text{cm}] - (-15)\,[\text{cm}] = 10\,[\text{cm}]$$

$$\lambda_d = 40\,[\text{cm}]$$

Hence $\phi = -\pi/2\;[\text{radians}]$

$$\Rightarrow \Gamma_L = \frac{1}{3} e^{-j\pi/2} = -j/3$$

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We then have

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right)$$

so

$$Z_L = 50\left(\frac{1+(-j/3)}{1-(-j/3)}\right)$$

Therefore

$$Z_L = 40 - j30\;[\Omega]$$